IES PRACTICE GUIDE

WHAT WORKS CLEARINGHOUSE

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

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Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

Students come to kindergarten with a rudimentary understanding of basic fraction concepts. They can share a set of objects equally among a group of people (i.e., equal sharing)[21] and identify equivalent proportions of common shapes (i.e., proportional reasoning).[22]

By using this early knowledge to introduce fractions, teachers allow students to build on what they already know. This facilitates connections between students' intuitive knowledge and formal fraction concepts. The panel recommends using sharing activities to develop students' understanding of ordering and equivalence relations among fractions.

Sharing activities can introduce children to several of the basic interpretations of fractions discussed in the introduction. Sharing can be presented in terms of division—such as by partitioning 12 candies into four equally numerous groups. Sharing also can be presented in terms of ratios; for example, if three cakes are shared by two children, the ratio of the number of cakes to the number of children is 3:2.

Although fractions are typically introduced by 1st or 2nd grade, both the sharing and the proportional reasoning activities described in this recommendation can begin as early as preschool or kindergarten.

Summary of evidence: Minimal Evidence

This recommendation is based on studies showing that students have an early understanding of sharing and proportionality,[23] and on studies of instruction that use sharing scenarios to teach fraction concepts.[24] However, none of the studies that used sharing scenarios to teach fraction concepts met WWC standards. Despite the limited evidence, the panel believes that students' informal knowledge of sharing and proportionality provides a foundation for introducing and teaching fraction concepts.

Equal sharing. Children have an early understanding of how to create equal shares. By age 4, children can distribute equal numbers of equal-size objects among a small number of recipients, and the ability to equally share improves with age.[25] Sharing a set of discrete objects (e.g., 12 grapes shared among three children) tends to be easier for young children than sharing a single object (e.g., a candy bar), but by age 5 or 6, children are reasonably skilled at both.[26]

Case studies show how an early understanding of sharing could be used to teach fractions to elementary students.[27] In two studies, teachers posed story problems with sharing scenarios to teach fraction concepts such as equivalence and ordering, as well as fraction computation. The studies reported positive effects on fraction knowledge, but they do not provide rigorous evidence on the impact of instruction based on sharing activities.

Proportional relations. The panel believes that instructional practices can build on young children's rudimentary knowledge of proportionality to teach fraction concepts. This early understanding of proportionality has been demonstrated in different ways. By age 6, children can match equivalent proportions represented by different geometric figures and by everyday objects of different shapes.[28] One-half is an important landmark in comparing proportions; children more often succeed on comparisons in which one proportion is more than half and the other is less than half, than on comparisons in which both proportions are more than half or both are less than half (e.g., comparing $\frac{1}{3}$ to $\frac{3}{5}$ is easier than comparing $\frac{2}{3}$ to $\frac{4}{5}$).[29] In addition, children can complete analogies based on proportional relations—for example, half circle is to half rectangle as quarter circle is to quarter rectangle.[30]

Although there is evidence that describes young children's knowledge of proportionality, no rigorous studies that met WWC standards have examined whether this early-developing knowledge can be used to improve teaching of fraction concepts.

How to carry out the recommendation

1. Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.

The panel recommends that teachers offer a progression of sharing activities that builds on students' existing strategies for dividing objects. Teachers should begin with activities that involve equally sharing a set of objects among a group of recipients and progress to sharing scenarios that require partitioning an object or set of objects into fractional parts. In addition, early activities should build on students' halving strategy (dividing something into two equal sets or parts) before having students partition objects among larger numbers of recipients. Students should be encouraged to use counters (e.g., beans, tokens), create drawings, or rely on other representations to solve these sharing problems; then teachers can introduce formal fraction names (e.g., *one-third*, *one-fourth*, *thirds*, *quarters*) and have children label their drawings to name the shared parts of an object (e.g., $\frac{1}{3}$ or $\frac{1}{8}$ of a pizza). For optimal success, children should engage in a variety of such labeling activities, not just one or two.

Sharing a set of objects. Teachers should initially have students solve problems that involve two or more people sharing a set of objects (see Figure 1). The problems should include sets of objects that can be evenly divided among sharers, so there are no remaining objects that need to be partitioned into fractional pieces.

In these early sharing problems, teachers should describe the number of items and the number of recipients sharing those items, and students should determine how many items each person receives.[31] Teachers might then pose the same problem with increasing numbers of recipients.[32] It is important to emphasize that these problems require sharing a set of objects equally, so that students focus on giving each person the same number of objects.

Partitioning a single object. Next, teachers should pose sharing problems that result in students dividing one or more objects into equal parts. The focus of these problems shifts from asking students *how many* things each person should get to asking students *how much* of an object each person should get. For example, when one cookie is shared between two children, students have to think about how much of the cookie each child should receive.

Teachers can begin with problems that involve multiple people sharing a single object (e.g., four people sharing an apple) and progress to problems with multiple people sharing a set of objects that must be divided into smaller parts to share equally (e.g., three people sharing four apples). Problems that involve sharing one object result in shares that are *unit fractions* (e.g., $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{9}$), whereas scenarios with multiple people and objects often result in *non-unit fractions* (e.g., $\frac{3}{4}$).[33] This distinction between unit and non-unit fractions is important, because when fractions are reduced to lowest terms, non-unit fractions are composed of unit fractions (e.g., $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$), but the opposite is not the case. Sharing situations that result in unit fractions provide a useful starting point

Figure 1. Sharing a set of objects evenly among recipients

Problem

Three children want to share 12 cookies so that each child receives the same number of cookies. How many cookies should each child get?

Examples of Solution Strategies

Students can solve this problem by drawing three figures to represent the children and then drawing cookies by each figure, giving one cookie to the first child, one to the second, and one to the third, continuing until they have distributed 12 cookies to the three children, and then counting the number of cookies distributed to each child. Other students may solve the problem by simply dealing the cookies into three piles, as if they were dealing cards.

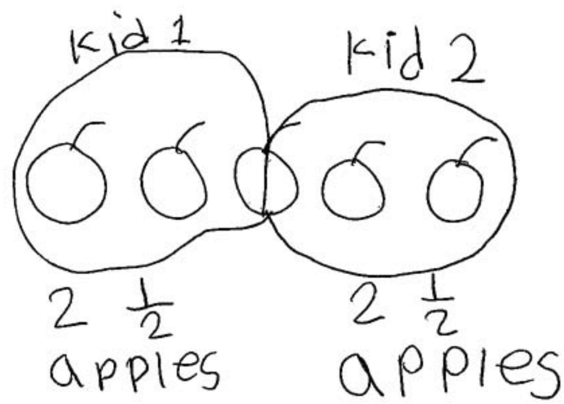
Figure 2. Partitioning both multiple and single objects

Problem

Two children want to share five apples that are the same size so that both have the same amount to eat. Draw a picture to show what each child should receive.

Examples of Solution Strategies

Students might solve this problem by drawing five circles to represent the five apples and two figures to represent the two children. Students then might draw lines connecting each child to two apples. Finally, they might draw a line partitioning the final apple into two approximately equal parts and draw a line from each part to the two children. Alternatively, as in the picture to the right, children might draw a large circle representing each child, two apples within each circle, and a fifth apple straddling the circles representing the two children. In yet another possibility, children might divide each apple into two parts and then connect five half apples to the representation of each figure.



for introducing fraction names, especially because some children think that all fractional parts are called one-half.[34]

The panel also suggests starting with problems that involve sharing among two, four, or eight people (i.e., powers of two).[35] This allows students to create equal parts by using a halving strategy—dividing an object in half, dividing the resulting halves in half, and so on, until there are enough pieces to share (see Figure 2).[36] Eventually, students should solve sharing problems for which they cannot use a halving strategy. Partitioning a brownie into thirds, for example, requires that students anticipate how to slice the brownie so that it results in three equal parts. Students may be tempted to use repeated halving for all sharing problems, but teachers should help students develop other strategies for partitioning an object. One approach is to have students place wooden sticks on concrete shapes, with the sticks representing the slices or cuts that a student would make to partition the object.[37]

2. Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.

Teachers can extend the types of sharing activities described in the previous step to develop students' understanding of ordering and identifying equivalent fractions. The overall approach remains the same: teachers pose story problems that involve a group of people sharing objects, and students create drawings or other representations to solve the problems. However, teachers use scenarios that require fraction *comparisons* or identification of equivalent fractions and focus on different aspects of students' solutions.

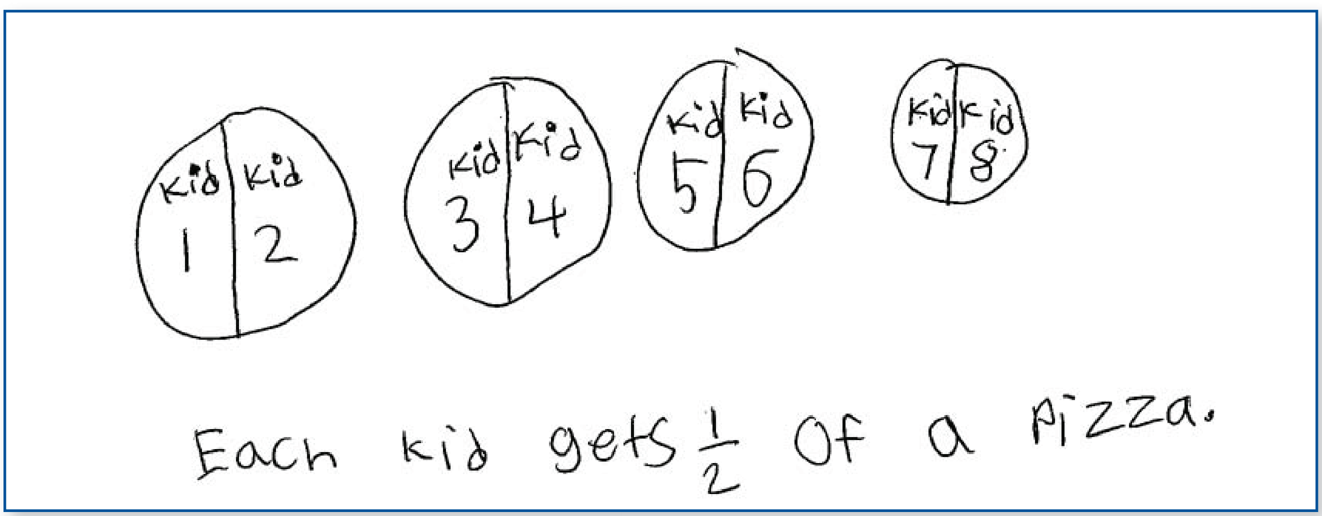
Sharing activities can be used to help students understand the relative size of fractions. Teachers can present sharing scenarios with an increasing number of recipients and have students compare the relative size of each resulting share. For example, students can compare the size of pieces that result when sharing a candy bar equally among three, four, five, or six children.[38] Teachers should encourage students to notice that as the number of people sharing the objects increases, the size of each person's share decreases; they should then link this idea to formal fraction names and encourage students to compare the fractional pieces using fraction names (e.g., ⅓ of an object is greater than ¼ of it).

When using sharing scenarios to discuss equivalent fractions, teachers should consider two approaches, both of which should be used with scenarios in which the number of sharers and the number of pieces to be shared have one or more common factors (e.g., four pizzas shared among eight children):

- **Partition objects into larger or smaller pieces.** One way to understand equivalent shares is to discuss alternative ways to partition and receive the same shares.[39] Students can think about how to solve a sharing scenario using different partitions to produce equal shares. Such partitioning may require trial and error on the part of students to identify which groupings result in equal shares. Students might combine smaller pieces to make bigger ones or partition bigger ones into smaller pieces. For example, to solve the problem of eight children sharing four pizzas, students might partition all four pizzas into eighths and then give each child four pieces of size ⅛. Alternatively, students could divide each pizza into fourths and give each person 2⁄4, or divide each pizza into halves and distribute ½ to each child. Students should understand that although there are different ways to partition the pizza, each partitioning method results in equivalent shares.

- **Partition the number of sharers and the number of items.** Another way to help students understand equivalence is to partition the number of sharers and objects.[40] For example, if students arrive at 4⁄8 for the problem in the previous paragraph, the teacher could ask how the problem would change if the group split into two tables and at each table four children shared two pizzas. Students can compare the new solution of 2⁄4 to their original solution of 4⁄8 to show that the two amounts are equivalent (see Figure 3). To drive home the point, the eight children could then sit at four tables, with two children at each table sharing a single pizza—and reaching the more familiar concept of ½.

Figure 3. Student work for sharing four pizzas among eight children



Another way to teach equivalent fractions with sharing scenarios is to pose a missing-value problem in which children determine the number of objects needed to create an equivalent share. For example, if six children share eight oranges at one table, how many oranges are needed at a table of three children to ensure each child receives the same amount?[41] The problem could be extended to tables with 12 children, 24 children, or 9 children. To solve these problems, students might identify how much one child receives in the first scenario and apply that to the second scenario. Alternatively, they could use the strategy described above and partition the six children and eight oranges at the original table into two tables, so that the number of children and oranges at the first new table equal the number of children and oranges at the second new table.

Here is another example that allows students to explore the concept of equal partitioning: if 24 children are going out for sandwiches, and 16 sandwiches have been ordered, what are the different ways the children could sit at tables and divide the sandwiches so they would all receive the same amount? Options might include having one big table of 24 children and 16 sandwiches, having four tables of six children and four sandwiches at each, eight tables of three children and two sandwiches at each, and so on.

3. Build on students' informal understanding to develop more advanced understanding of proportional-reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Early instruction can build on students' informal understanding to develop basic concepts related to proportional reasoning. Teachers should initially pose problems that encourage students to think about the proportional relations between pairs of objects, without necessarily specifying exact quantities. For example, teachers could use the story of *Goldilocks and the Three Bears* to discuss how the big bear needs a big chair, the medium-sized bear needs a medium-sized chair, and the small bear needs a small chair.[42]

The following list provides examples of different relations relevant to early proportional reasoning that can be explored with students:

- **Proportional relations.** Teachers can discuss stories or scenarios that present basic proportional relations that are not quantified. For example, a class could discuss the number of students it would take to balance a seesaw with one, two, or three adults on one end. Creating more and less saturated liquid mixtures with lemonade mix or food coloring can facilitate discussions comparing the strength or concentration of different mixtures.
- **Covariation.** Teachers should discuss problems that involve one quantity increasing as another quantity increases. Examples could include the relation between height and clothing size or between foot length and shoe size.[43]
- **Patterns.** Simple repeating patterns can be useful for discussing the concept of ratio. For example, students could complete a pattern such as blue star, blue star, red square, blue star, blue star, red square, blue star, blue star, red square, and so on.[44] Teachers can then discuss how many blue stars there are for every red square, have students arrange the stars and squares to show what gets repeated, have students change the pattern to a different ratio (e.g., three blue stars to one red square), or have students extend the pattern.[45]

Potential roadblocks and solutions

Roadblock 1.1. *Students are unable to draw equal-size parts.*

Suggested Approach. Let students know that it is acceptable to draw parts that are not exactly equal, as long as they remember that the parts should be considered equal.

Roadblock 1.2. *Students do not share all of the items (non-exhaustive sharing) or do not create equal shares.*

Suggested Approach. Although children have an intuitive understanding of sharing situations, they sometimes make mistakes in their attempts to solve sharing problems. Students may not share all of the items, especially if a sharing scenario requires partitioning an object. Teachers should help students understand that sharing scenarios require sharing all of the objects—possibly even noting that each child wants to receive as much as he or she possibly can, so no objects should remain unaccounted for.

Students also might not create equal shares because they do not understand that dealing out equal-size objects results in an equal amount for each person.[46] In this case, teachers can discuss how dealing out objects ensures that each person receives an equal amount and can encourage students to verify that they divided the items equally.

Equal sharing is important because it lays a foundation for later understanding of equivalent fractions and equivalent magnitude differences (e.g., understanding that the difference between 0 and ½ is the same as the difference between 1 and 1½ or between 73 and 73½).

Roadblock 1.3. *When creating equal shares, students do not distinguish between the number of things shared and the quantity shared.*

Suggested Approach. Younger students in particular may confuse equal numbers of shares with equal amounts shared.[47] For example, if students are asked to provide equal amounts of food from a plate with both big and small pieces, a child might give out equal numbers of pieces of food rather than equal amounts. This misunderstanding may stem from limited experience with situations in which entities of different sizes are dealt out or shared.

One way to address this misconception is to use color cues to help students distinguish between the quantity being shared and the number of items being shared.[48] For example, in a scenario in which both of two identical toy dogs are said to be hungry, children could be asked whether the dogs would have the same amount to eat if one dog received five large red pieces of pretend food and the other dog five small green pieces of pretend food.