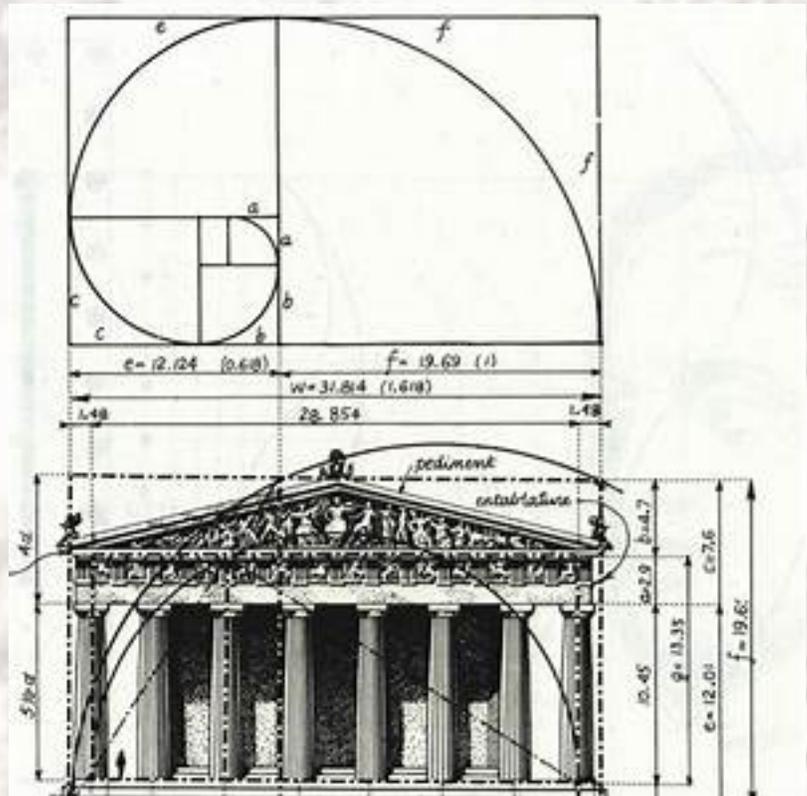
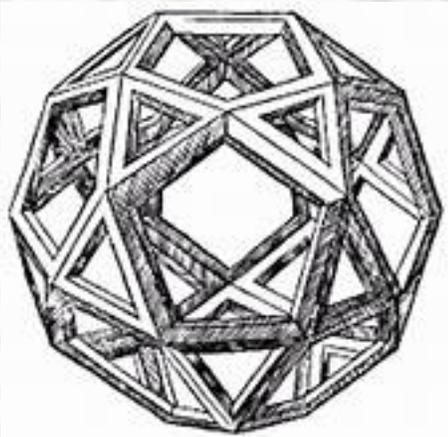


Qubits®

BUILDING SET

The Golden Polyhedra Lesson Plan



The **Qubits Building Set** is capable of building impressive 3D spherical structures. When you study these structures you might wonder what polyhedra they relate to? We wondered ourselves what those polyhedra would look like so we made a polyhedra for each of the 7 major shapes that our toy is able to construct. This document allows you to build them yourself for free. Just print out the pages with the “**GEOMETRICAL NETS**”. The nets represent the 3D shape when it is folded flat. After printing on stiff paper, cut them out, fold them along all the lines and tape them together into a 3D shape with small pieces of tape in the inside. It is a craft called “**Paper Polyhedra**”.

In doing this exercise we quickly realized that the polyhedra formed were “**GOLDEN POLYHEDRA**”. That is to say, there are line lengths that are **PHI - 1.618...** included in the polyhedra. This is a terrific educational development because it allows us to introduce students of all ages to this fascinating aspect of **Mathematics**.

This number **1.618** was called the “**GOLDEN RATIO**” by the **Ancient Greeks**. The Golden Ratio is also known as the **Golden Section**, the **Golden Mean** or the Golden Rectangle. The Ancient Egyptians used the Golden Ratio to build the pyramids. In fact the **Great Pyramids** show the first example of using the **Golden Ratio in Architecture**. The Egyptians accomplished all this back in around 205 - 3100 BC.

In India the Golden Mean was used in the construction of the **Taj Mahal** which was completed in 1648. Leonardo da Vinci utilized the Golden Ratio intensively while creating inventions. He even included it in many of his paintings. He called it the **Divine Proportion**. The architect **Le Corbusier** applied the Golden Ratio specifically with his modular system which he saw as a continuation of the traditions found in the work of the renaissance architect **Leon Battista Alberti**.

PHI 1.618... is not to be confused with PI which is 3.14... The ratio of a circle's circumference to its diameter. Both are important to mathematics and NATURE. **Nautilus Shells** are perhaps the most famous example of PHI geometric beauty. PHI is a number without an arithmetic solution, the digits simply continue for eternity without repeating themselves. PI is similar however it is an irrational number, just not an infinite number.

So sit down with a scissors and tape. The manipulative effect of working with small pieces of paper will improve the dexterity of nearly anyone. The result will be 7 of the coolest 3D shapes our toy can geometrically describe. Two of these shapes were already discovered by **Archimedes** back in 287-212 bc. The **Cuboctahedron** and **Icosidodecahedron** were two of the thirteen shapes he created. It is fun to realize that our toy made over two thousand years later is harkening back to this moment of discovery. It was a “**Eureka Moment**”.

NECESSARY TOOLS FOR MAKING PAPER POLYHEDRA

#1 - STIFF PAPER - We recommend **PRINTWORKS BRAND** White Cardstock 67 lbs. This bright white paper works well with most inkjet printers without jamming. We find ours at the **Ingles Grocery Store**, you can also find it on **Amazon**. Printworks is a “Made in the USA” company who is a member of the **Sustainable Forestry Initiative**.

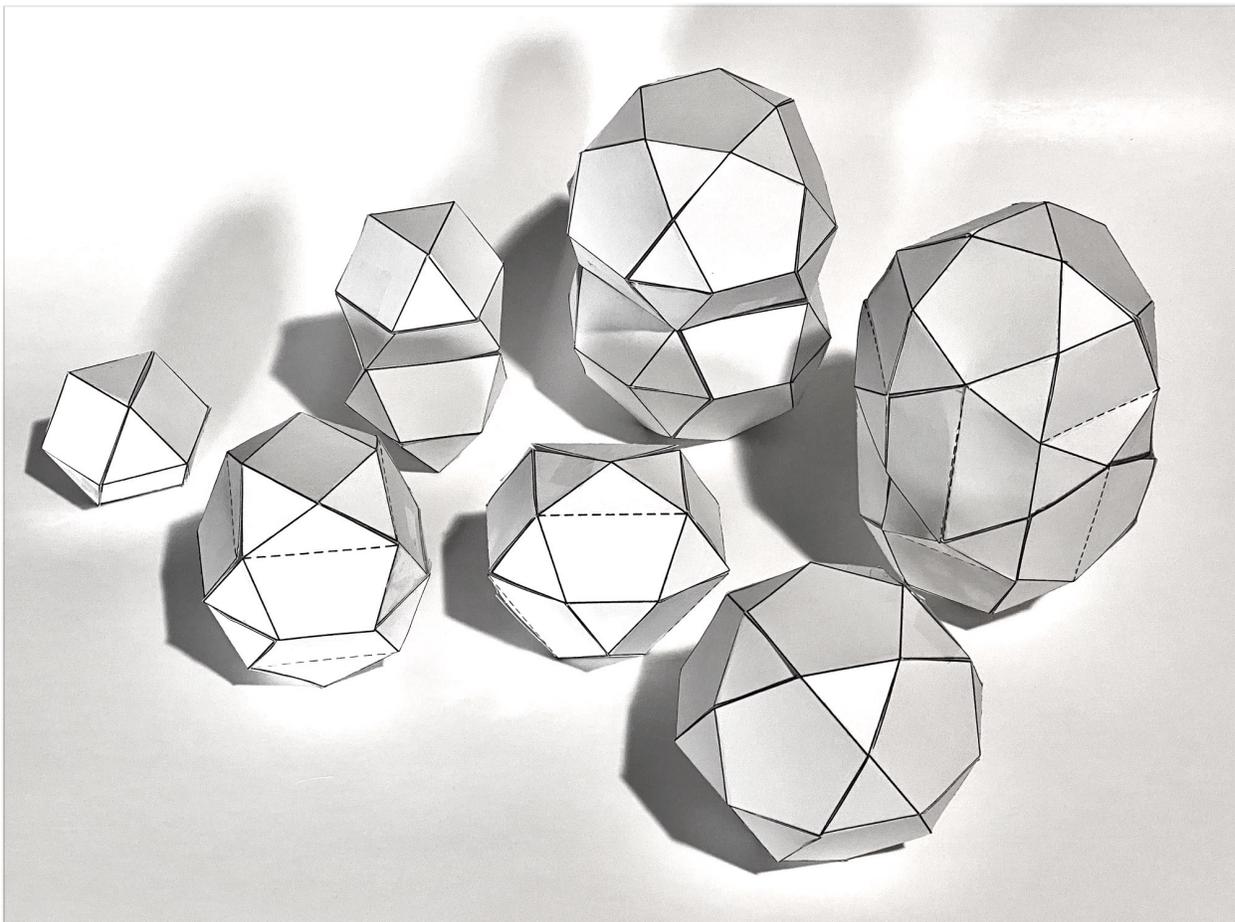
#2 - SCISSOR - We recommend **WESTCOTT BRAND SCISSORS**. These scissors have been “Made in the USA” since 1872. Precise cutting with an easy to handle shape. These scissors are also available on Amazon and their social media handle on Twitter is **@WestcottBrand**. Give them a shout out about your **Paper Polyhedra Project** by using the hashtag **#WestcottBrand**

#3 - TAPE - We have found that **SCOTCH BRAND** Tape “**Super-Hold**” to be very easy to work with. This product is made in St. Paul, Minnesota. You can talk about this “Made in the USA” product on Twitter at **@SCOTCH** with the hashtag **#SCOTCHTAPE**.

Not required, but useful to making professional Paper Polyhedra, we have found that one of the best tools are **Gorilla Brand** products. The **Gorilla Glue Sticks** and the famous **Gorilla Dual Temp Mini Hot Glue Gun** make perfect seams inside of enlarged NETS. This quick setting glue will allow you to build oversize Paper Polyhedra or even polyhedra made from Cardboard. Remember, these nets can be “**SCALED UP**” to any size you want. You can find out more about Gorilla Brand products on Twitter at **@GorillaGlue**.

Qubits Building Sets are also “**Made in the USA**” (Hendersonville, North Carolina to be exact). These **#STEM** favorites can be purchased on **Amazon** or at the factory website www.QubitsToy.com. The Qubits Nets that allow for making the the shapes to match these 7 polyhedra are available with a separate Lesson Plan. This plan is devoted to the art of Paper Polyhedra and it is intended as a very inexpensive way to introduce **STEM/STEAM/Makerspace** students to the fine art of making 3D paper shapes.

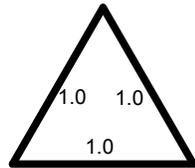
A FAMILY PHOTO OF THE SEVEN POLYHEDRA



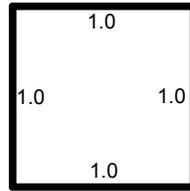
Enjoy this document and realize that the lessons learned here are good for kids from 4 to 104. Everybody learns from solid geometry.

After you build one of these polyhedrons, take a close-up picture of it in your hand. We will RT your photo on Twitter at @Qubits_Toy

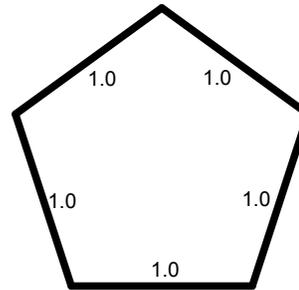
THE SIX POLYGONS OF MODULAR BUILDING ELEMENT GEOMETRY



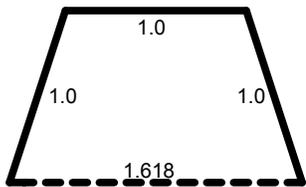
EQUILATERAL
TRIANGLE



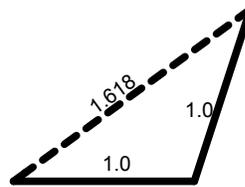
REGULAR
SQUARE



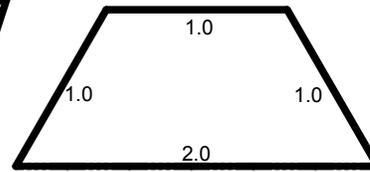
REGULAR
PENTAGON



ISOSCELES
TRAPIZOID



ISOSCELES
TRIANGLE



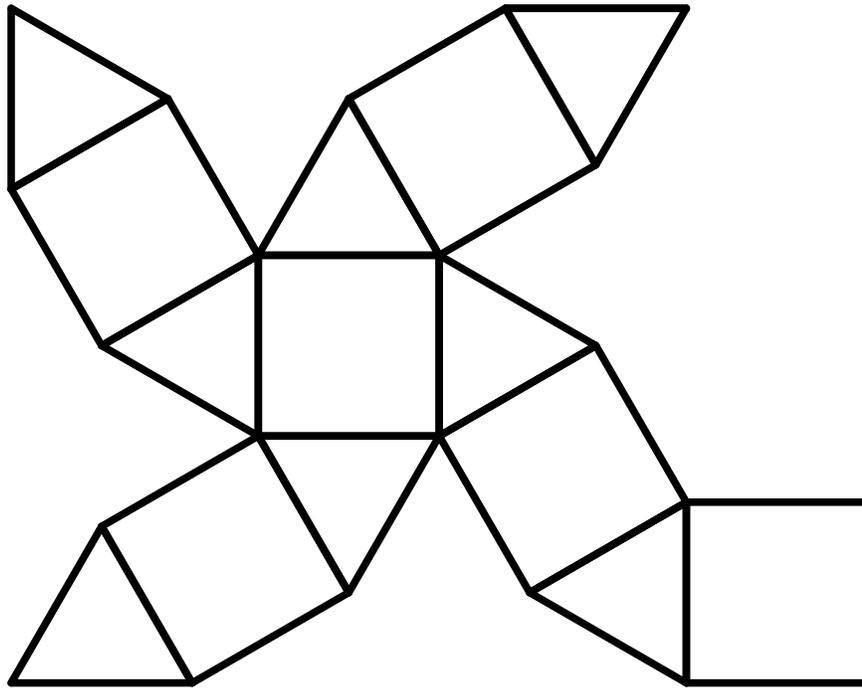
REGULAR
TRAPIZOID

NOTE: ADDITIONAL POLYGONS ARE GENERATED
AT THE CUT PLANES WITHIN THE POLYHEDRA
PROPOSED BY THIS GEOMETRY EXERCISE.

SOME OF THE ADDITIONAL GEOMETRIC FIGURES
GENERATED WHILE BUILDING THESE NETS ARE:

THE GOLDEN TRIANGLE
THE REGULAR HEXAGON
THE GOLDEN SQUARE
THE GOLDEN PENTAGON
THE REGULAR OCTAGON
THE REGULAR DECAGON
THE 2x SIZE SQUARE

THIS SET OF 6 POLYGONS WILL CONSTRUCT THE 7 POLYHEDRA
SHOWN WITHIN THIS GEOMETRY EXERCISE. MANY OTHER
POLYHEDRA CAN BE MADE FROM THESE 6 POLYHEDRONS,
HOWEVER AT THIS TIME WE ARE ONLY CONCERNING OURSELVES
WITH THESE UNIQUE SHAPES AND HOW THEY DIRECTLY RELATE
TO THE PATENTED MODULAR BUILDING ELEMENT GEOMETRY
THAT IS SOLD ON AMAZON AS A STEM TOY CALLED "QUBITS®".



Cuboctahedron

by Archimedes
214 BC

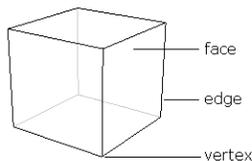
NOTICE THAT EACH POLYHEDRA HAS A FORMULA WRITTEN UNDER IT. THE FORMULA SAYS, THE NUMBER OF VERTICES MINUS THE NUMBER OF EDGES PLUS THE NUMBER OF FACES EQUALS 2.

FOR EXAMPLE FOR THE CUBOCTAHEDRON THE FORMULA IS:

$$12 - 24 + 14 = 2$$

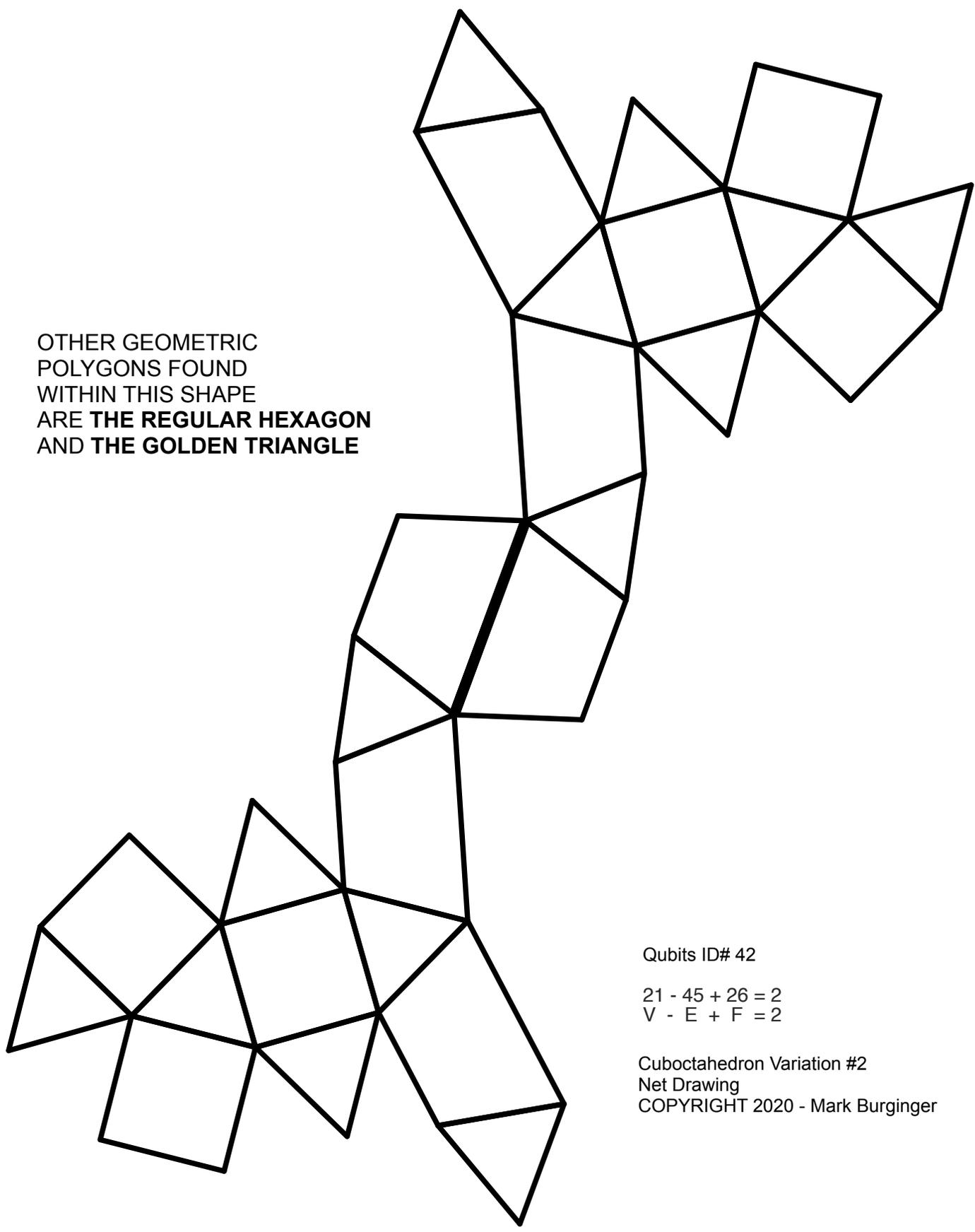
$$V - E + F = 2$$

THIS FORMULA IS CALLED THE EULER'S POLYHEDRON FORMULA AFTER THE SWISS MATHEMATICIAN - LEONHARD EULER 1707-1783



Qubits ID# 24

Cuboctahedron Net Drawing
COPYRIGHT 2020 - Qubits Toy, Inc.

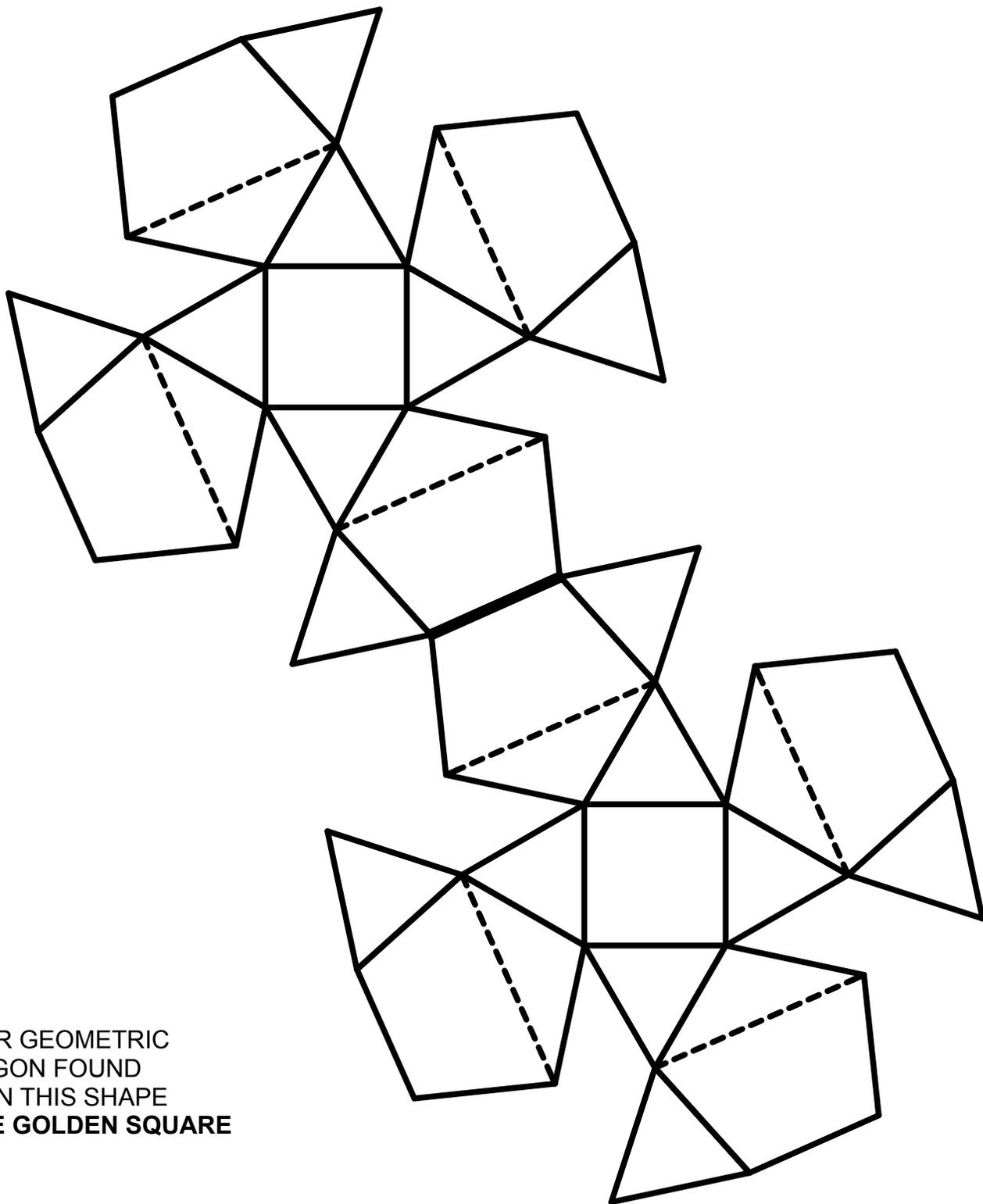


OTHER GEOMETRIC
POLYGONS FOUND
WITHIN THIS SHAPE
ARE **THE REGULAR HEXAGON**
AND **THE GOLDEN TRIANGLE**

Qubits ID# 42

$$21 - 45 + 26 = 2$$
$$V - E + F = 2$$

Cuboctahedron Variation #2
Net Drawing
COPYRIGHT 2020 - Mark Burginger



OTHER GEOMETRIC
POLYGON FOUND
WITHIN THIS SHAPE
IS **THE GOLDEN SQUARE**

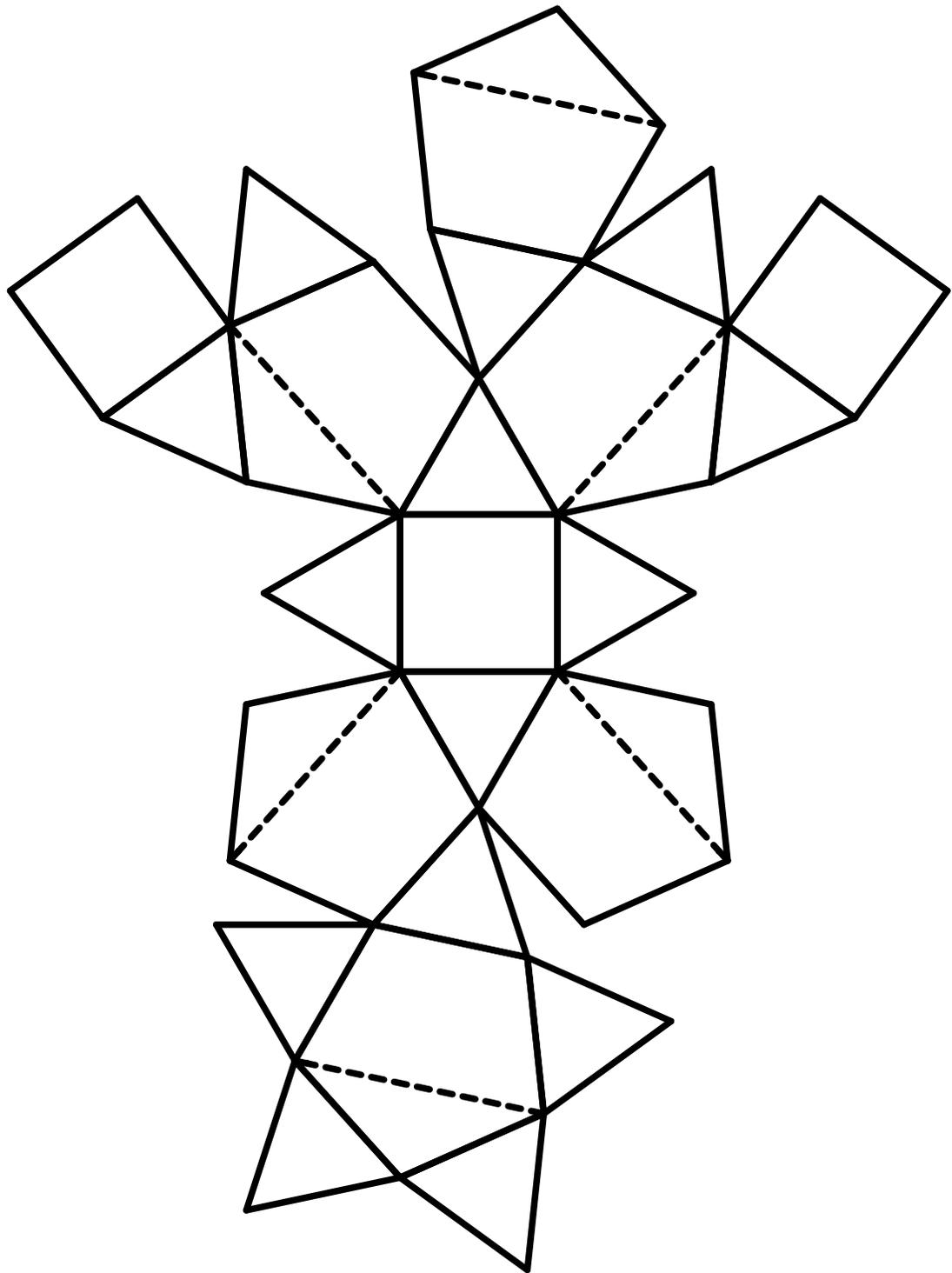
Qubits ID# 48

$$24 - 56 + 34 = 2$$
$$V - E + F = 2$$

Cuboctahedron Variation #2

Net Drawing

COPYRIGHT 2020 - Qubits Toy, Inc.

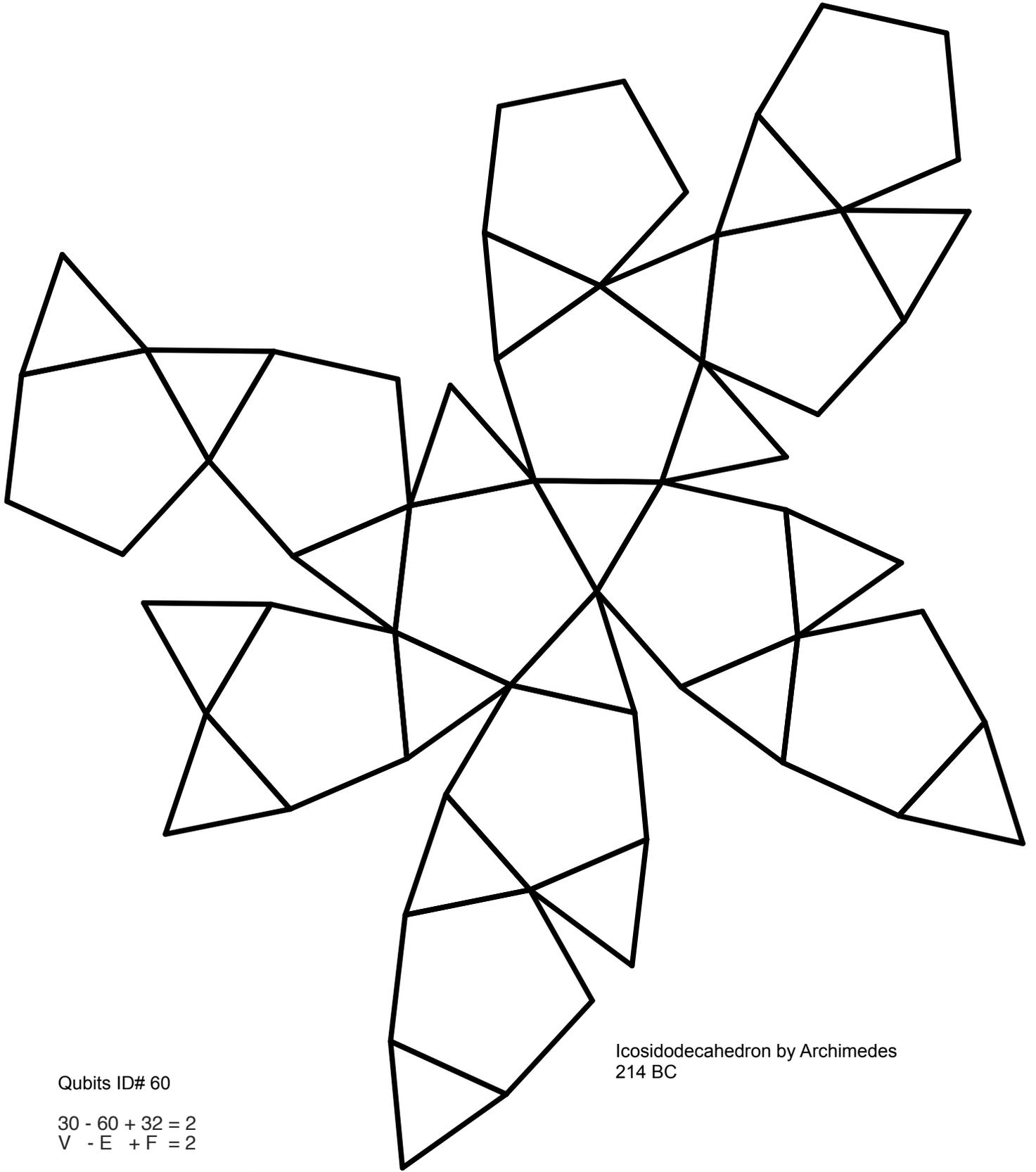


Qubits ID# 42A

Icosienneahedron

Icosienneahedron Net Drawing
and name.
COPYRIGHT 2020 - Qubits Toy, Inc.

$$21 - 48 + 29 = 2$$
$$V - E + F = 2$$

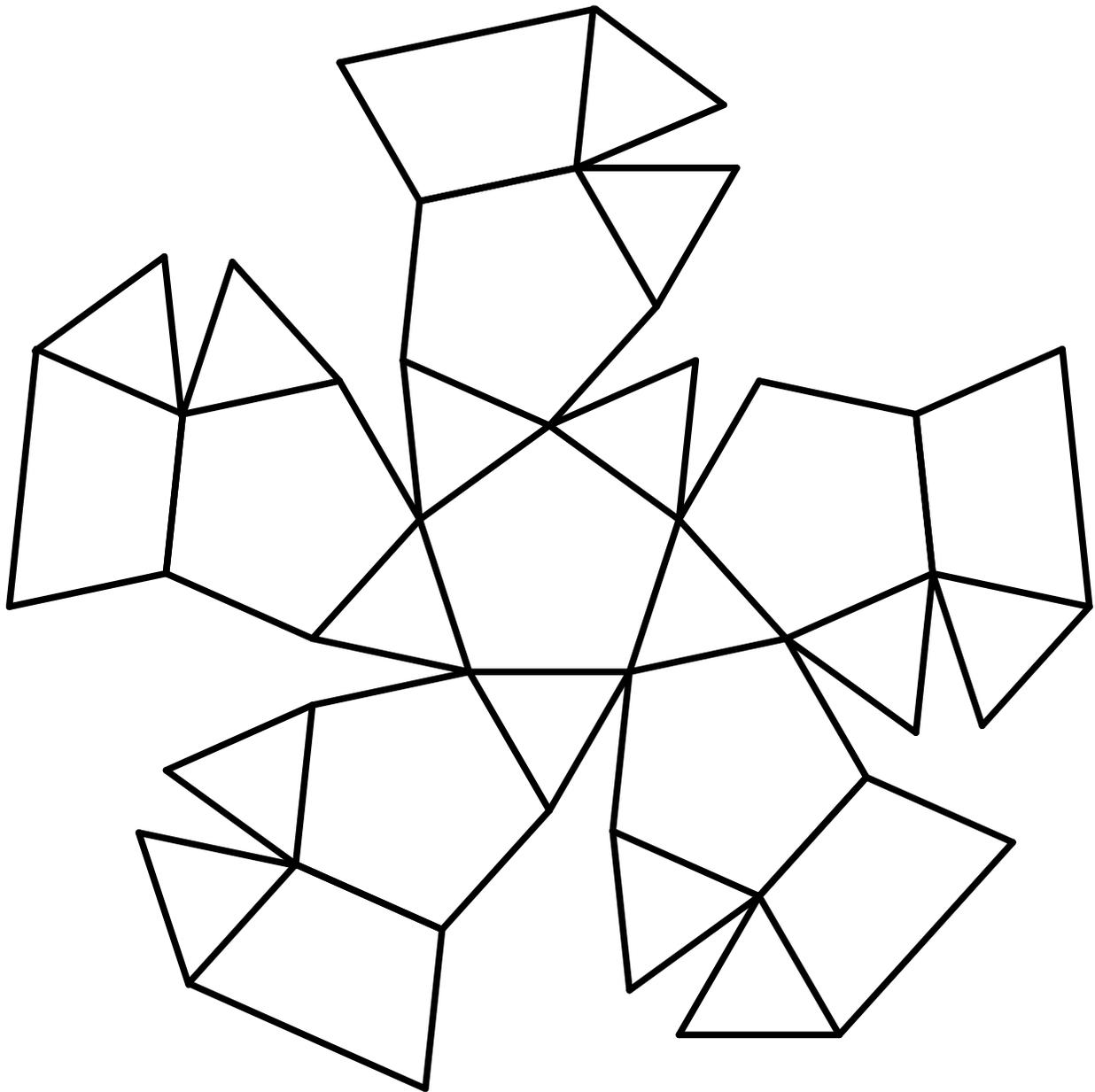


Qubits ID# 60

$$30 - 60 + 32 = 2$$
$$V - E + F = 2$$

Icosidodecahedron Net Drawing
COPYRIGHT 2020 - Qubits Toy, Inc.

Icosidodecahedron by Archimedes
214 BC



OTHER GEOMETRIC
POLYGONS FOUND
WITHIN THIS SHAPE
ARE **THE GOLDEN PENTAGON**
AND **THE REGULAR DECAGON**

Qubits ID# 90

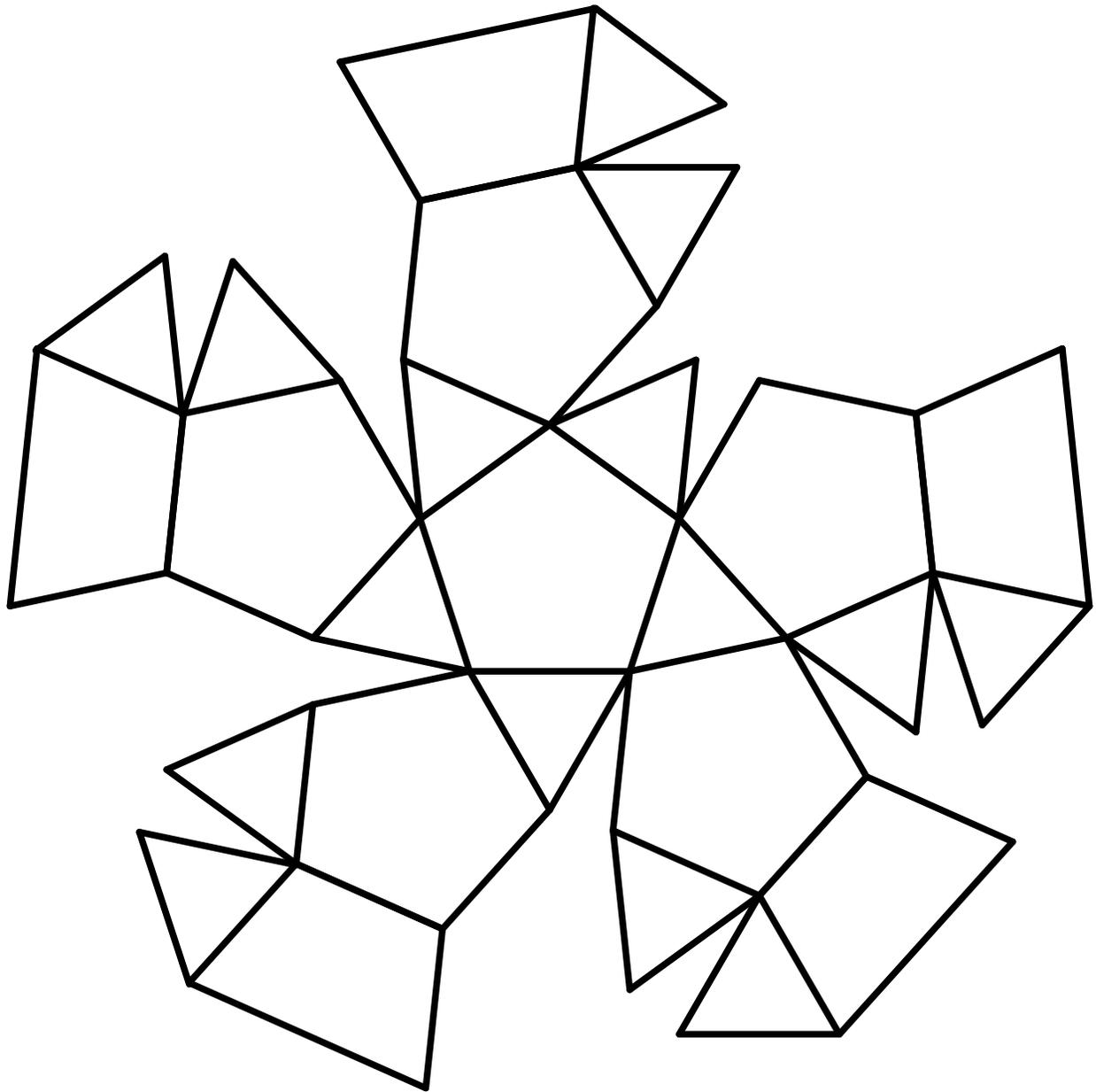
$$45 + 95 - 52 = 2$$
$$V - E + F = 2$$

Icosidodecahedron Variation #1
Net Drawing

COPYRIGHT 2020 - Qubits Toy, Inc.

Sheet 1 of 2

THIS SHEET MAKES 1/2 THE
ENTIRE SHAPE



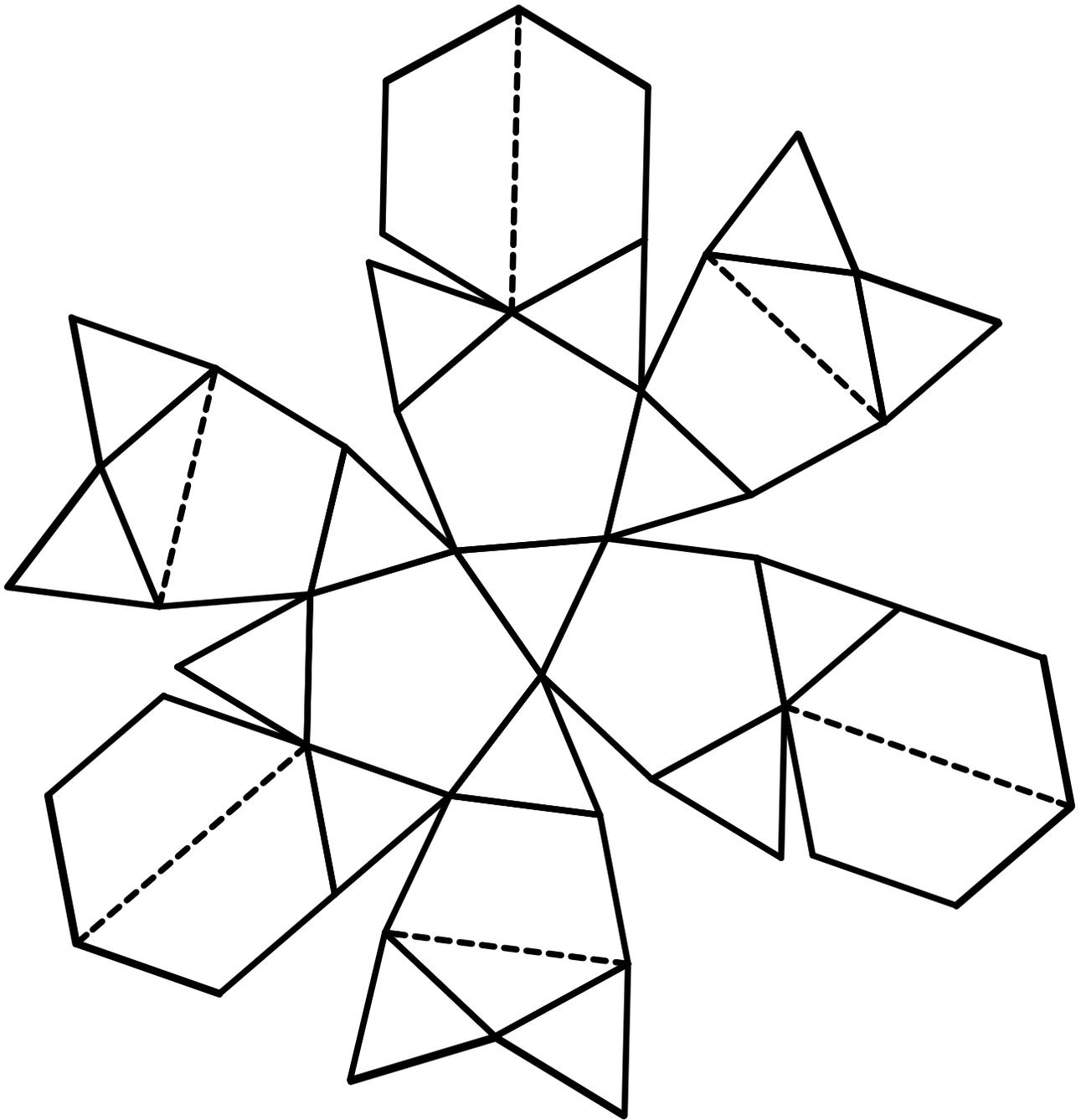
Qubits ID# 90

$$45 + 95 - 52 = 2$$
$$V - E + F = 2$$

Icosidodecahedron Variation #1
Net Drawing

COPYRIGHT 2020 - Qubits Toy, Inc.

Sheet 2 of 2



Qubits ID# 96

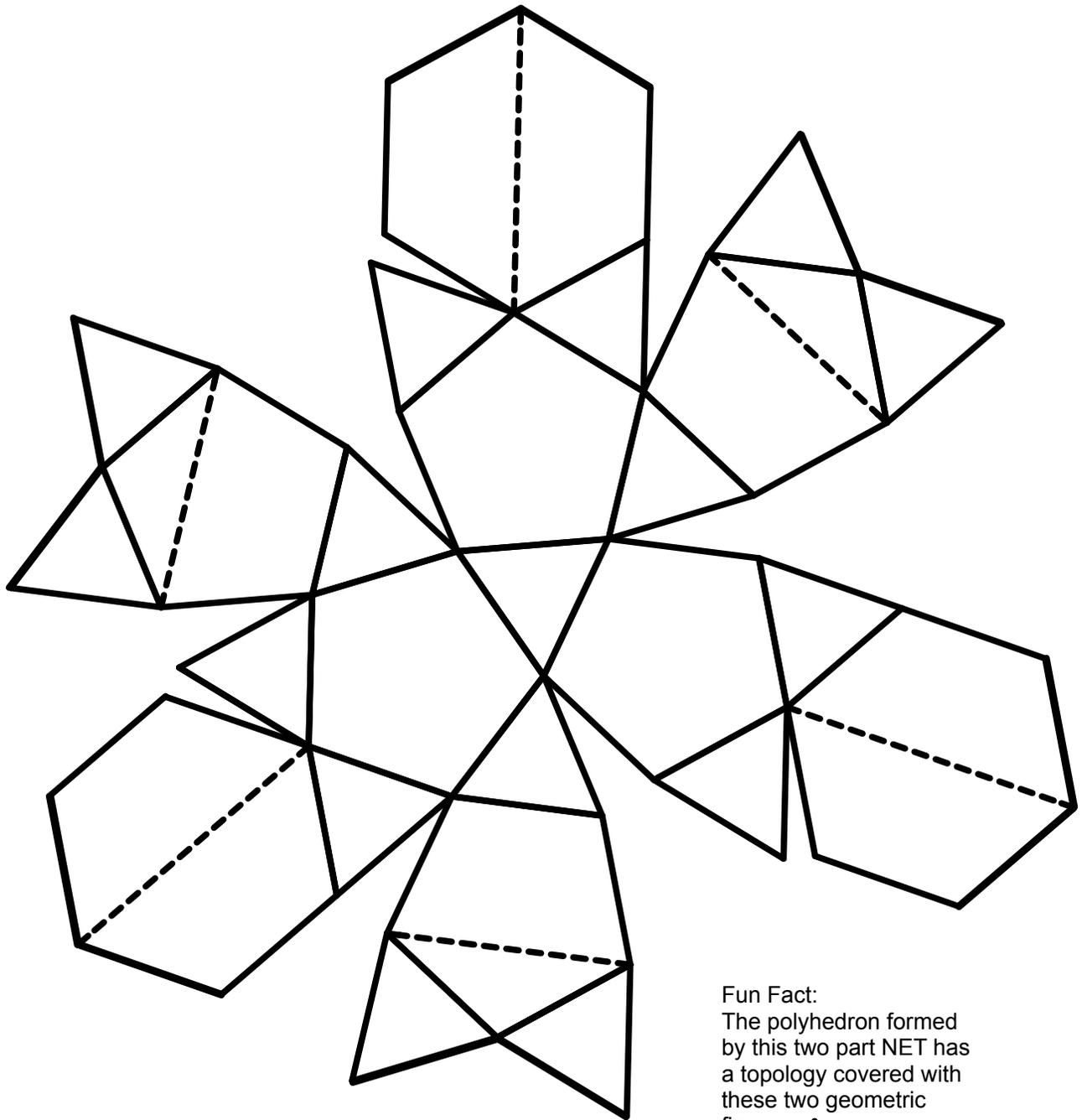
$$48 - 108 + 62 = 2$$
$$V - E + F = 2$$

Icosidodecahedron Variation #2
Net Drawing

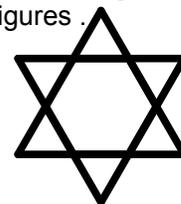
COPYRIGHT 2020 - Qubits Toy, Inc.

Sheet 1 of 2

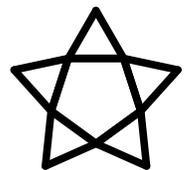
THIS SHEET MAKES 1/2 THE
ENTIRE SHAPE



Fun Fact:
 The polyhedron formed
 by this two part NET has
 a topology covered by
 these two geometric
 figures .



6 of the
6 Pointed Stars



5 of the
5 Pointed Stars

Qubits ID# 96

$$48 - 108 + 62 = 2$$

$$V - E + F = 2$$

Icosidodecahedron Variation #2
 Net Drawing

COPYRIGHT 2020 - Qubits Toy, Inc.

Sheet 2 of 2

RECOMMENDED BOOKS AND LINKS

The Golden Ratio: The Divine Beauty of Mathematics
by Gary B. Meisner and Rafael Araujo

The Golden Section: Nature's Greatest Secret (Wooden Books)
by Scott Olsen

The Golden Ratio: The Story of PHI, the World's Most Astonishing Number
by Mario Livio

The Power of Limits: Proportional Harmonies in Nature, Art, and Architecture
by Gyorgy Doczi

Legends of the Ancient World: The Life and Legacy of Archimedes
by Charles River Editors

A Geometric Analysis of the Platonic Solids and Other Semi-Regular Polyhedra:
With an Introduction to the Phi Ratio, 2nd Edition 2nd ed. Edition
by Kenneth J M MacLean

For Kids:

Archimedes and His Numbers - Biography Books for Kids 9-12 | Children's Biography Books
by Baby Professor

Archimedes: The Man Who Invented The Death Ray
by Shoo Rayner

Web Links

<https://plus.maths.org/content/eulers-polyhedron-formula>

Math is Fun
<https://www.mathsisfun.com>

Geometry Junkyard
<https://www.ics.uci.edu/~eppstein/junkyard/euler/>