ON ARCHING SHAPE AND VIOLIN TONE

Part 1
A qualitative analysis of the relationship between longitudinal string vibration, arching shape and violin sound

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Abstract
The key to understanding the effect of arching shape on violin sound is to understand the role of longitudinal string vibrations (LSV). The sources of LSV are discussed. The body deformations caused by changes in the static string tension are shown by a qualitative analysis to be dependent on the shape of the arching of the plates. The deformation of a violin under string tension is measured.
Arching shape features that are common to all classical violin models would enable LSV to drive the end bouts cross arches of the violin in a way that is likely to radiate sound efficiently. Two geometric parameters are introduced, which determine those aspects of the arching shape that control the effectiveness of this driving mechanism. A subjective assessment of the tonal effect of varying these arching shape parameters is given. By controlling these geometric parameters makers can influence violin tone in a rational and predictable way.
A hypothesis is proposed that would explain dynamically how some of the input energy from the transversely vibrating string is transformed by LSV to bend the violin in its length and drive the end bouts cross arches.

Introduction
It is natural for violinmakers to visualise how a violin body might deform when the string moves slowly, at quasi-static speeds. This leads to concepts of the bridge rocking about the sound post foot and raising and lowering the bass bar foot. The transverse string vibration (TSV) is also associated with longitudinal string vibrations (LSV). In this paper the significance of LSV is discussed and the body deformations caused by LSV operating at quasi-static speeds are qualitatively examined. This leads to pointers as to how the shape of the arching might influence the dynamic behaviour of the body and affect the radiated sound.

The relationship between TSV and LSV
The TSV puts a transverse force (TSV force) on the bridge, which tends to vibrate it in its own plane. The strings also vibrate in their length. This longitudinal string vibration (or LSV) comes from two causes. "Primary" LSV arises when the transversely vibrating string stretches, thus increasing the string tension. Primary LSV only occurs in the transversely vibrating string. "Secondary" LSV is generated by the dynamic activity of the body and involves all four strings. The motions of the violin body can alter the distance between the string supports (the nut, the bridge and the saddle) raising or lowering the tension in all four strings. An increase in string tension, from either primary or secondary LSV applies a downward force on the

bridge and an upwards force on the nut and saddle, thus tending to press the bridge downwards vertically in its own plane and bend the violin in its length.

Primary LSV can be shown to apply only small forces to the body. If the string displacement amplitude at the nth harmonic is a_n, the ratio of the vertical force on the bridge from primary LSV to the transverse force on the bridge from TSV (at the nth harmonic) is given by $\frac{F_{primary\,LSV}}{F_{TSV}} = \frac{k_{st}}{87T}\frac{n\pi}{(L+L_1)}\frac{(a_{n/2})^2}{a_n}$, where L is the playing string length, L_1 is the string length between the tailpiece and the bridge, T is the static string tension, and k_{st} is the spring stiffness of the string (the force required to stretch unit length of string by 1 unit). This simplified expression does overlook the effect of string waves but is accurate enough for the purposes of this discussion. For a fuller analysis see [1]. By leaving out the constants we can write that, $\frac{F_{primary\,LSV}}{F_{TSV}} \propto n\frac{(a_{n/2})^2}{a_n}$. So the contribution is non-linear, the ratio increases with the string displacement. At the odd numbered harmonics, since there can be no contribution from $a_{n/2}$, the ratio is zero. In the bowed string the value of a_n declines as the inverse of n (saw tooth wave), so the even numbered harmonics have a ratio $\frac{F_{primary\,LSV}}{F_{TSV}}$ that is a constant. This ratio can be evaluated (for example) for the second harmonic of a bowed open G string. If the string is strongly bowed, the transverse displacement in the first harmonic might realistically be 1.5mm and therefore the second harmonic displacement is 0.75mm. Assuming, k_{st}=2804N, T=40N, $(L+L_1)$ 383mm, and n=2, the ratio $\frac{F_{primary\,LSV}}{F_{TSV}}$ is 0.04, for all even numbered harmonics. A similar figure for a D string might be 0.035. See figure 1.

The analysis neglects the fact that the bowed string is dragged sideways at the bridge and does not vibrate equally either side of its straight-line position. A more realistic estimate of primary LSV can be found by experiment. To generate primary LSV in a violin string without generating secondary LSV, all movement of the nut and saddle, and vertical in plane movement of the bridge must be disallowed. This was achieved by using a bridge with no holes in it, placed on a rigid base (rather than on a violin) so as to only be free to move normally to its surface, and by using a nut and saddle also firmly fixed to a rigid base. The primary LSV was recorded on a force gauge located where the tailpiece would normally be. The geometry of the set-up was the same as that of a string on a real violin. There was only one string (violin D) on the bridge and it was strongly bowed. This experiment showed that the relative strength of the first two harmonics of the LSV was quite dependent on the distance of the bow from the bridge and the bow speed and weight. Figure 1 shows that the ratio of the primary LSV on the bridge to the TSV force on the bridge was small at about 0.008, except at the second harmonic where it was about 0.045, which agrees well with the theoretical figure, although there was less agreement at the other even numbered harmonics.

The total LSV (primary plus secondary) was measured in a real violin, with a normal bridge, when the open G string was strongly bowed. The LSV was measured at the tailgut and represents the net result of LSV in each of the four strings (the method of its measurement is given in Part 2 of this paper). Figure 1 shows a ratio of vertical LSV (primary plus secondary) to TSV forces on the bridge. This averages about 0.4 up to 5000Hz, falling to 0.1 in the higher harmonics. The LSV force in the group of four strings would be 2.7 times greater than the vertical component downwards at the bridge, shown in figure 1, and this larger force is applied to the body at the nut and at the saddle.

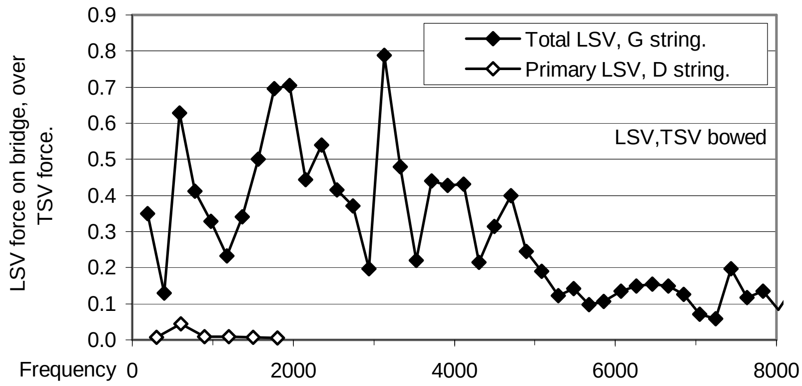


Figure 1. Ratio of vertical LSV force on the bridge to TSV force on the bridge, shown for primary LSV in a bowed D string and total LSV in a bowed G string. The peak heights at the harmonics are shown as points (at 196Hz intervals for the G string and 298Hz for the D), and the points are joined by lines to improve the clarity of presentation.

Primary LSV has a non-linear relationship with the TSV. Secondary LSV has a linear relationship with the dynamic activity in the body. But, since the dynamic activity of the body results in part from the effects of primary LSV, secondary LSV does not have a linear relationship with the TSV. However, because the non-linear contribution from primary LSV is relatively small, the degree of non-linearity is small. The secondary LSV is not small in relation to the TSV and so it is worth considering what effect it might have on the dynamic behaviour of the body.

The quasi-static displacements of the body resulting from the string forces

The violin complete with its strings vibrates in patterns called modes. The modal displacements peak at resonance frequencies. At any one frequency the 'operating shape' will in general be made up of a combination of several modes. An oscillatory force applied to a body will best excite those modes that are close to the force in natural frequency and have large modal displacements in the direction of the force. Therefore, the spatial arrangement of the applied forces is important. It is therefore helpful to look at the static displacements on the body caused by the static application of the string forces.

When a TSV force is applied to the bridge (as when the string is gripped by the rosin in a down bow motion) it tends to move the top of the bridge sideways. This causes the bridge to rock in its own plane by lifting the bass bar foot of the bridge and slightly pushing down the sound post foot. The effect of this bridge motion is to lift the belly along the bass bar and depress a small area of the belly at the sound post, and through the sound post to move the back away from the belly. As the string swings sideways in the opposite direction, these motions are reversed. The net effect is a sort of breathing action (at quasi-static speed), the violin sucks in air when the string goes one way and expels it when the string swings the other way.

When an LSV force (caused by an increase in string tension) is applied to a violin the resulting displacements are similar to those caused by screwing up the tuning pegs to bring the violin up to pitch. A qualitative analysis of body displacements caused by the tension in the strings will now be given.

Figure 2. The violin as a plane frame.

Figure 2 shows the principal structural components of the violin represented as force vectors in a plane frame. Others [2] have proposed alternative representations of the violin as a plane frame. If the compression down the length of the violin imposed by the string tension were to be applied in a line about mid height of the ribs the violin would not bend in its length. The string tension vector is shown leaving this line and diverting to the top of the bridge. The bending this causes is balanced by a vector that passes through the bottom of the sound post. The compression of the body is balanced by a vector that runs to the base of the bridge. For a fuller discussion of this model see [3]. By assuming a string tension of 1, the relative forces in the vectors can be found. Figure 3 shows the computed forces.

Figure 3. The force vectors in a violin.

This shows that the strings push down on the bridge with a force of about 37% of the tension in the strings. The sound post carries only about 1/5 of the bridge force the remaining 4/5 being carried by the compressive force in the belly, which tends to make the plate push up to the underside of the bridge. The load in the sound post is resisted by a tension in the back of the instrument, which is only about 1/3 as great as the compression in the belly.

To investigate the deformations of the body, I will use a method of structural analysis that was well known to engineers from the time of the renaissance in Italy and was used to analyse the complex structural forms seen in the vaulting of the roofs of cathedrals and other buildings. The basic unit of renaissance constructions was the arch. Complex structures such as a violin plate can be represented as a series of arches. It is quite possible that the early Cremonese violinmakers understood the qualitative analysis that follows.

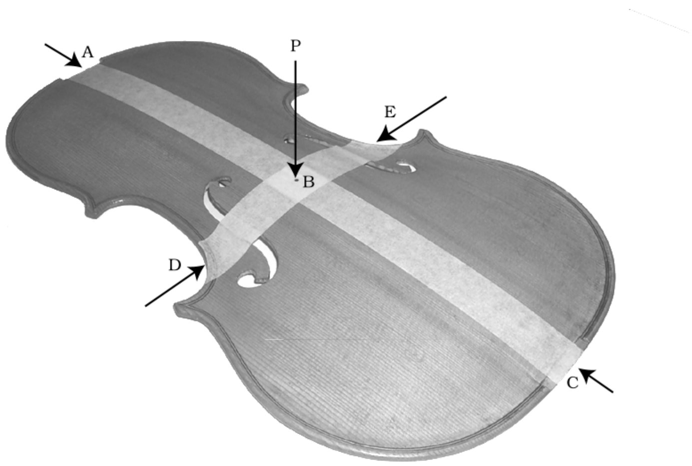


Figure 4. The bridge load splitting into the long arch and the cross arch.

If a load is applied to the crown of an arch it must be resisted by a reaction at the abutments, which can be represented as a vertical reaction and a sideways reaction. Consider a violin belly placed upon a frictionless table with a load P applied in a direction normal to the belly in the bridge position. This is shown in figure 4.
The force P at the crown can be carried either on the "long arch" ABC, or on the "cross arch" DBE. The force in the cross arch DBE can get around the sound holes. In fact what happens is that the long and cross arches both share in the support. The proportion being carried by each depends on the relative stiffness of the two systems. Let us call the load carried on the cross arch system C, and that on the long arch L. Such that P = C+L.

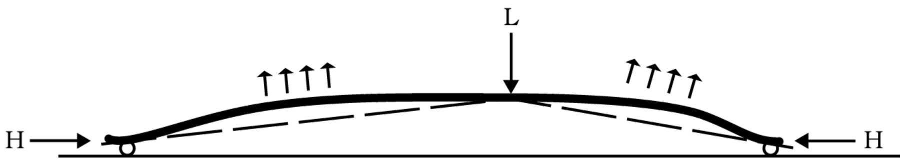


Figure 5. The forces and movements in the long arch.

Considering first the long arch support system. Figure 5 shows a long section through the plate carrying a load L at the bridge, which is resisted by a horizontal reaction at the ends provided by the compression in the belly. A line of action of the thrust at the ends is shown going in a straight line to the bridge. This line of thrust is eccentric to the line of the wood and the eccentricity will induce the wood to move upward in the end bouts. The result of carrying a load on the long arch alone is that the crown of the end bouts cross arches rises and the plate edges in the end bouts pull in.

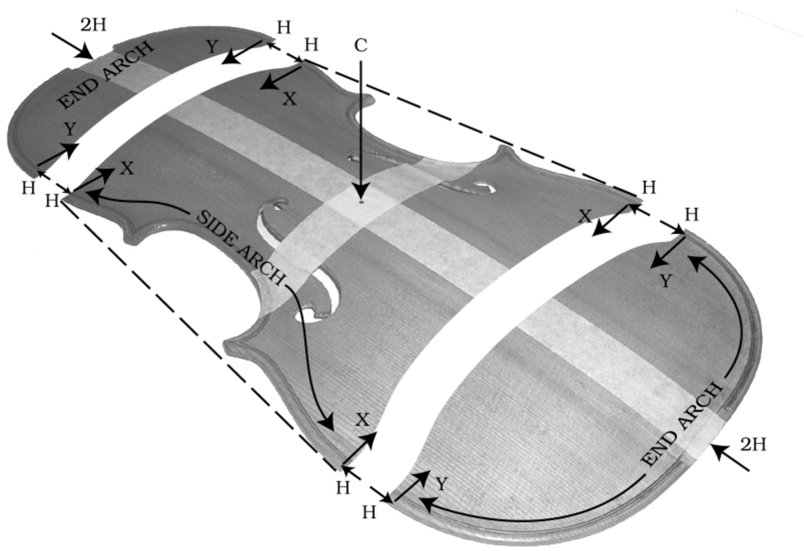
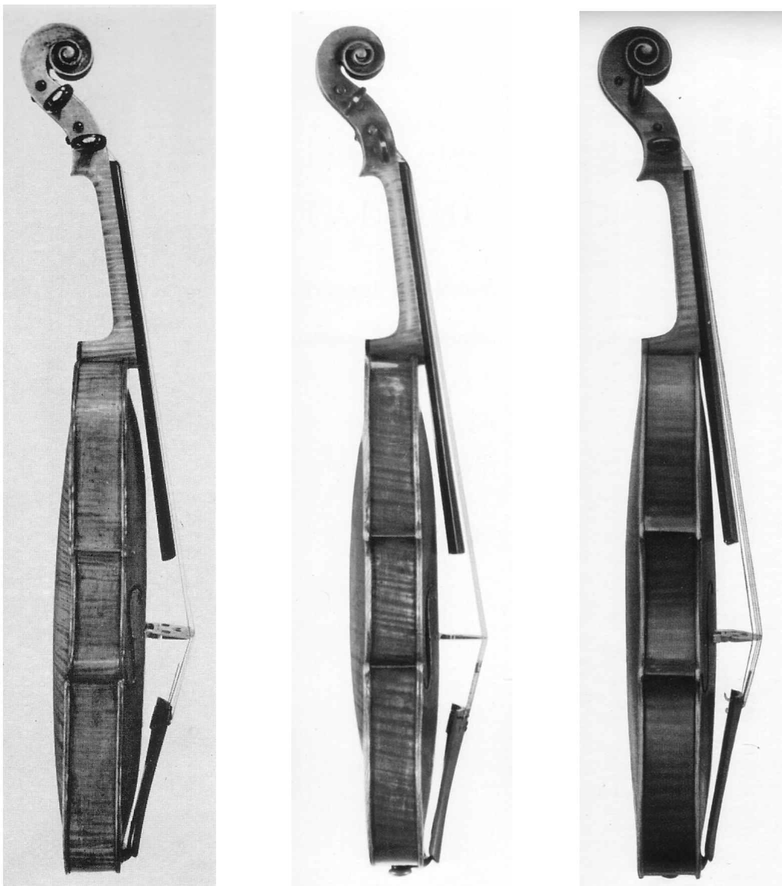


Figure 6. The forces in the cross arch support of the bridge.

Now consider the cross arch support system. Figure 6 shows a violin belly with a cut made across the plate at the end bouts cross arches. The load C at the bridge is carried on the centre bouts cross arch to the two side arches. The sound holes intervene in the most direct path to the side arch but the forces can spread out and get around the sound hole. To support the abutments of the side arches requires a horizontal force, which can be split into two components H and X. Force H can be provided by the end thrusts at the neck and saddle, but not without requiring an additional horizontal force Y in the same direction as X. Thus the end bouts must be cross-tied to provide the forces X+Y necessary to hold the system together. The end bouts of a violin are of course cross tied by the end bouts cross arches, which

provides the force X+Y necessary to stabilise the system. This tension in the end bouts cross arch will pull the crown of the arch down and widen the end bouts.
So then, the load L which goes into the long arch system, lifts the end bouts cross arch (ebx arch for short), and the component C which goes into the cross arch system lowers the ebx arch. Does it go up or does it go down? That depends on the height of the ebx arch. If the ebx arches were very low compared to the centre bouts cross arch (cbx arch), they would approach the line of thrust of the long arch in figure 5, and the upward moving tendency in the long arch would be very weak. Although the downwards moving tendency caused by the spreading of the ebx arches in figure 6 would also be weakened, it would not be weakened to the same extent, since the ebx arch would have to go completely flat to reduce the downwards moving tendency to zero. So we reach the important conclusion that *when the string tension increases, if the ratio of the ebx arch height to the cbx arch height is low, the ebx arch will drop. If this ratio is high, the ebx arch will rise.*

Now consider the back plate. If one ignores the fact that the sound post is off centre, in all other respects the plate is loaded in the same way as the belly, except that the direction of the load is reversed and the reaction at the ends of the plate is a tension rather than a compression. It therefore behaves in exactly the same way as the belly but with all the forces and movements being reversed in direction. Note then that *if the ratio of ebx/cbx arch height is high, the ebx arch height will drop, and if it is low, it will rise.* This is the exact opposite to what happens in the belly. If the belly were made high in the end bouts and the back low, an increase in string tension would make the back and belly move apart, thus reinforcing the breathing action initiated by the rocking motion of the bridge.



Jacob Stainer, 1672. Stradivari, 1702. Guarneri Del Gesu, 1742.
Ashmoleum Museum ex David Oistrakh. ex Paganini

Figure 7. Side view of three classical violins, showing the back and belly long arch.

Classic violins, almost without exception, have the belly end bouts cross arch as a relatively high proportion of the centre bouts height and the back end bouts cross arch as a low proportion of the centre bouts height. The violins in figure 7 show this. The effect of increasing the string tension would then be to make the end bouts cross arches of the back and belly rise thus inhaling air into the body through the sound holes. By using the above analysis we might predict with some confidence that increasing the string tension will raise the height of the back and belly end bouts cross arches, but we can be less certain how the plate edges might move.

Experimental confirmation by the measured static displacements in a violin under string tension

To verify the analysis and to find out how the plate edges move, the static deflection was measured, as the string tension is taken from zero to normal tension. This was done to high precision, at the National Physical Laboratory (UK), using a coordinate measuring machine, and on a violin made in our workshop (No. 156).

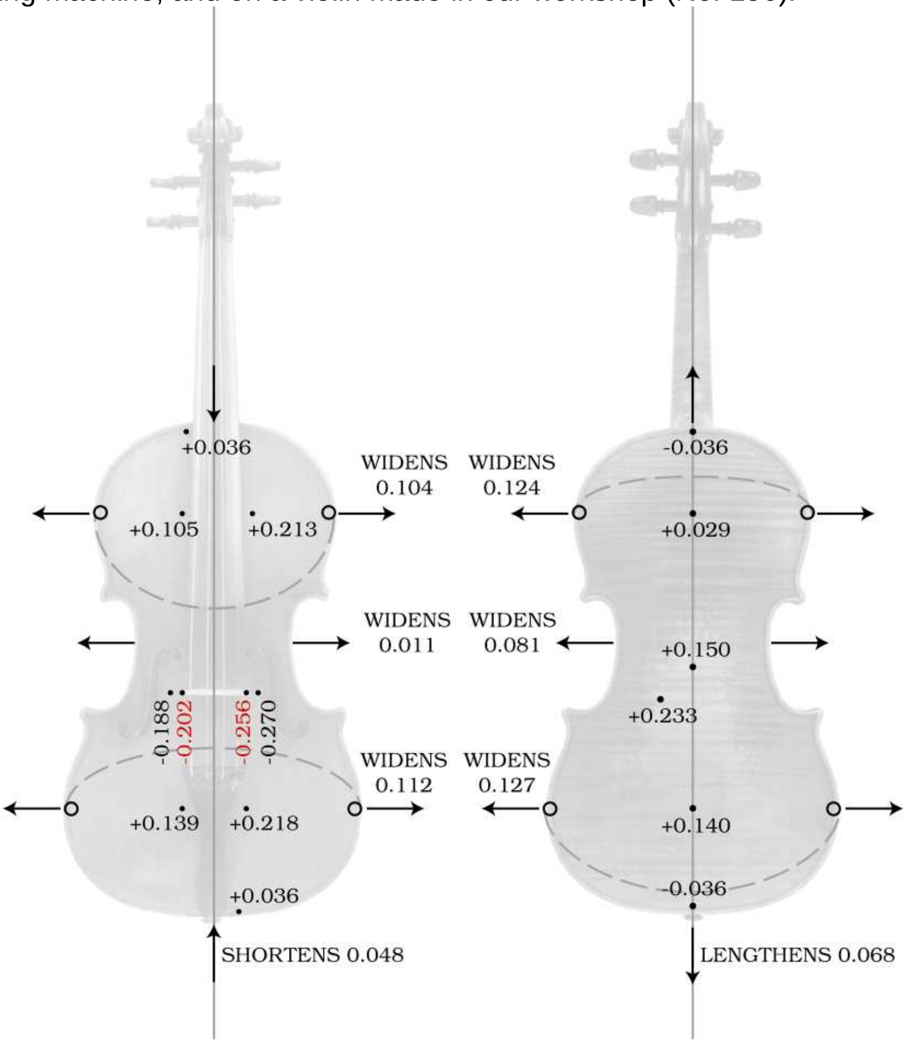


Figure 8. Static deformation caused by raising string tension. The deformation is shown in mm. Positive measurements are away from the violin towards the reader.

Figure 8 shows the displacement at a number of points on the violin surface, in mm., caused by raising the string tension from zero to normal tension. The machine gave measurements to 0.0001mm, but the standard deviation does justify quoting them to

0.001mm. These displacements are relative to the average position in space of the four points at the sides of the end bouts cross arches which are marked O in figure 8. The datum for the measurement was taken at the sides of the belly end bouts cross arches because deformation of these arches is of particular interest. The plates are viewed from the outside looking at the back and belly. Positive measurements denote a movement away from the surface pictured, towards the reader. Negative measurements denote the opposite. It should first be noticed that the violin bends in its length, both the back and the belly centre bouts moving in the same direction. The length of the back increases and the belly shortens. As predicted by the static analysis, the end bouts cross arches of both the back and the belly lift. The violin widens in the upper bouts, the centre bouts and the lower bouts of both the back and the belly, and there is fairly close agreement between the movement of the back and belly, thus the edges move synchronously with little rib twisting being involved (except at the centre bouts where the sound holes allow for a difference between the back and belly movements). Inspection of the figures each side of the belly centre line show that the bass bar resists the bending in the length and the rise in the end bouts cross arches, these motions being greater on the sound post side. The dotted lines added to the diagrams of the back and front separate areas of outwards movement from areas of inwards movement in the plates. The static deformation of the violin under string tension has also been measured by McLennan [4]. He used the top and bottom of the back over the end blocks as a datum for the displacements. These measurements also showed the violin bending in its length, the belly endbouts cross arches rising and the back rising over nearly all its length. It also showed similar assymetrical effects due to the bass bar.

It is quite clear from the diagram that an increase of string tension causes an expansion of the body and an inhaling of air, and the release of string tension causes an expelling of air. If we take the deformed shape shown in figure 8, and to it we add the deformation caused by the rocking bridge, a large chunk of the centre belly will move upwards. This means that there will be very little of the violin surface that does not move in a direction that increases the volume of the body. As can be seen, the internal forces and the direction of their application and the direction of the resulting movements, caused by a change in string tension, depends on the height of the ebx arches in relation to the cbx arches.

In a vibrational system, inertia plays a significant role in modifying the static displacements, but the forces on the system will influence the combination of modes excited and these forces depend on the relative height of the cbx and ebx arches.

The role of LSV in the production of violin sound

From the earliest times, violins have been made with the arching of the belly relatively higher in the end bouts than it is in the back. The belly tends to rise more steeply into the long arch at the ends than the back, and has a longer flat area in between. The violin would not have evolved into this shape if experiments with small changes in arching shape had not been rewarded with perceptible tonal results. Since these tonal results are dependent on features of the arching shape that affect the violin's response to a variation of the string tension there are strong grounds for expecting that LSV (longitudinal string vibration) plays a significant role in the production of violin sound, and that the effectiveness of this contribution is dependent on the shape of the plate arching. The sound post is an essential structural component of the mechanism that enables LSV forces to move the end bouts cross arches. Pluck a violin string and notice how quickly the energy radiates compared with flat plate instruments like the guitar or harpsichord that do not have this mechanism. I now present a hypothesis as to how LSV influences violin sound dynamically.

The input power to the violin comes from the bowing of the string. The string holds a reservoir of energy releasing small quantities to the body at each cycle of TSV, where it dissipates as mechanical loss or radiated sound. This transfer of energy to the body creates a complex vibrational response in the violin. This involves forces in the string (secondary LSV). The string must react and apply equal and opposite constraining forces to the body. Thus the modes in the violin are driven by power from TSV but determined by forces from both TSV and LSV acting together on the body. These forces will excite best those modes that are nearby in frequency and have large displacements in the direction of the forces. In this way LSV influences the modal behaviour of the violin.

For example, we have already seen that the rocking of the bridge tends to initiate a breathing action, and this will in turn cause a dynamic variation in the tension of the string. The resulting LSV will excite those modes that move the end bouts cross arches in the direction of the applied force. So, the TSV-driven bridge rocking energy that initially concentrated its dynamic activity at the bridge area, can through LSV, transfer its energy to excite modes that drive the end bouts cross arches to radiate. Secondary LSV does not add any energy to the body, but redirects energy from TSV to improve the radiation efficiency. The body's response to LSV can be optimised by careful control of the arching height in the end bouts compared to that in the centre bouts. To take an extreme case, if the belly was low in the end bouts and the back high, the end bouts cross arches would be driven in the opposite direction and would excite an entirely different combination of modes.

Defining arching shape by parameters

The qualitative static analysis of the effect of raising the string tension showed that the resulting body motions would be dependant on the ratio of the height of the plates in the end bouts to the height of the plates at the centre bouts, and on the difference in this ratio between the back and the belly. These features of the plate arching shape can be fully described by two parameters. I have called these parameters the 'EAR' and the 'Deviation'. It is important to understand what these terms mean. The term EAR stands for the 'End Arch Ratio', (pronounced 'ear', as in the ear we listen with). The violin has two values for EAR, one for the upper bouts and one for the lower bouts. $\text{Upper bouts EAR} = \frac{1}{2}\left(\frac{\text{belly height at upper bouts}}{\text{belly height at bridge}} + \frac{\text{back height at upper bouts}}{\text{back height at highest point}}\right)$

Thus the EAR is the rate at which the average of the back and belly heights fall from the bridge to the end bouts. The 'Deviation' is the amount by which the ratio $\frac{\text{belly height at upper bouts}}{\text{belly height at bridge}}$ is greater than the EAR, which is the same as the amount by which the ratio $\frac{\text{back height at upper bouts}}{\text{back height at highest point}}$ is less than the EAR. Thus the deviation is the amount by which the belly rate of fall is greater than the EAR. This is the same as the amount by which the rate of fall of the back is lower than the EAR. The EAR and deviation of the lower bouts is found in the same way.

It should be understood that a group of violins may have differing values of C/L (the ratio of cross arch to long arch support of the bridge) and would require differing values of EAR in order to have the same tonal effect. The EAR must relate to the C/L ratio. The relationship between the EAR and the C/L ratio is complex but in the range we are going to be concerned with, it is close to a direct linear relationship.

This means that if the C/L ratio is increased the EAR must be increased in about the same proportion. These matters are dealt with in a forthcoming paper [6].

Experiments by playing and subjective tonal assessment

A violin of high EAR could equally be described as having a normal EAR but a large deviation in the belly and a small deviation in the back. In this case the belly ebx arches would be driven much more than the back ebx arches. The reverse would be so for a low EAR violin. One would reasonably expect these "imbalances" to affect the tone. By making and playing a number of instruments with controlled variations to the EAR, I made the following observations about the tone.

1. The EAR of the lower bouts strongly affects the sound of the G string, and has some effect on the D string. The EAR of the upper bouts most strongly affects the A string, and has some effect on the D string and the E string. Thus the tone on the D string is influenced by both the upper and lower bouts EAR.
2. If the EAR is too low, the sound is woolly, fuzzy, and muffled. A player would say that it is not "open". It is also impure and identifiably hollow in quality, as though the sound is emanating from inside a large metal tank. It is interesting to note that this tonal quality is found on the lowest notes on the clarinet and has been shown to be due to a weakness of the second and fourth harmonics. In the violin, primary LSV may strengthen the second harmonic, and so a weakness may be due to a failure of the body to respond properly to the LSV. The higher notes on low EAR violins have a metallic quality to the sound.
3. If the EAR is too high, the sound is thinner, edgy, cold, astringent and perceptibly nasal. It could be described as sounding like a tenor who produces a tight sound from the throat.
4. As the EAR is raised from a low starting point, the characteristics in (2) above gradually fade away until a point is reached where, quite sharply, the characteristics in (3) above appear. There is an in between value of EAR that we work to in our workshop, which gives a very clean, open, powerful, untainted sound. I refer to this as medium EAR in what follows in this paper.
5. The tonal sensitivity to the EAR quite acute and in a making a violin, careful control of the EAR is necessary to maintain a consistent tonal quality.

By making violins with the same EAR and controlled differences in the deviation I found that the deviation has little or no effect on the tone (it will be shown in a forthcoming paper however, that it is important as a means of reducing the acute sensitivity of the tone to the EAR). I do not wish to assert that any particular tone is better or worse than any other, but I would assert however, that the choice of EAR in both the upper and lower bouts does certainly affect the tonal quality in a consistent and predictable way.

The EAR must relate to the C/L ratio, so in addition to maintaining a constant EAR from violin to violin it is also necessary to maintain a constant C/L ratio. This can be done by maintaining the same long and cross grain bending stiffnesses in the wood. I have proposed a method of achieving this in a previous paper [5]. In a forthcoming paper a practical method for controlling the EAR, deviation and the flexural stiffness will be given [6].

In Part 2 of this paper, quantitative vibration measurements are presented, which were done to see what support there may be for the hypothesis however a violinmaker who does not wish to follow the technical material in Part 2 will not be disadvantaged.

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