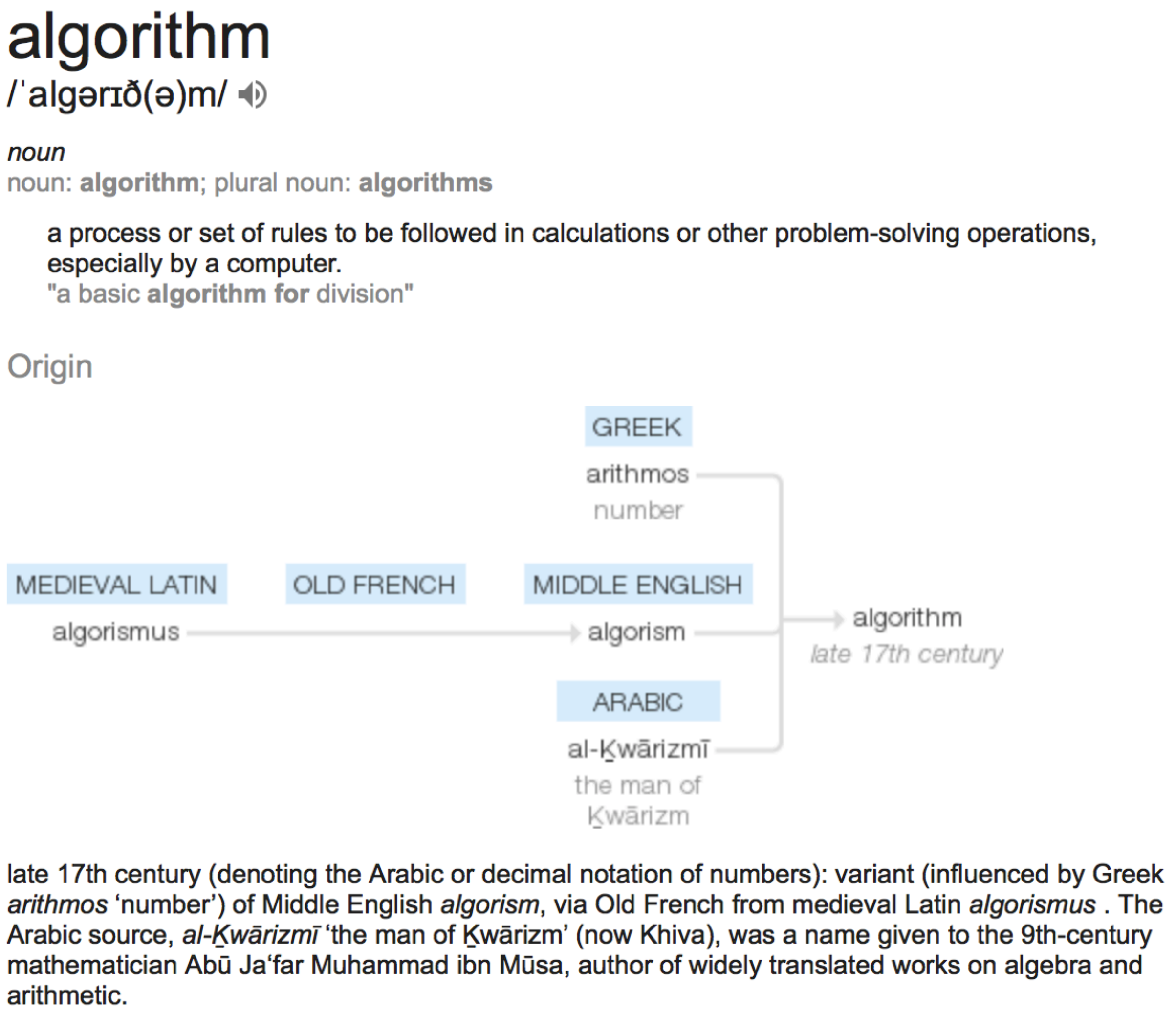
Introduction to Algorithms

What are algorithms?



algorithm

/ˈalɡərɪð(ə)m/ 🔊

noun

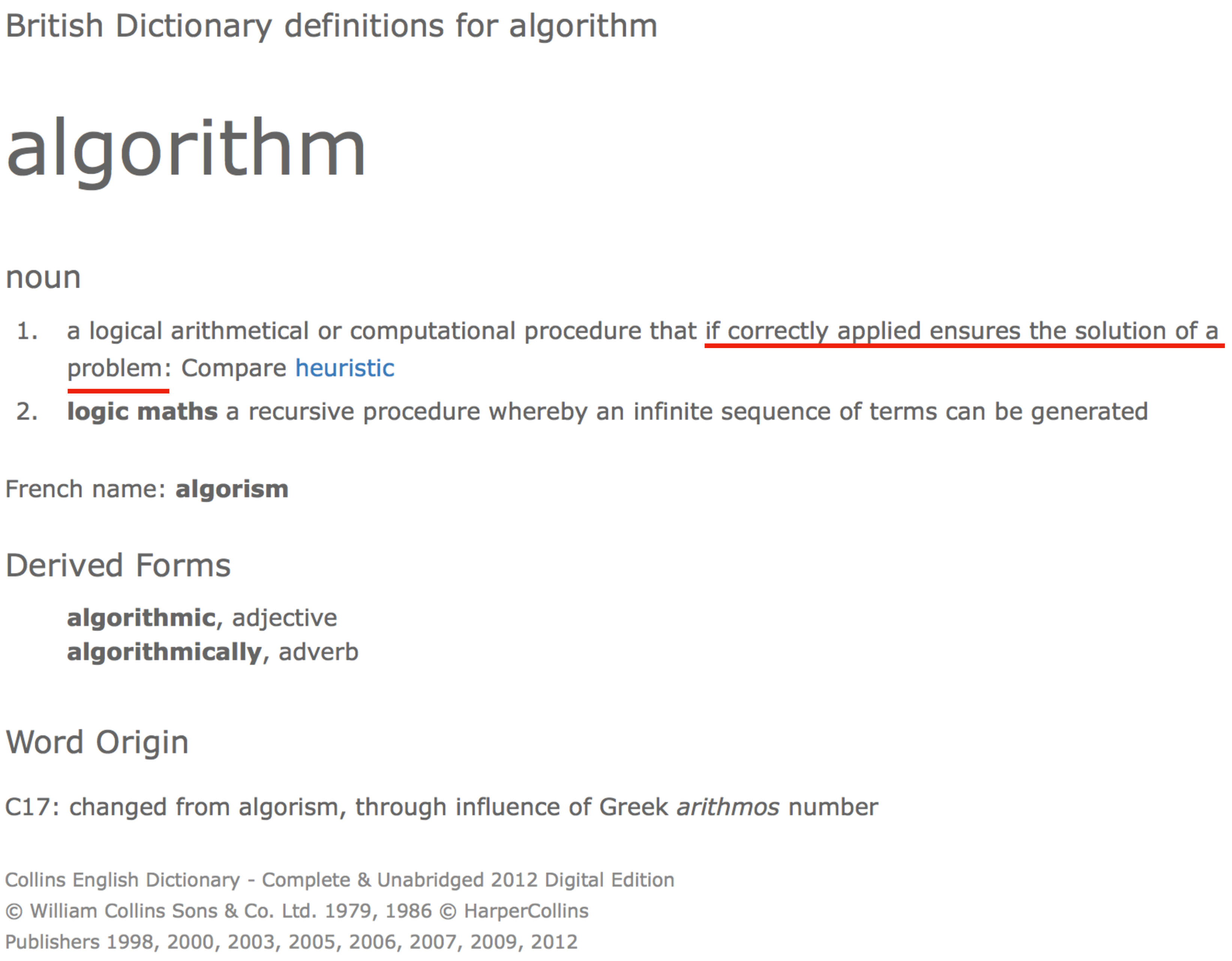
noun: **algorithm**; plural noun: **algorithms**

a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.
"a basic **algorithm for** division"

Origin

late 17th century (denoting the Arabic or decimal notation of numbers): variant (influenced by Greek *arithmos* ‘number’) of Middle English *algorism*, via Old French from medieval Latin *algorismus* . The Arabic source, *al-Ḵwārizmī* ‘the man of Ḵwārizm’ (now Khiva), was a name given to the 9th-century mathematician Abū Ja‘far Muhammad ibn Mūsa, author of widely translated works on algebra and arithmetic.

https://www.dictionary.com/browse/algorithm



British Dictionary definitions for algorithm

algorithm

noun

1. a logical arithmetical or computational procedure that if correctly applied ensures the solution of a problem: Compare heuristic
2. **logic maths** a recursive procedure whereby an infinite sequence of terms can be generated

French name: **algorism**

Derived Forms

algorithmic, adjective
algorithmically, adverb

Word Origin

C17: changed from algorism, through influence of Greek *arithmos* number

Collins English Dictionary - Complete & Unabridged 2012 Digital Edition
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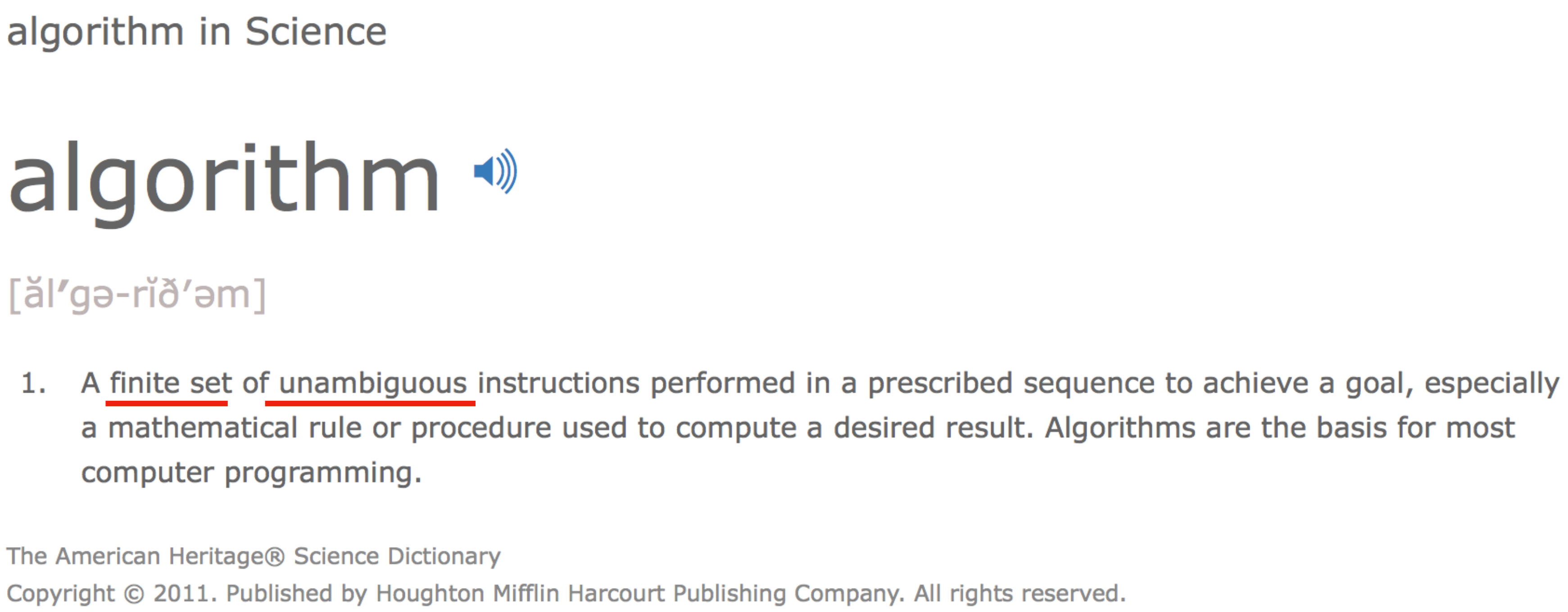
Publishers 1998, 2000, 2003, 2005, 2006, 2007, 2009, 2012



algorithm in Science

algorithm 🔊

[ăl′gə-rĭð′əm]

1. A finite set of unambiguous instructions performed in a prescribed sequence to achieve a goal, especially a mathematical rule or procedure used to compute a desired result. Algorithms are the basis for most computer programming.

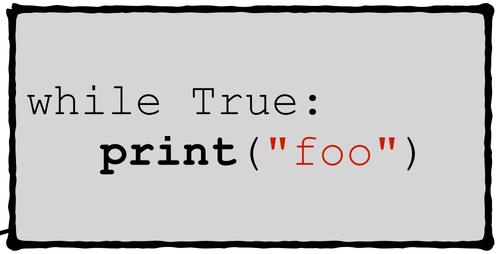
The American Heritage® Science Dictionary

Combining definitions…

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Combining definitions…

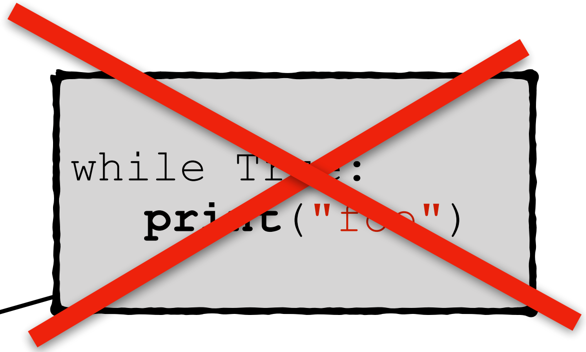


```
while True:
   print("foo")
```

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Combining definitions…



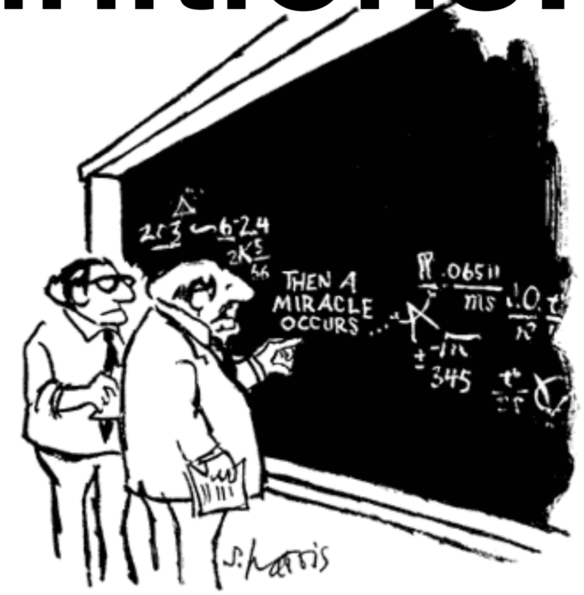
```
while True:
    print("foo")
```

We want our algorithms to **terminate!**

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Combining definitions…



"I think you should be more explicit here in step two."

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Cartoon from http://www.sciencecartoonsplus.com/gallery/math/math07.gif
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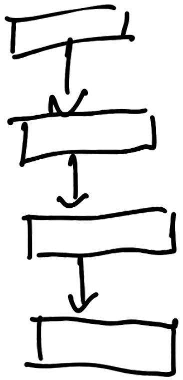
Combining definitions…

Algorithm (noun):

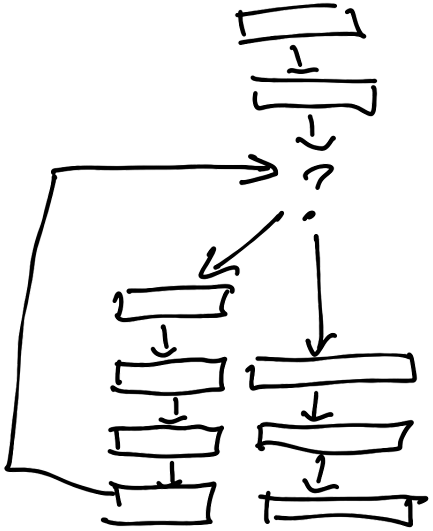
A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Building-blocks in Programming and in Algorithms

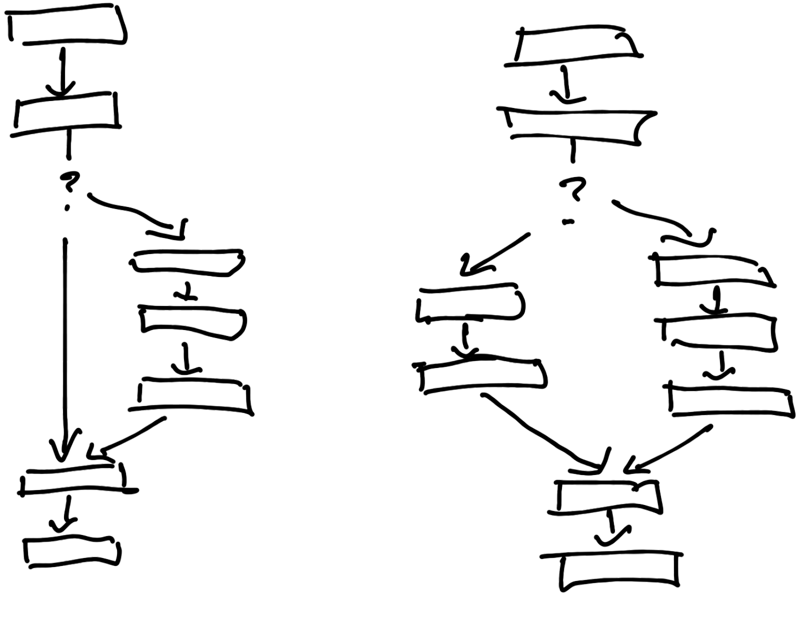
Sequential



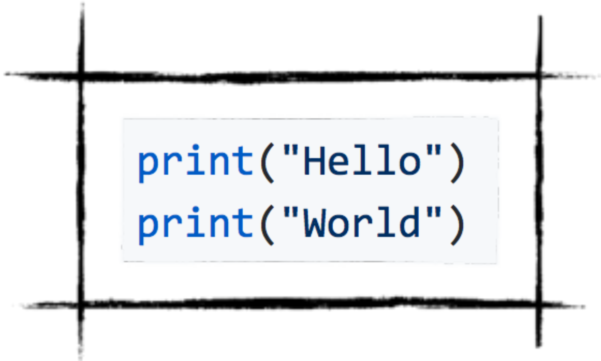
Looping



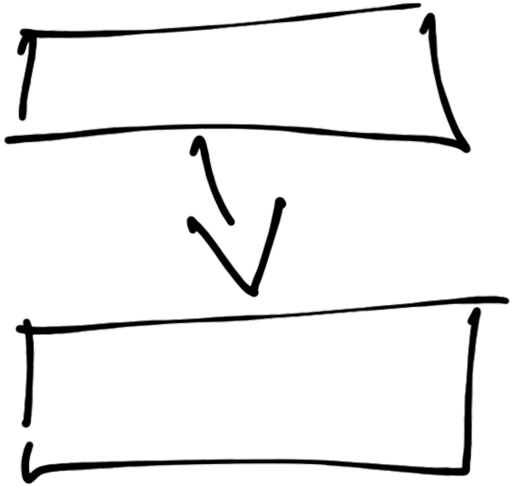
Branching



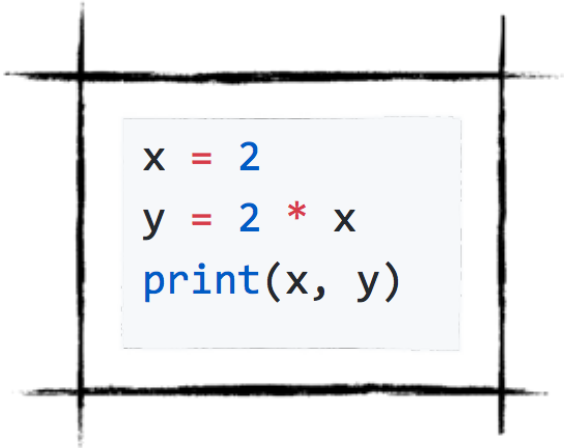
Sequential execution



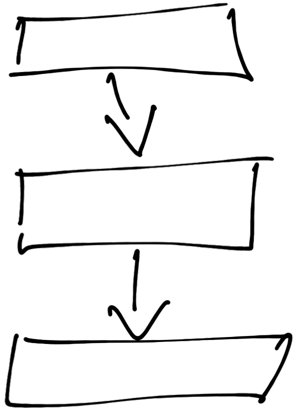
```
print("Hello")
print("World")
```

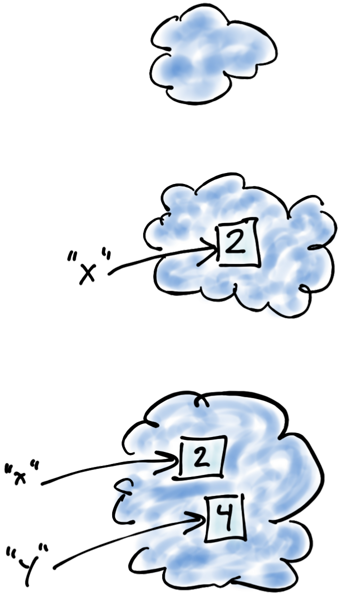


Variables (program state)

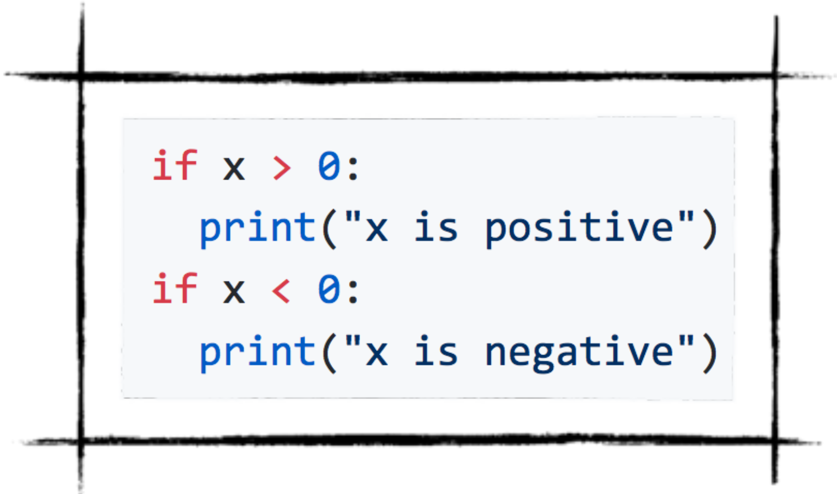


```
x = 2
y = 2 * x
print(x, y)
```

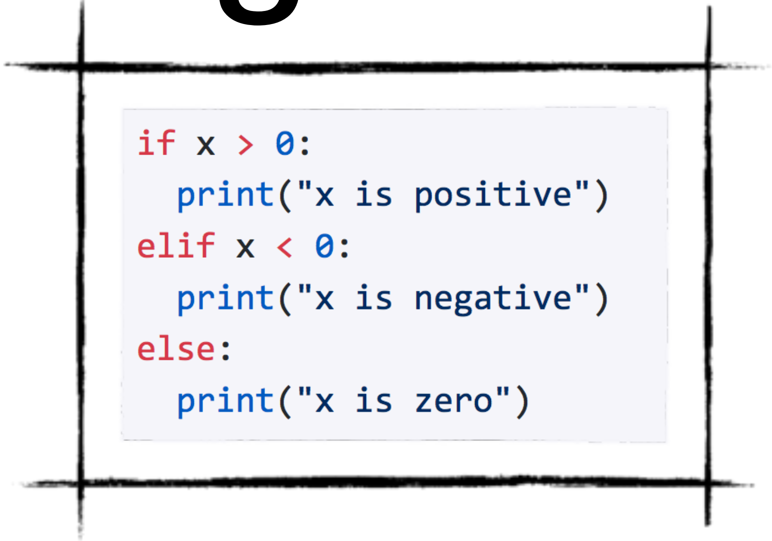




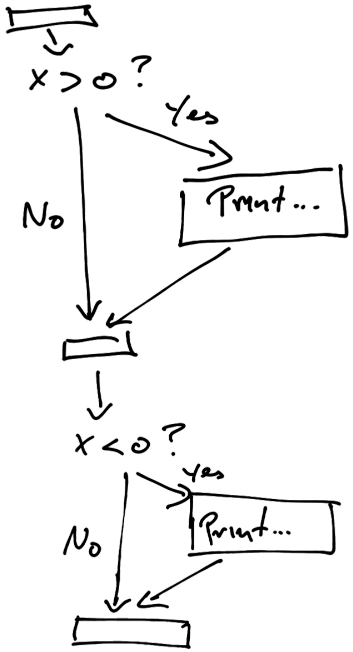
Branching

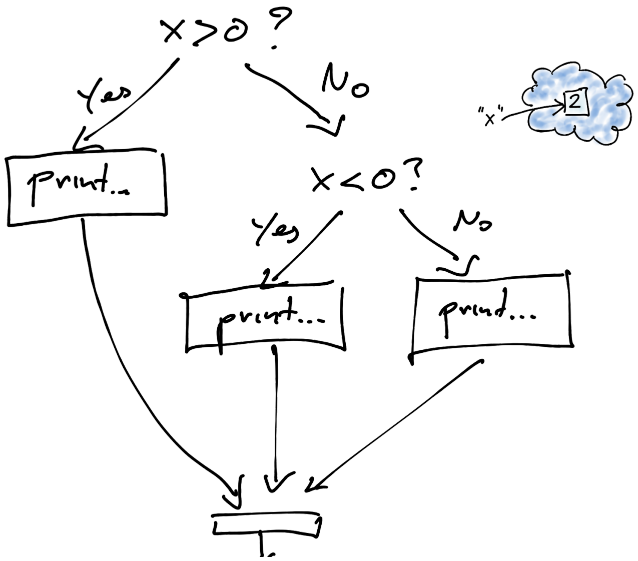


```python
if x > 0:
  print("x is positive")
if x < 0:
  print("x is negative")
```

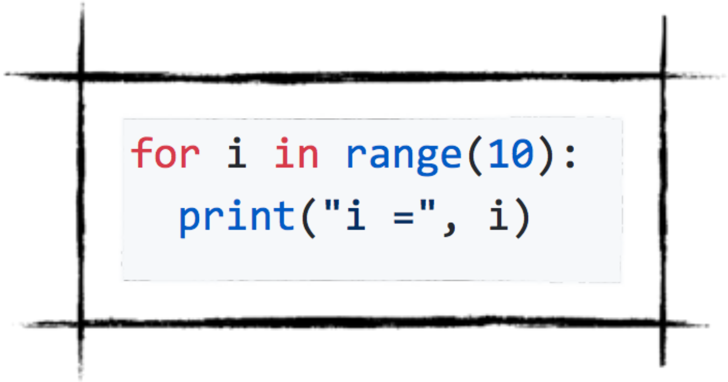


```python
if x > 0:
  print("x is positive")
elif x < 0:
  print("x is negative")
else:
  print("x is zero")
```

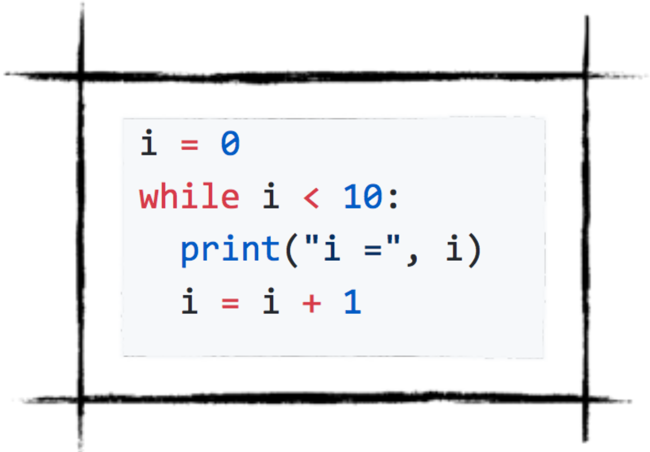




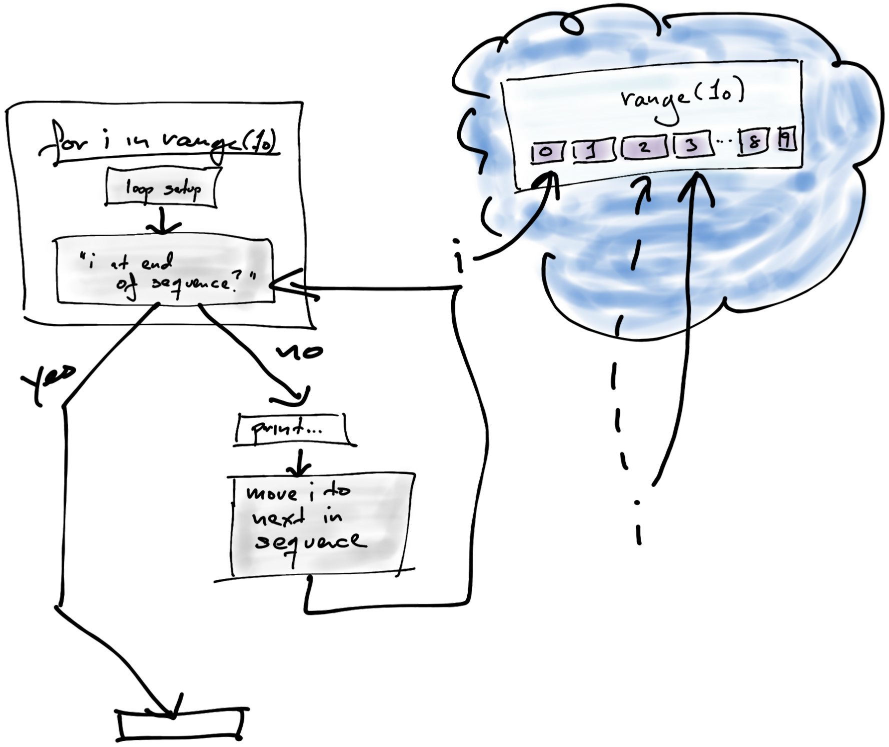
Looping

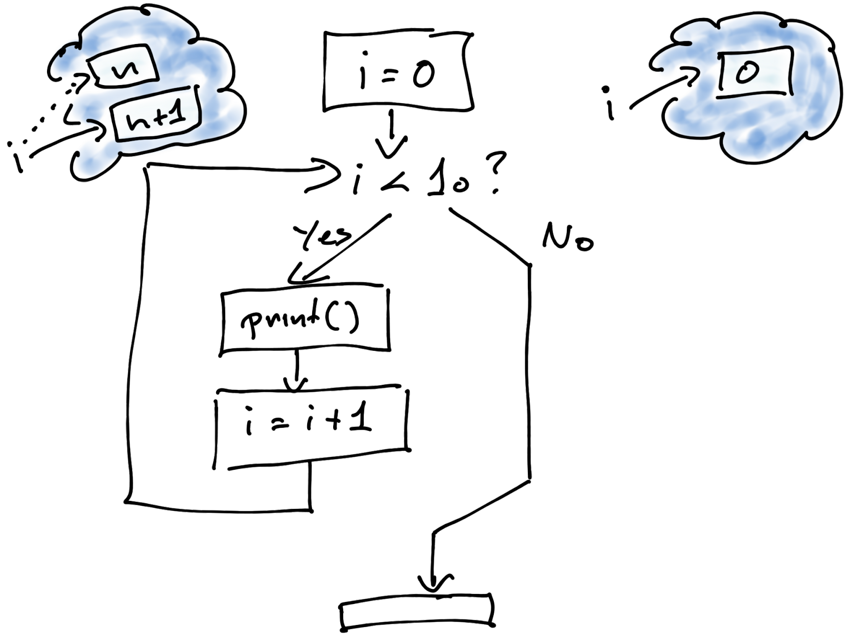


```python
for i in range(10):
  print("i =", i)
```



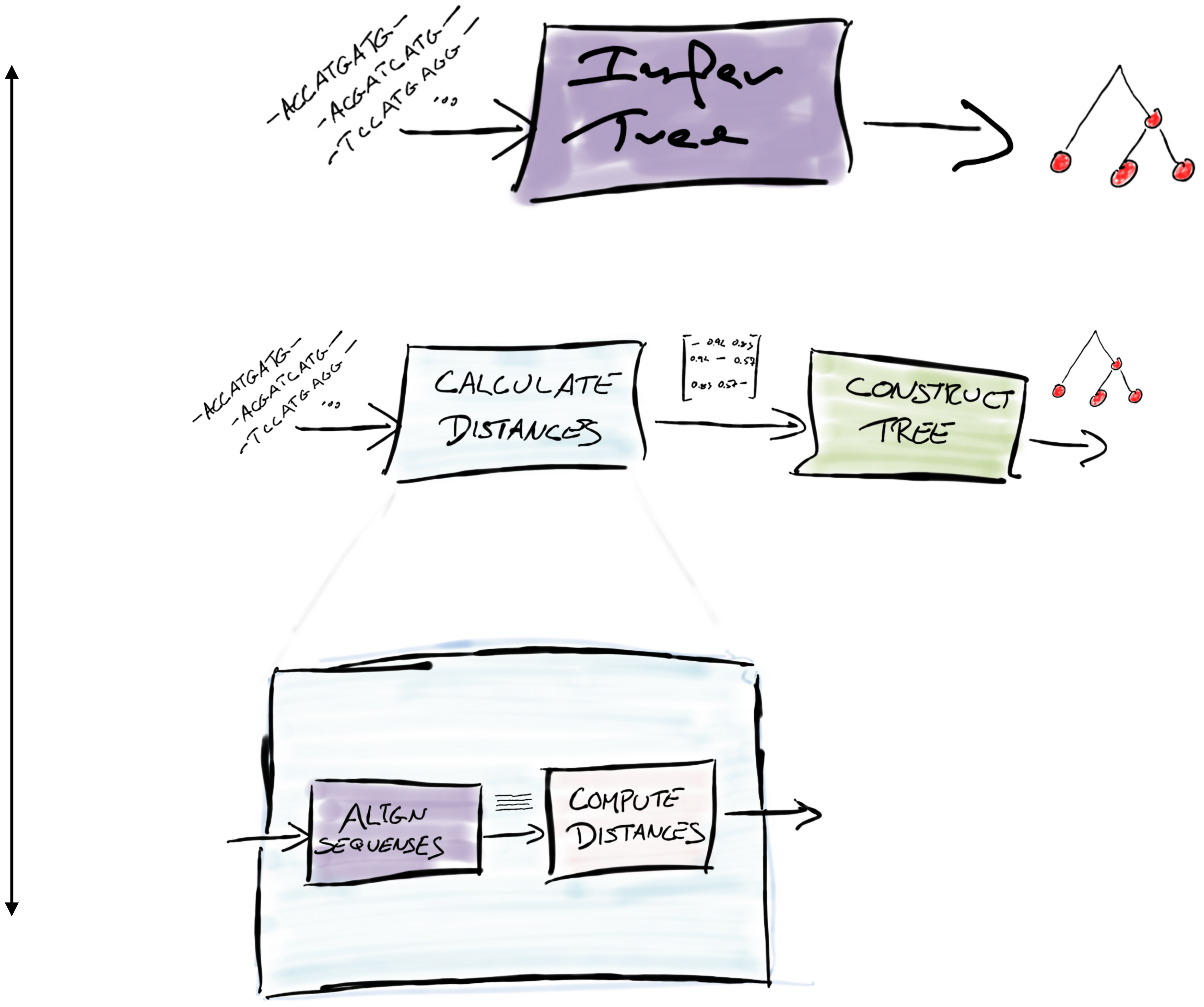
```python
i = 0
while i < 10:
  print("i =", i)
  i = i + 1
```



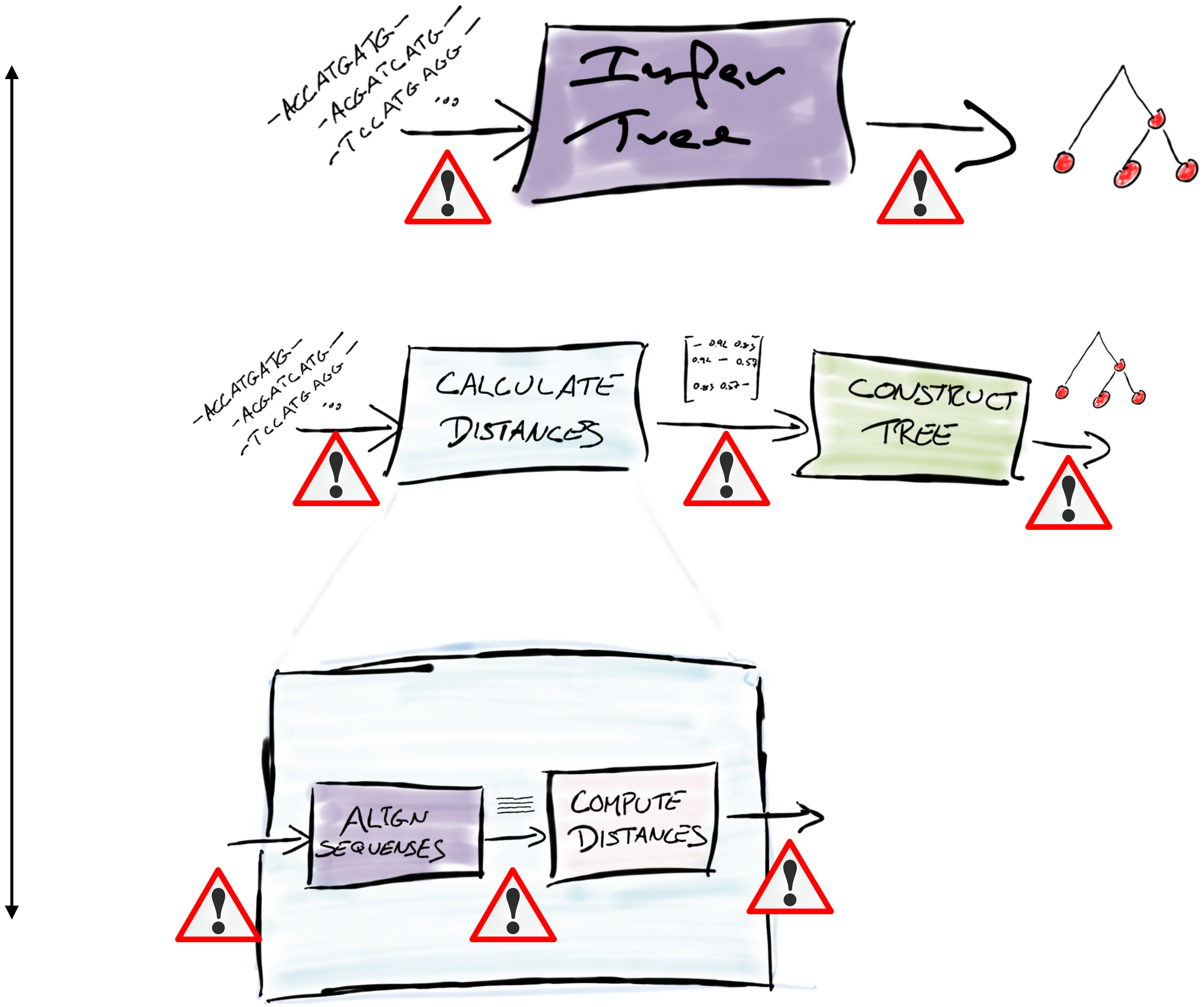


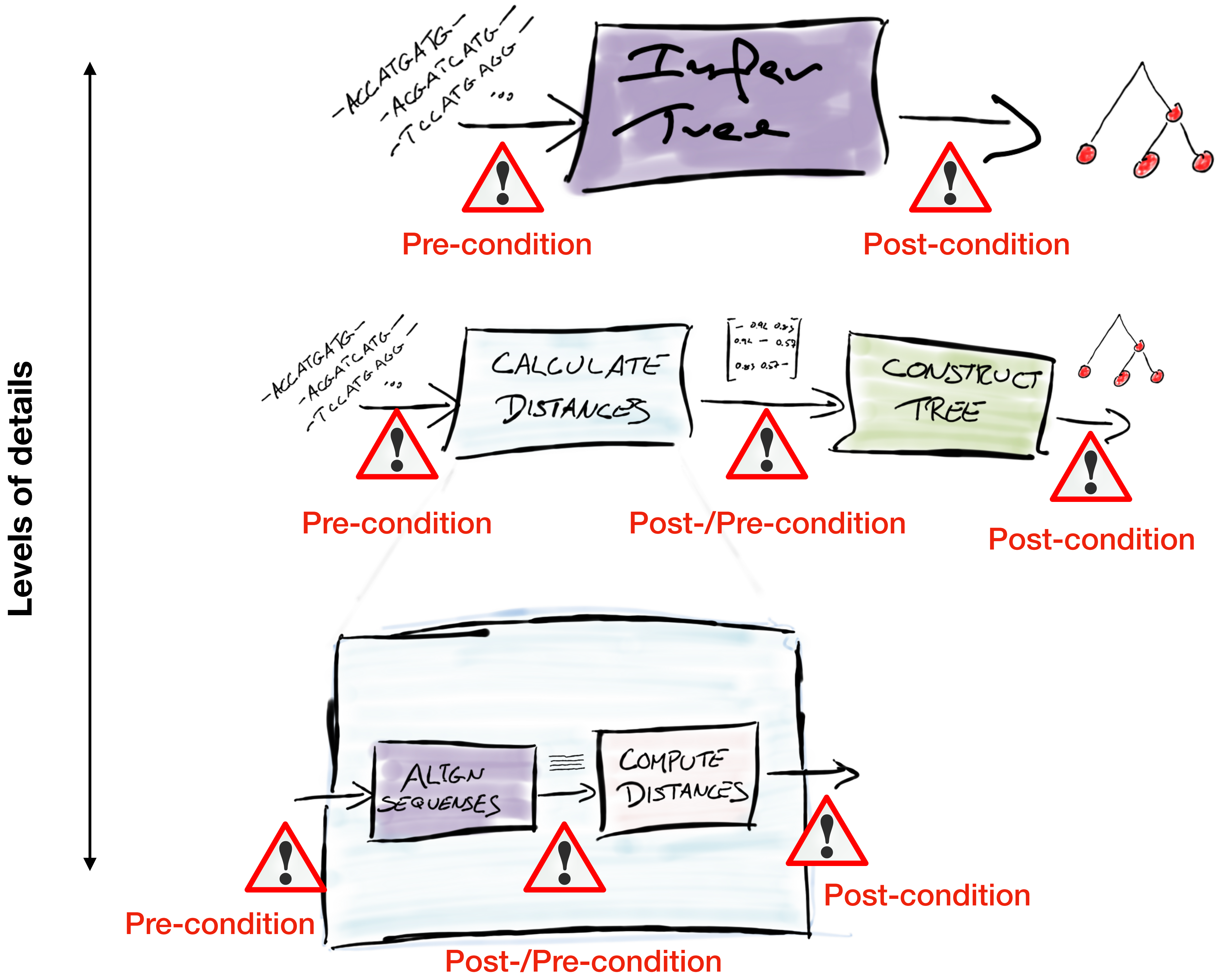
Designing algorithms: Breaking down problems to smaller parts

Levels of details



Levels of details





Levels of details

Pre-condition

Post-condition

Pre-condition

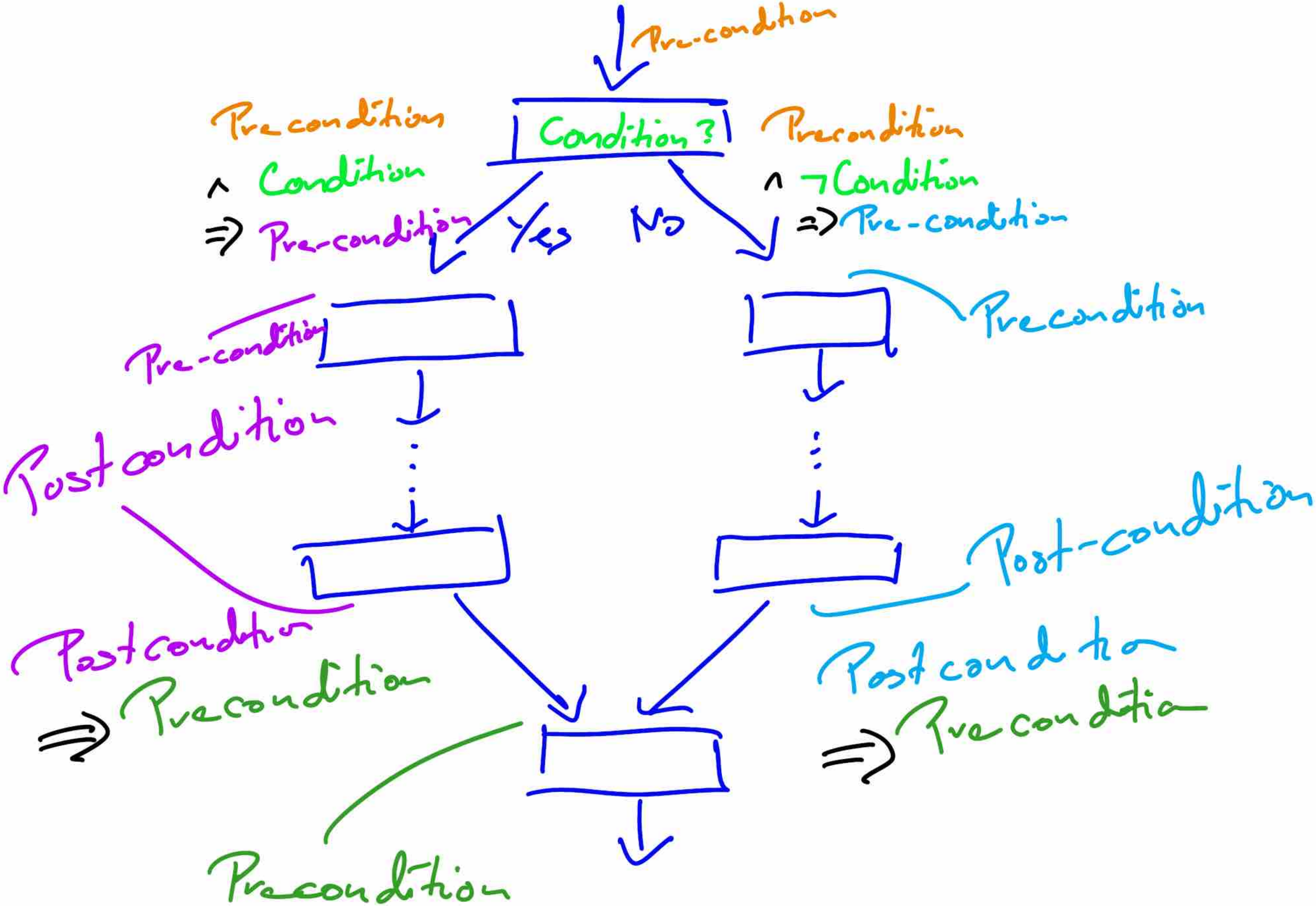
Post-/Pre-condition

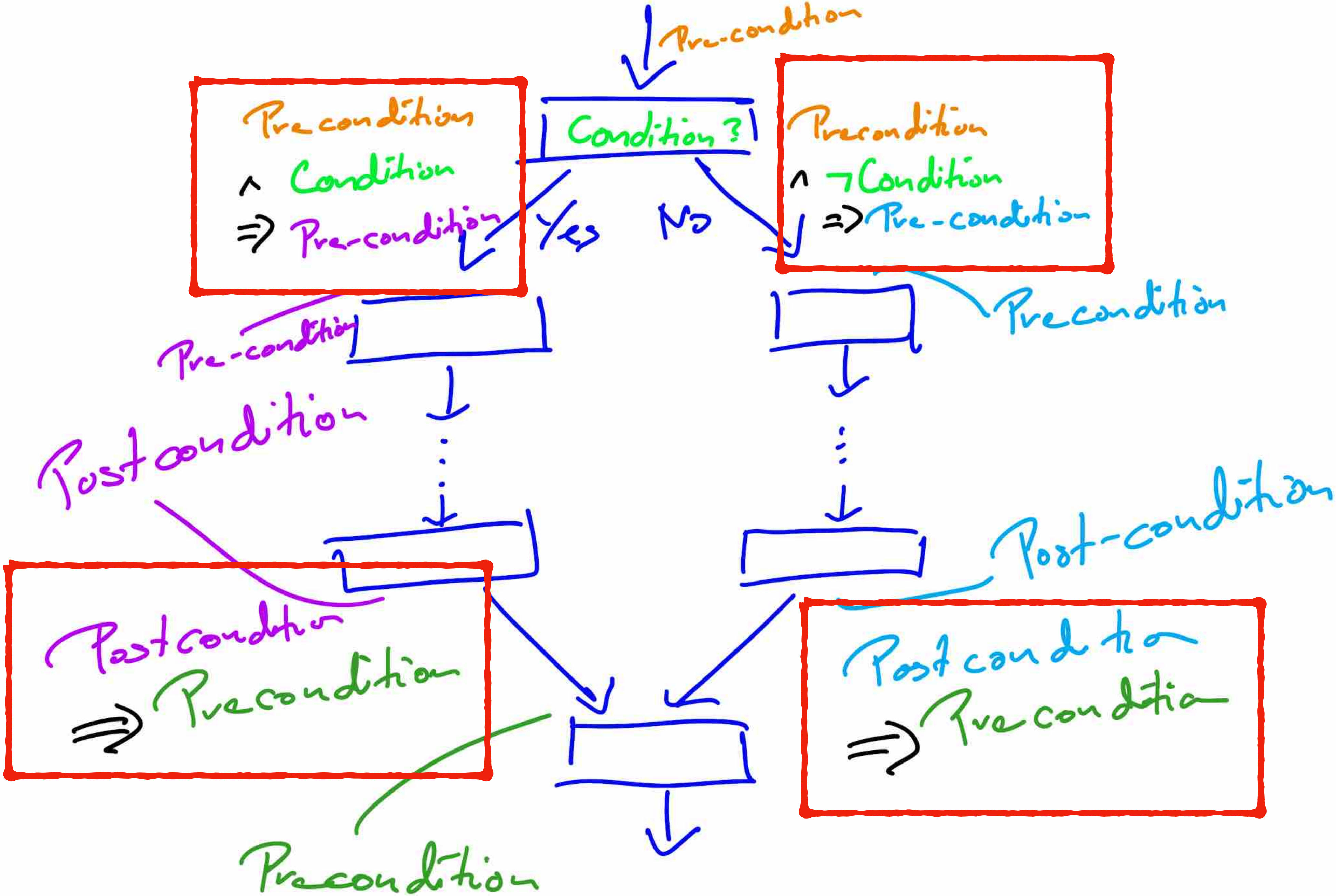
Post-condition

Pre-condition

Post-/Pre-condition

Post-condition


Pre-condition
Condition ?
Precondition
∧ Condition
⇒ Pre-condition
Yes
No
Precondition
∧ ¬Condition
⇒ Pre-condition
Pre-condition
Precondition
Postcondition
Post-condition
Postcondition
⇒ Precondition
Postcondition
⇒ Precondition
Precondition


Pre-condition
Condition ?
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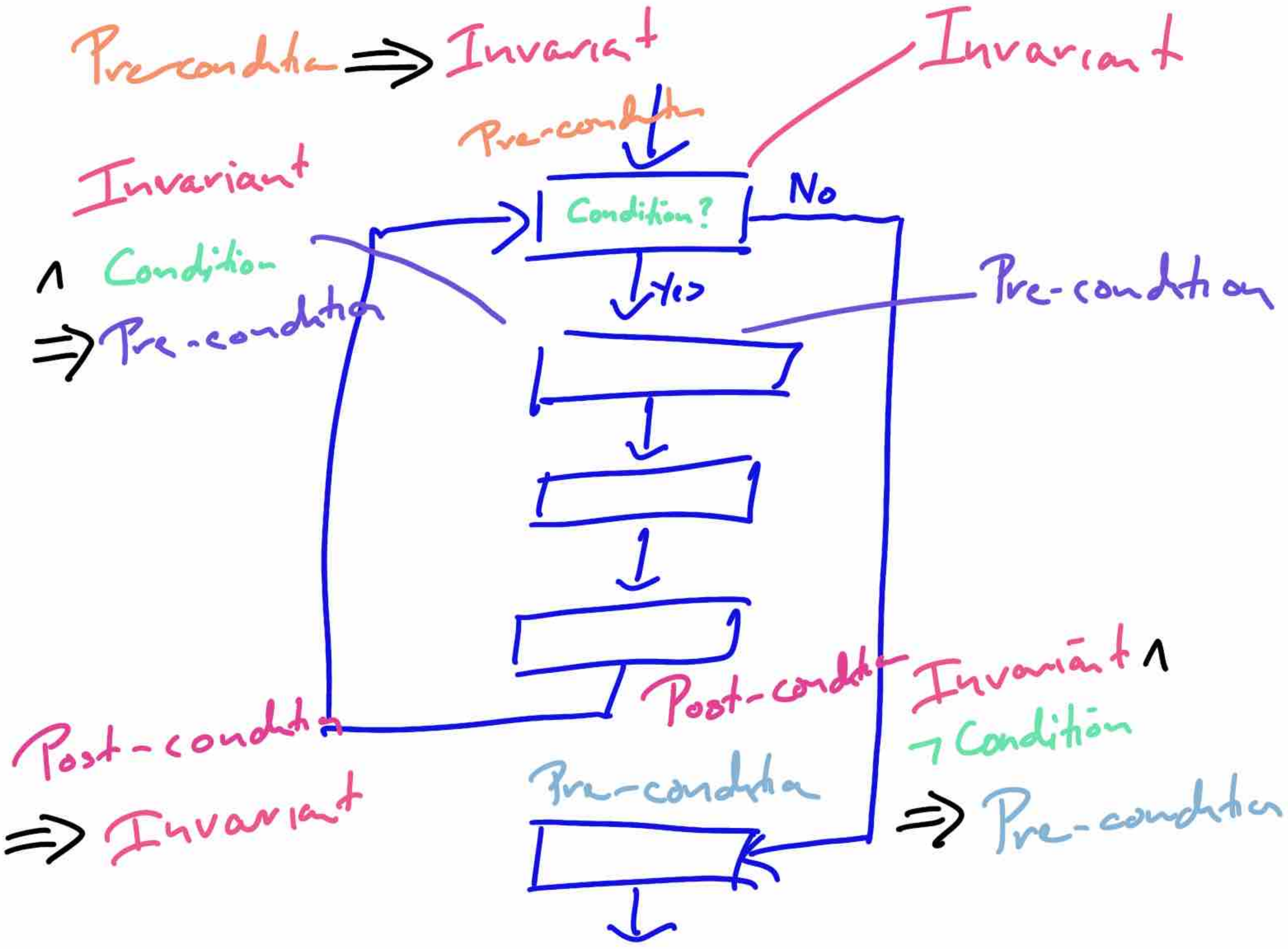
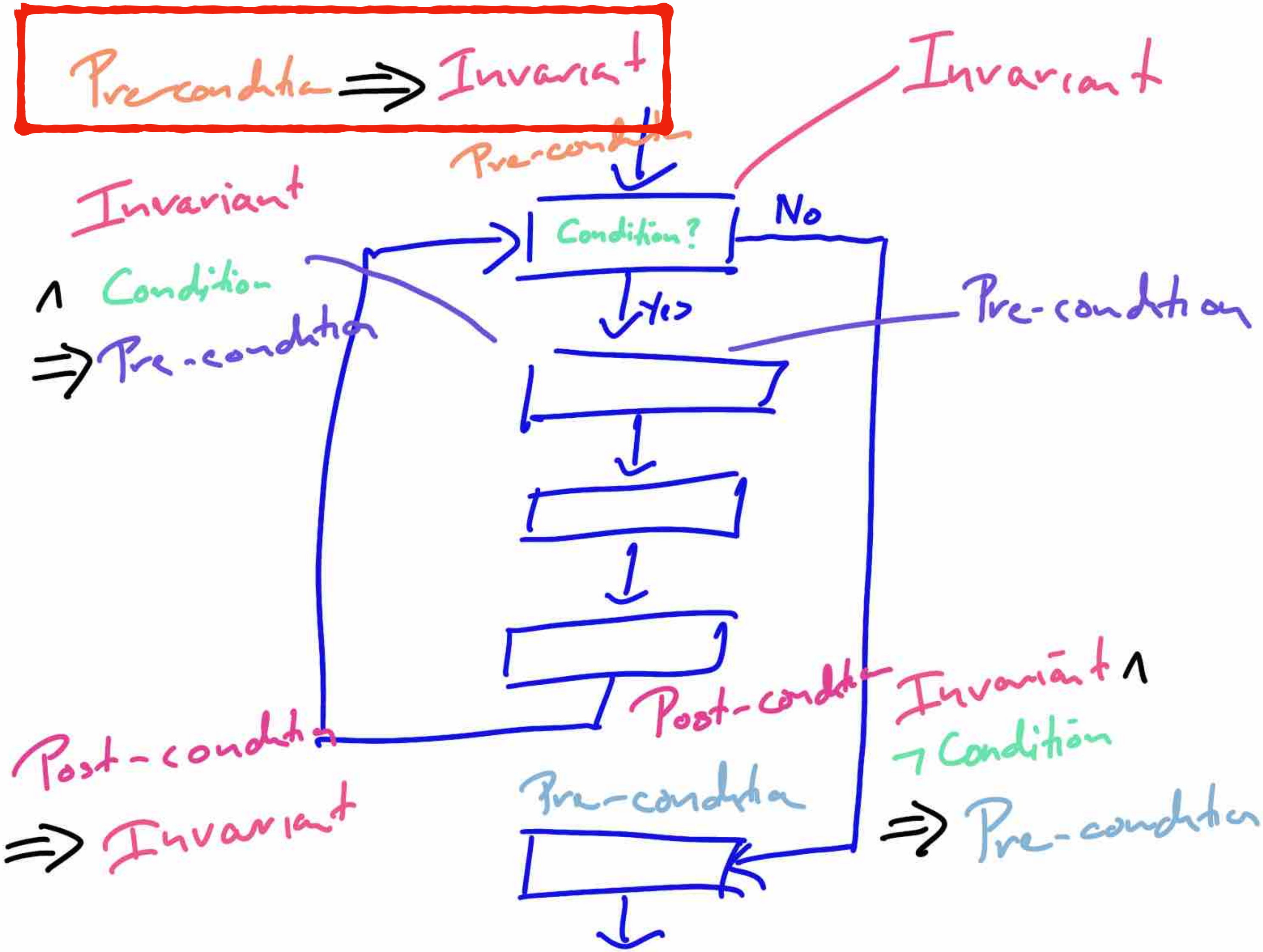
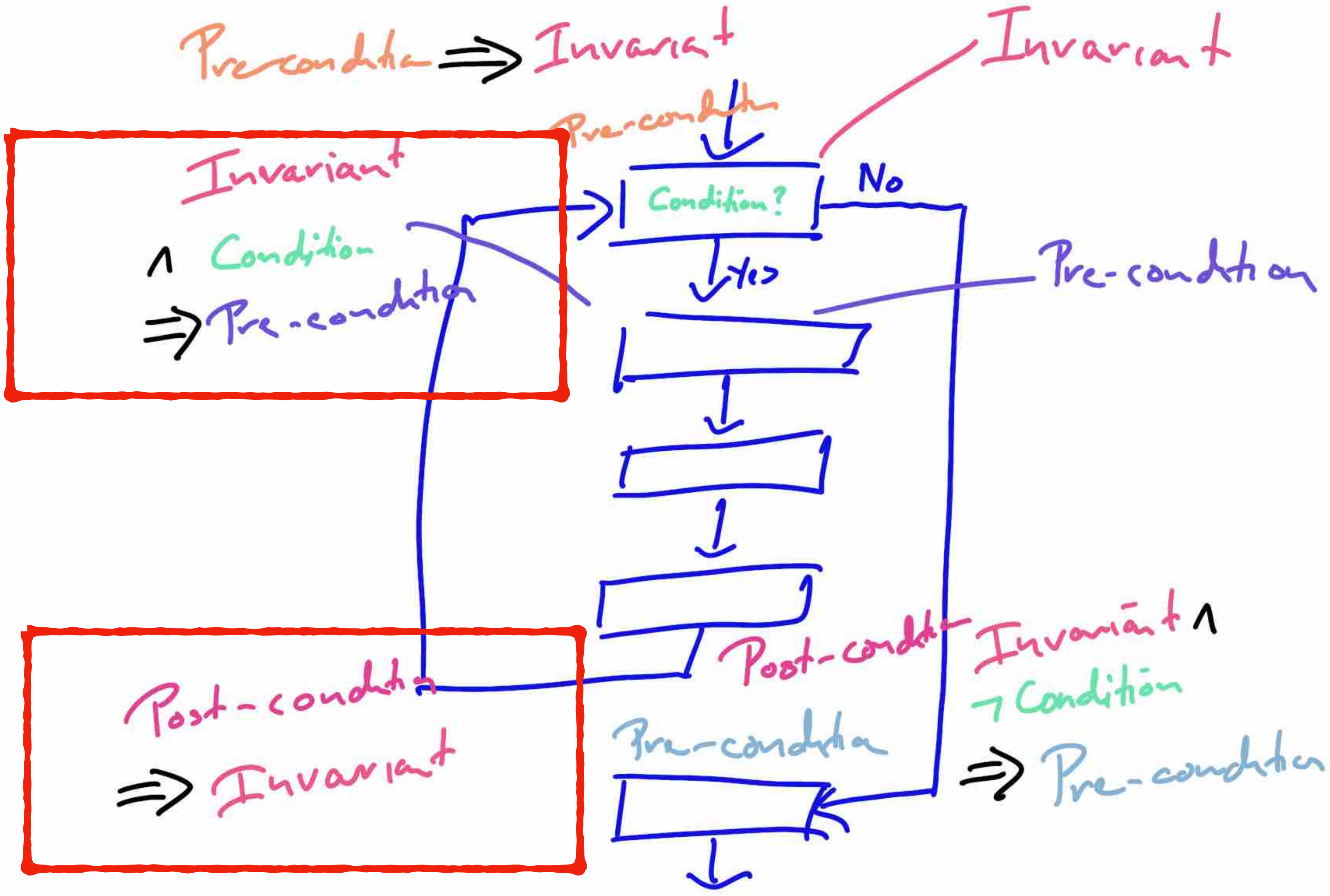
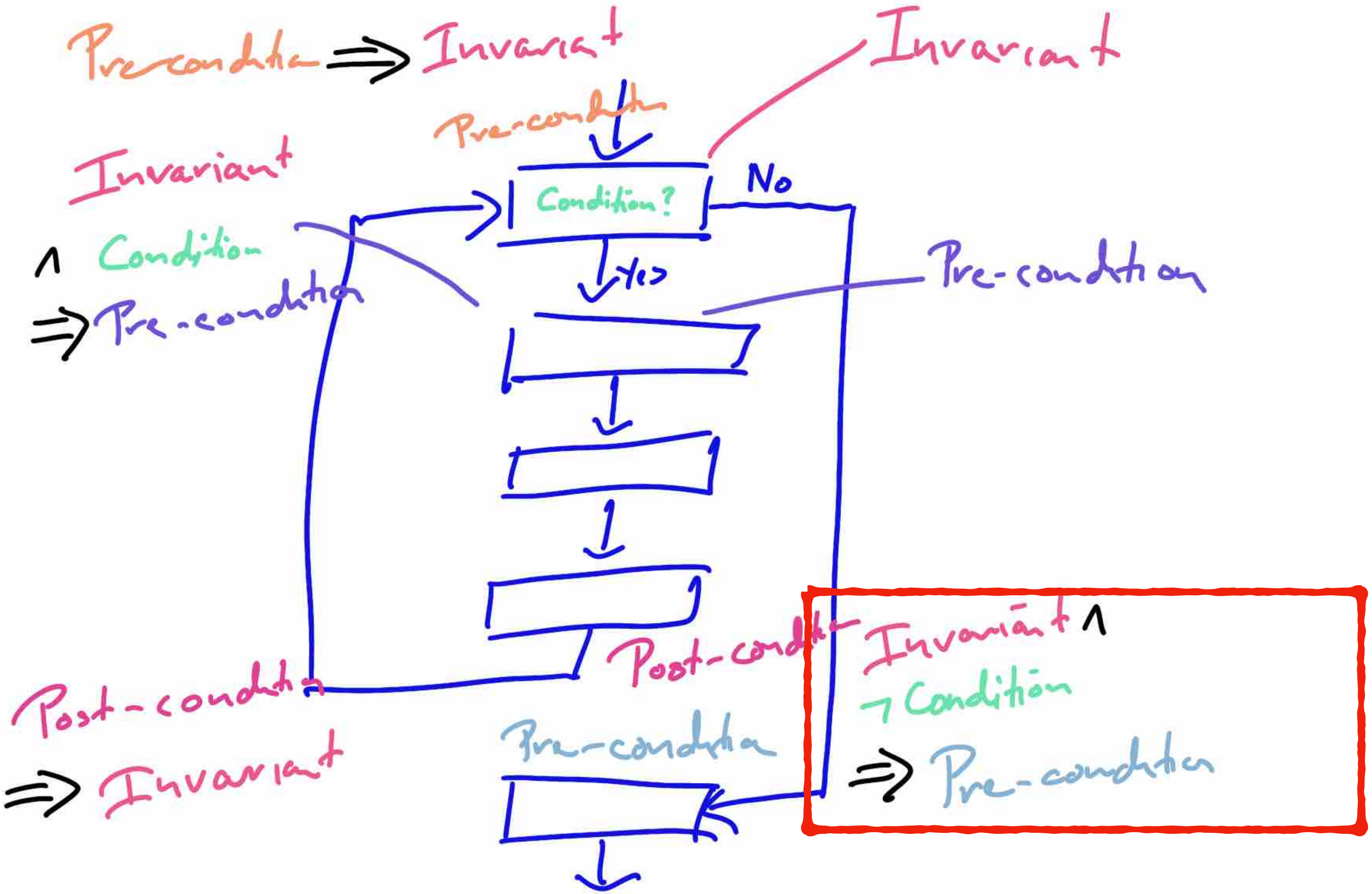
Precondition

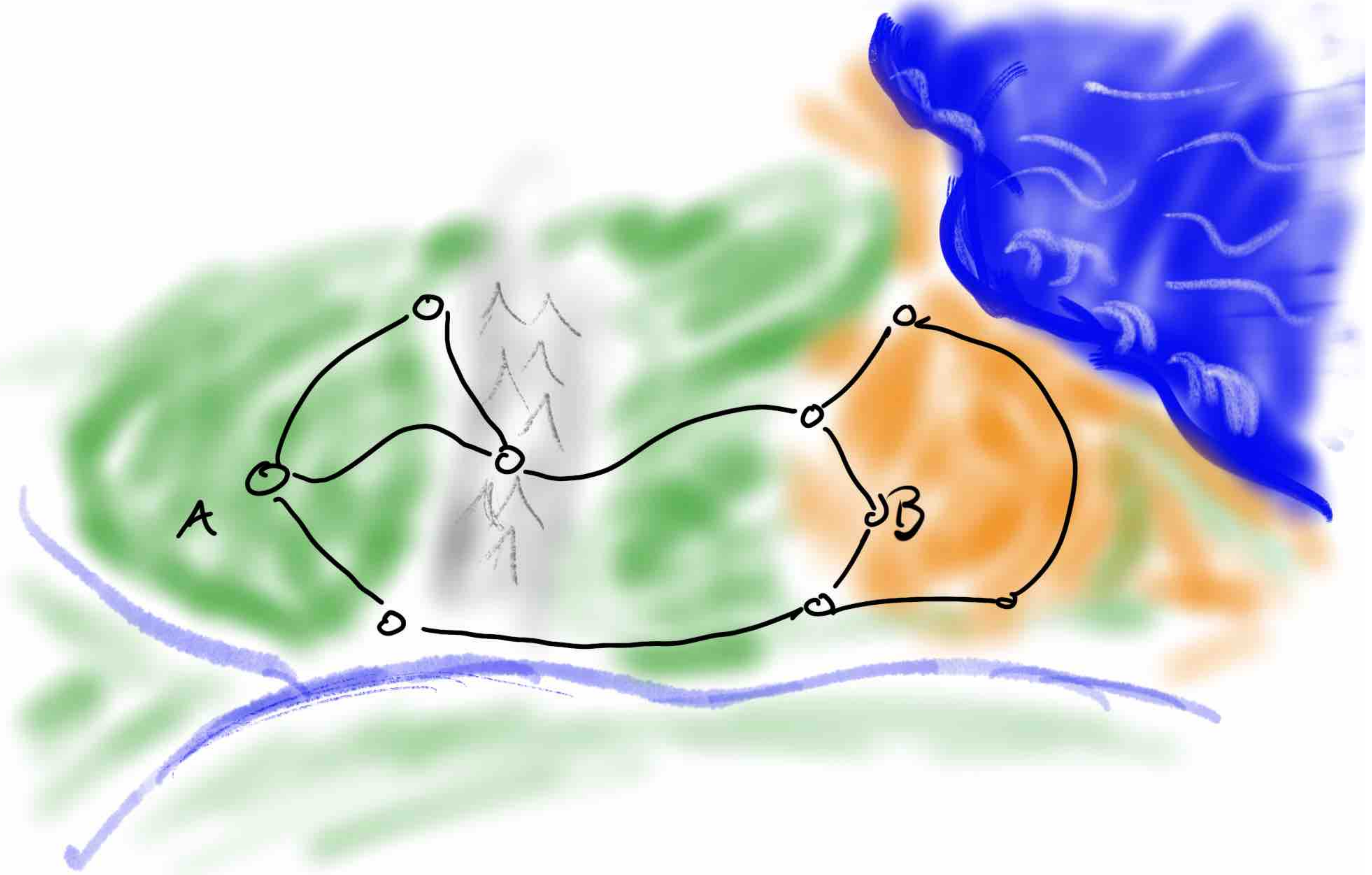

Pre-condition ⇒ Invariant
Invariant
Pre-condition
Invariant
Condition?
No
Yes
∧ Condition
⇒ Pre-condition
Pre-condition
Post-condition
Invariant ∧
¬ Condition
⇒ Pre-condition
Post-condition
⇒ Invariant
Pre-condition

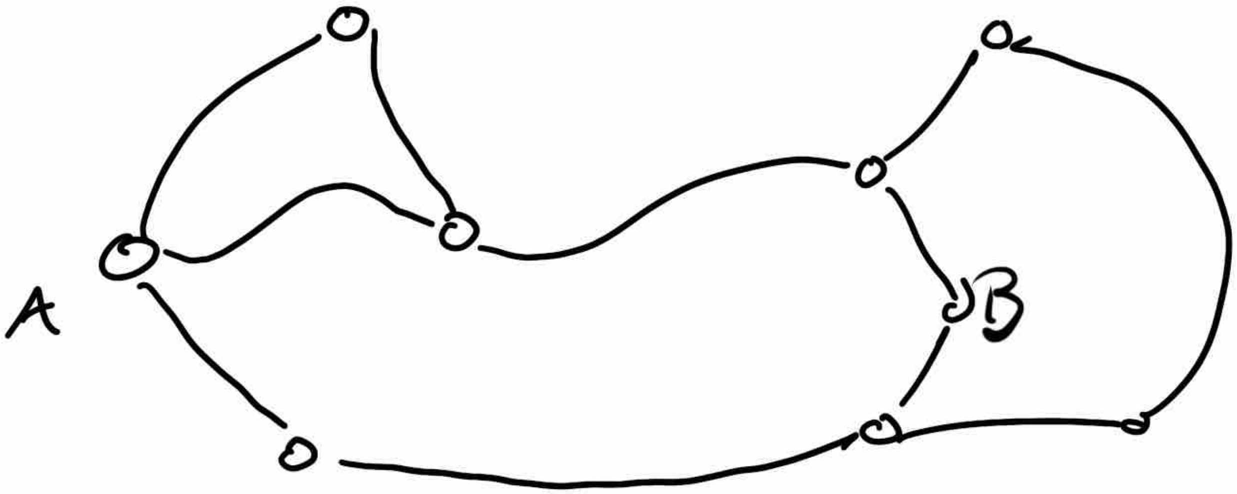

Pre-condition ⇒ Invariant
Invariant
Pre-condition
Invariant
∧ Condition
⇒ Pre-condition
Condition?
No
Yes
Pre-condition
Post-condition
Invariant ∧
¬ Condition
⇒ Pre-condition
Post-condition
⇒ Invariant
Pre-condition

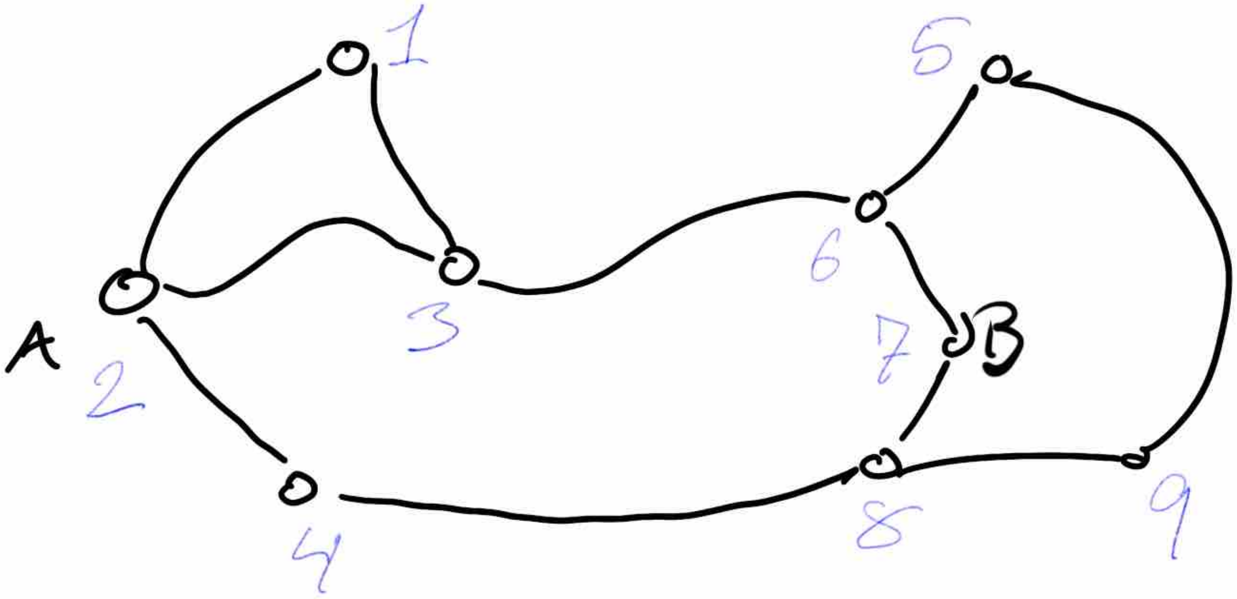

Precondition ⇒ Invariant
Invariant
Pre-condition
Condition?
No
Yes
Invariant
∧ Condition
⇒ Pre-condition
Pre-condition
Post-condition
Invariant ∧
¬ Condition
⇒ Pre-condition
Post-condition
⇒ Invariant
Pre-condition

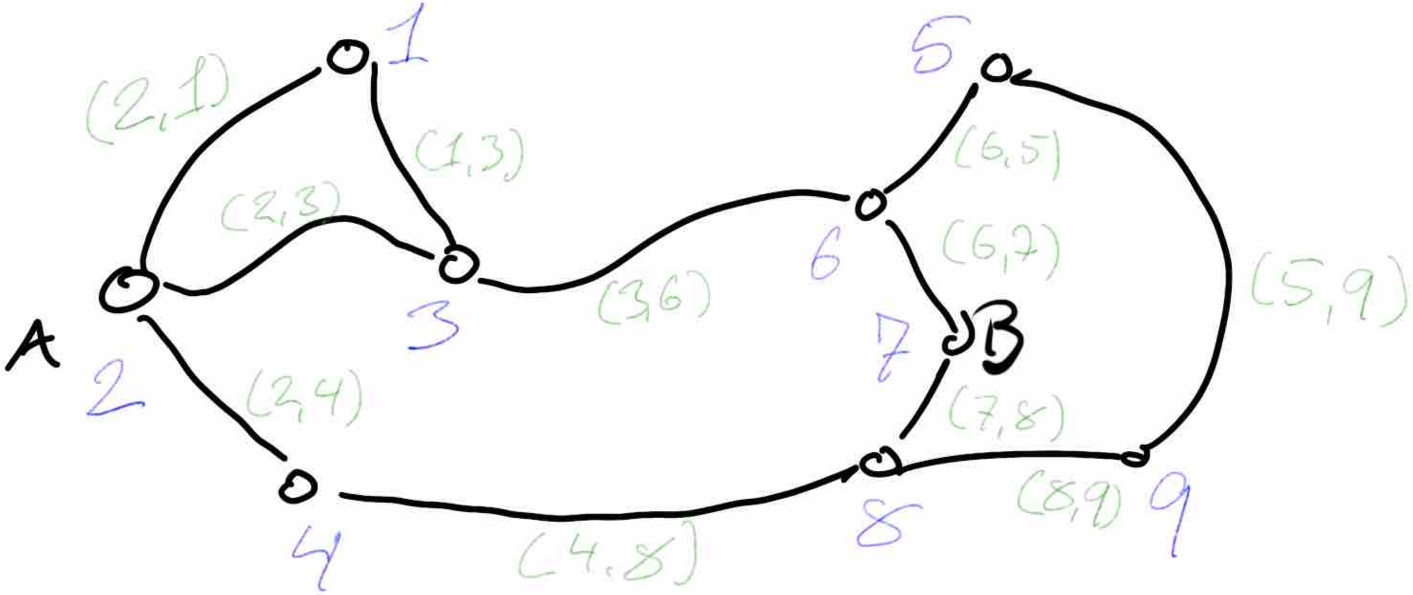

Precondition ⇒ Invariant
Invariant
Pre-condition
Invariant
∧ Condition
⇒ Pre-condition
Condition?
No
Yes
Pre-condition
Post-condition
Post-condition
⇒ Invariant
Pre-condition
Invariant ∧
¬ Condition
⇒ Pre-condition

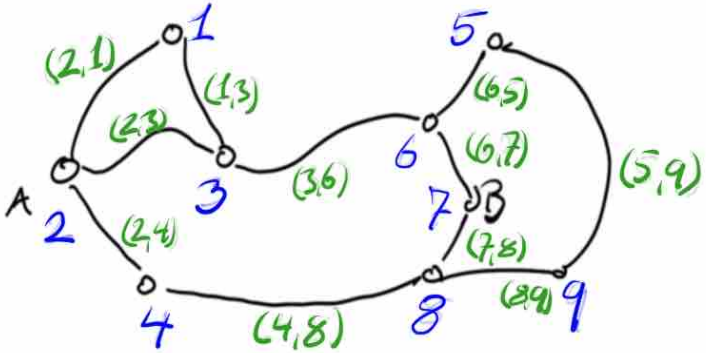
Example: are cities A and B connected?


A
B


A
B


1
5
6
3
A
2
7
B
8
9
4


1
(2,1)
(1,3)
5
(6,5)
(2,3)
6
(6,7)
(5,9)
A
2
3
(3,6)
7
B
(2,4)
(7,8)
4
8
(8,9)
9
(4,8)



	1	0
A	2	0
	3	0
	4	0
	5	0
	6	0
B	7	0
	8	0
	9	0

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

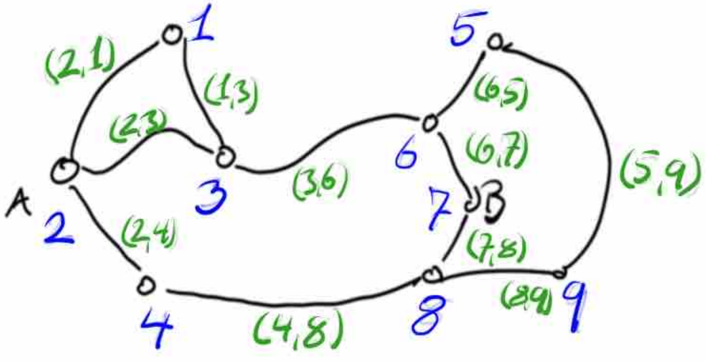
(6,5)

(6,7)

(7,8)

(8,9)

(5,9)



	1	0	→ 1
A	2	0	→ 2
	3	0	→ 3
	4	0	→ 4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

Seen

(2,1) Unseen

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

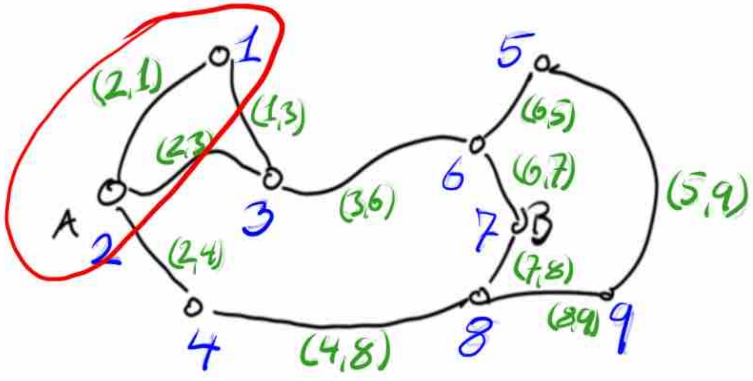
(6,5)

(6,7)

(7,8)

(8,9)

(5,9)



	1	0	→	1
A	2	0	↗	2
	3	0	→	3
	4	0	→	4
	5	0	→	5
	6	0	→	6
B	7	0	→	7
	8	0	→	8
	9	0	→	9

(2,1) Seen

(2,3) Unseen

(2,4)

(1,3)

(3,6)

(4,8)

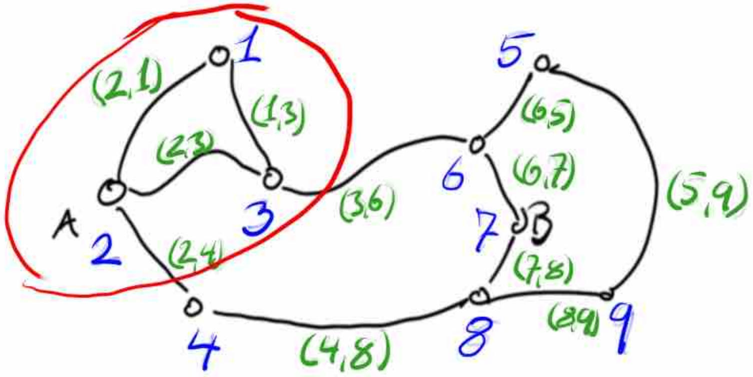
(6,5)

(6,7)

(7,8)

(8,9)

(5,9)



	1	0	→ 1
A	2	0	2
	3	0	3
	4	0	→ 4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)

(2,3) Seen

(2,4) Unseen

(1,3)

(3,6)

(4,8)

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(7,8)

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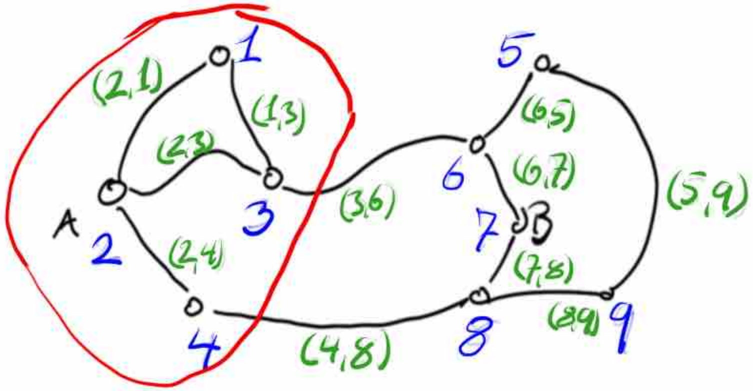

	1	0	→ 1
A	2	0	2
	3	0	3
	4	0	4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)

(2,3)

(2,4) Seen

(1,3) Unseen

(3,6)

(4,8)

(6,5)

(6,7)

(7,8)

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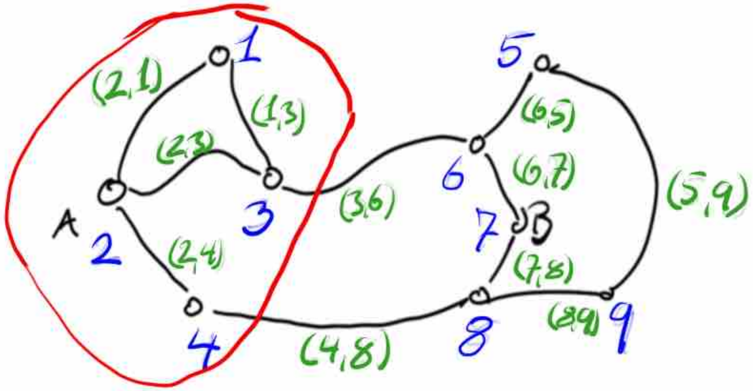

	1	0	→ 1
A	2	0	2
	3	0	3
	4	0	4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)

(2,3)

(2,4)

(1,3) Seen

(3,6) Unseen

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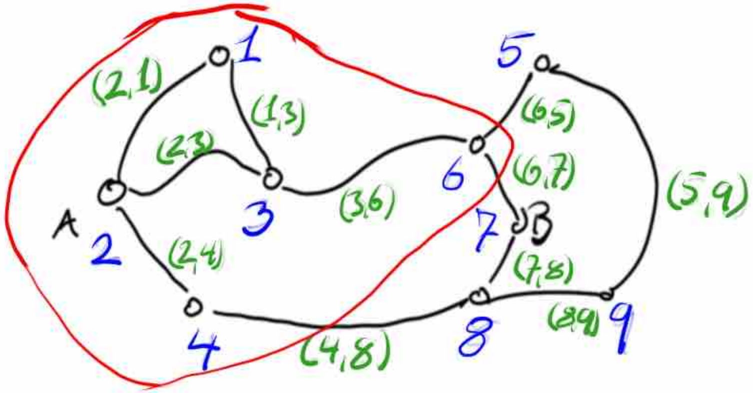

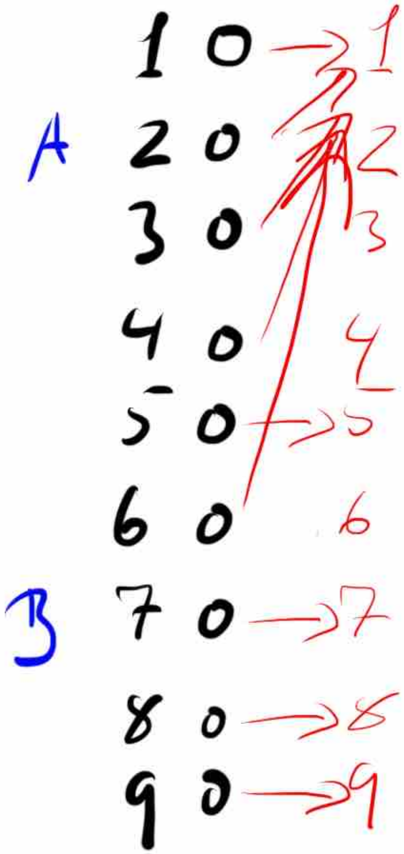

(2,1)

(2,3)

(2,4)

(1,3)

(3,6) Seen

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(6,5)

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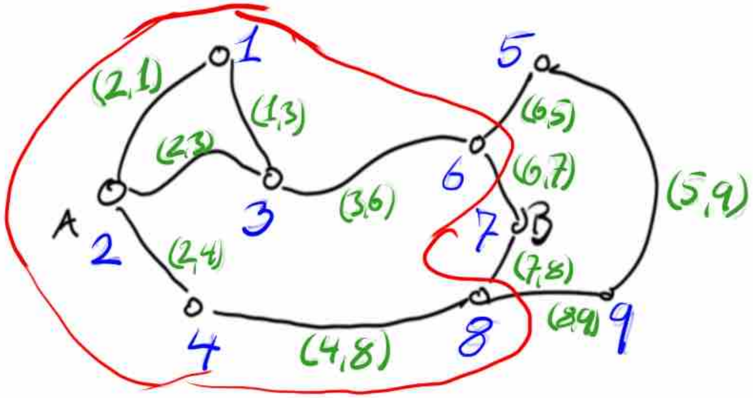

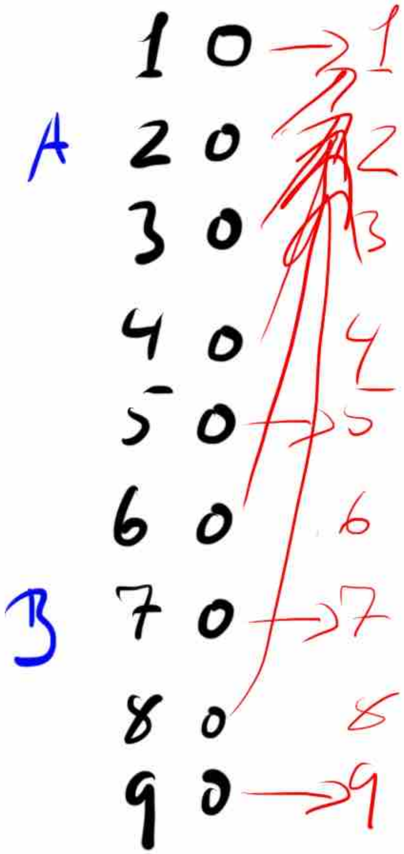

(2,1)

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(4,8) Seen

(6,5) Unseen

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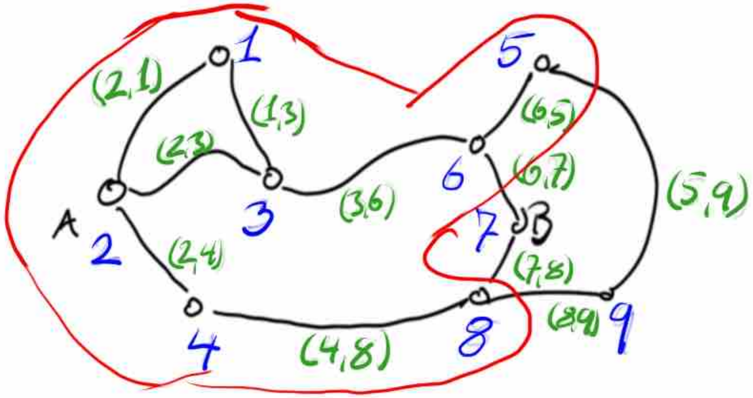

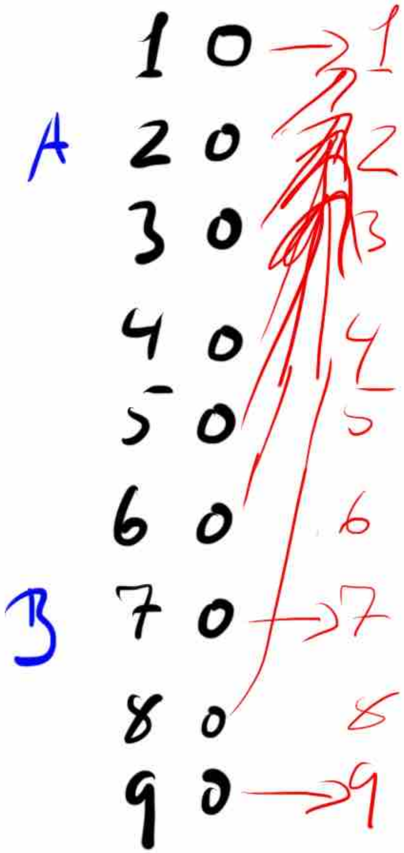

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

(6,5) Seen

(6,7) Unseen

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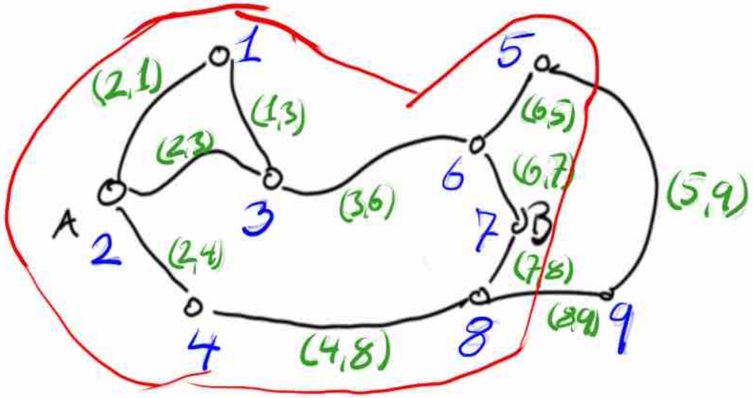
(5,9)



	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

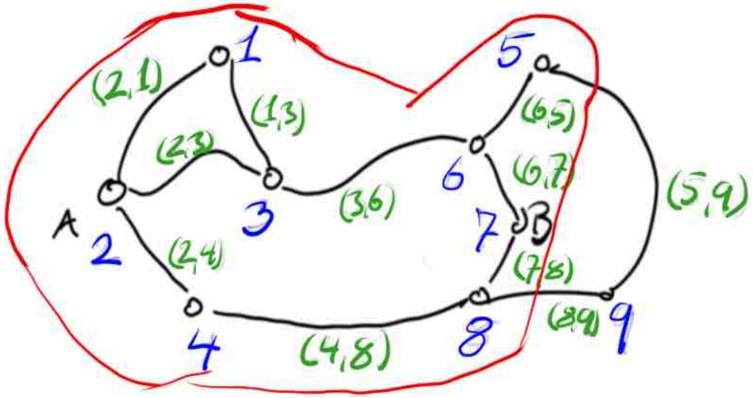
(6,5)

(6,7) Seen

(7,8) Unseen

(8,9)

(5,9)



	1	0	→ 1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	→ 9

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

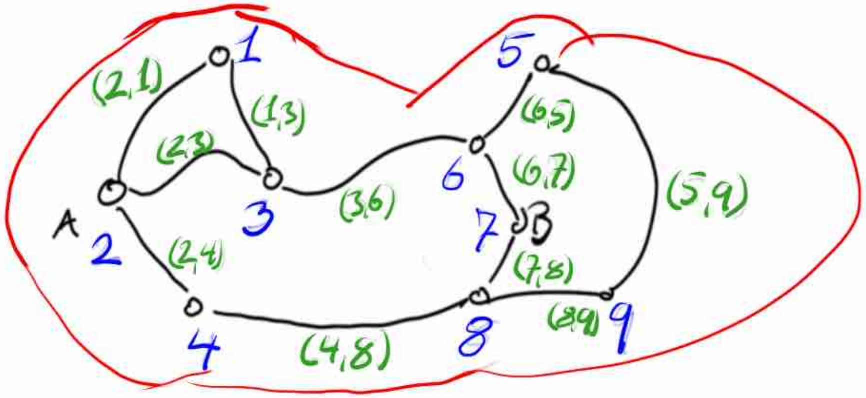
(6,5)

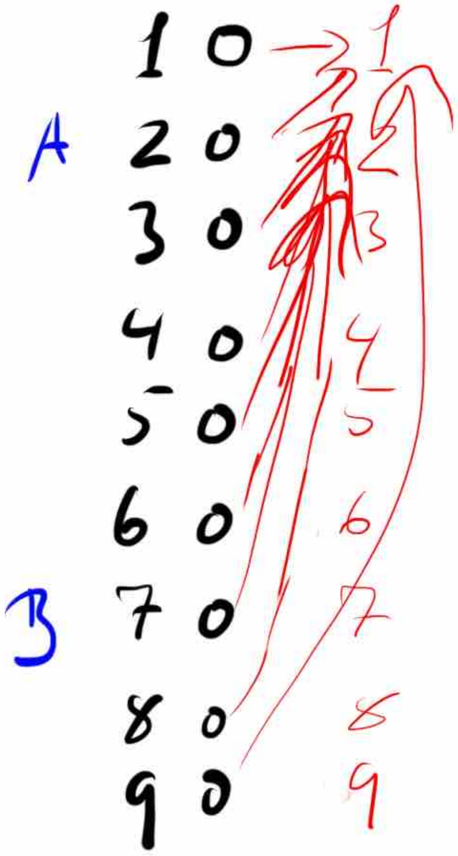
(6,7)

(7,8) Seen

(8,9) Unseen

(5,9)





	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

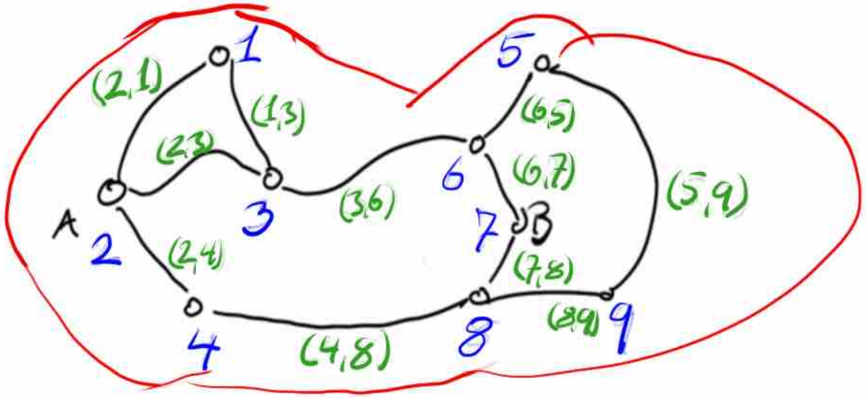
(6,5)

(6,7)

(7,8)

(8,9) Seen

(5,9) Unseen



	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

(6,5)

(6,7)

(7,8)

(8,9)

(5,9) Seen

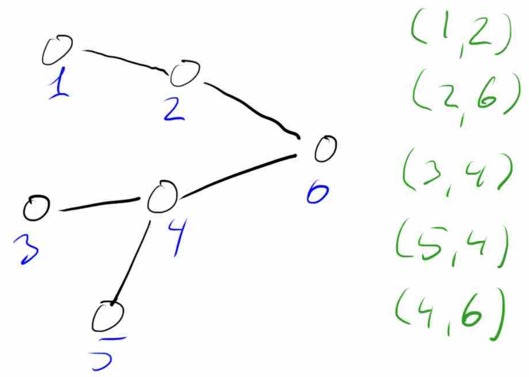
Unseen

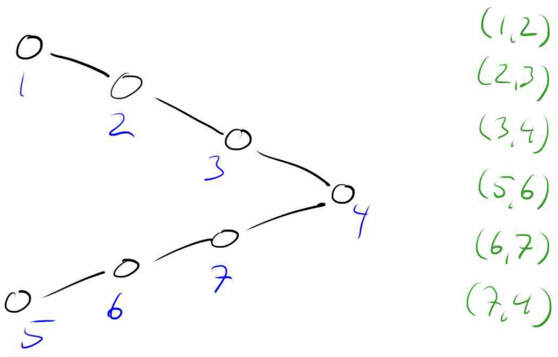
Intermezzo

- How would you represent the components in Python?
- How would you formalise the loop invariant to take into account the representation of the components?

Intermezzo

- Run the algorithm on these two graphs:



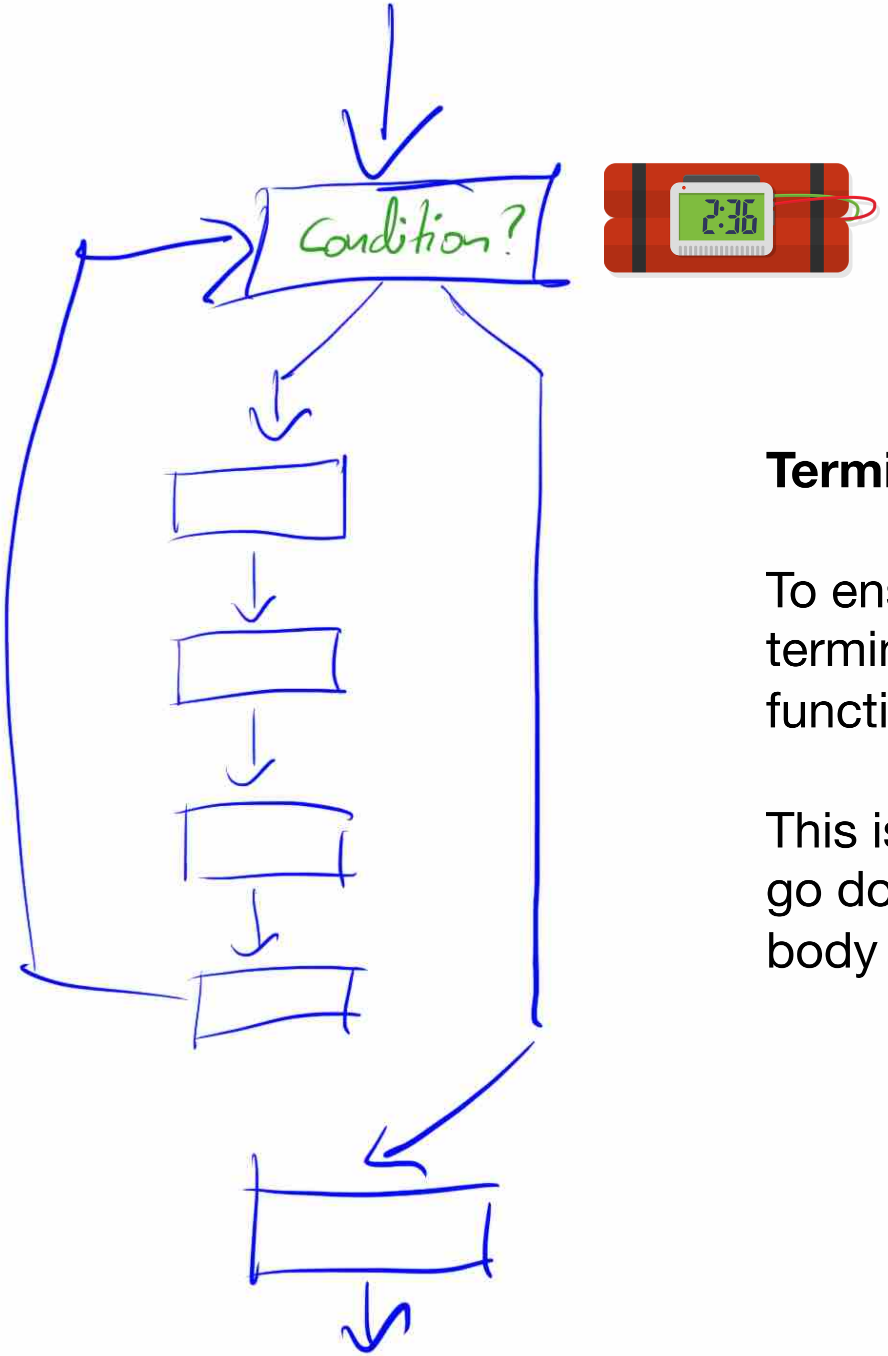


Correctness

How do we prove correctness?

- We prove that all pre- and post-conditions are satisfied through the steps in the algorithm.
- We prove that the post-condition of the last step implies that the overall problem is solved.
- *Correctness is just a special case of post-conditions*

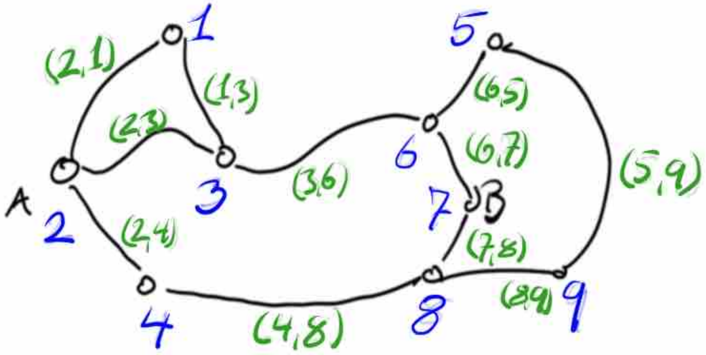
Termination



Termination functions:

To ensure that our algorithm terminations, we add a "termination" function to each loop.

This is a count-down that should go down each time we execute the body of the loop.



	1	0	→ 1
A	2	0	→ 2
	3	0	→ 3
	4	0	→ 4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

Seen

(2,1) Unseen

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

(6,5)

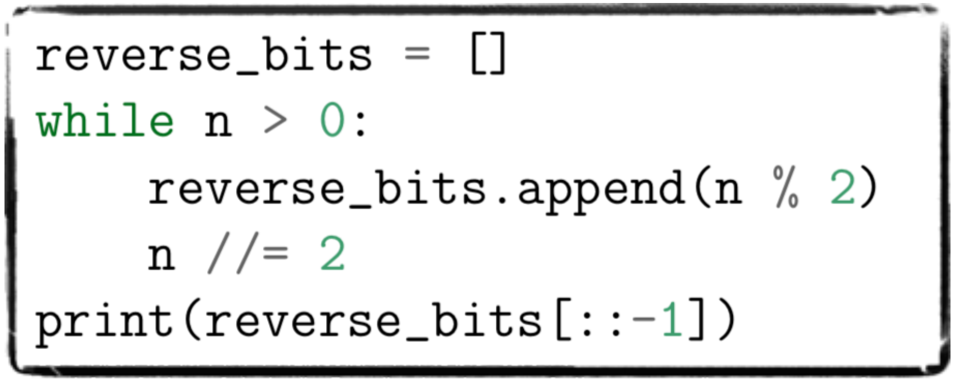
(6,7)

(7,8)

(8,9)

(5,9)

Get the binary representation of a number *n*



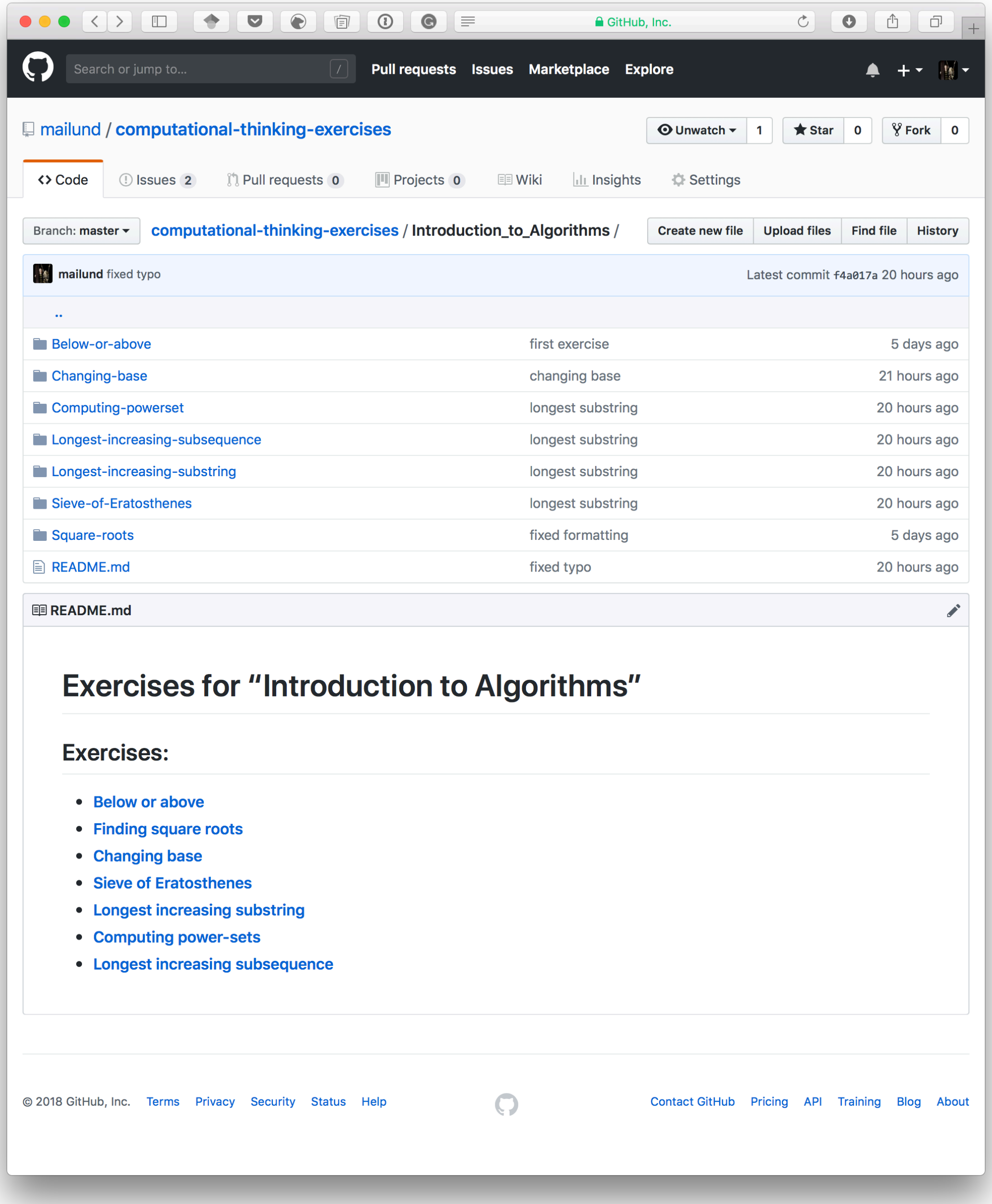
```
reverse_bits = []
while n > 0:
    reverse_bits.append(n % 2)
    n //= 2
print(reverse_bits[::-1])
```

Termination function:

$t(n) = n$

Thats it!

Now it is time to do the exercises to test that you now know how to construct algorithms



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mailund fixed typo — Latest commit f4a017a 20 hours ago

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Below-or-above	first exercise	5 days ago
Changing-base	changing base	21 hours ago
Computing-powerset	longest substring	20 hours ago
Longest-increasing-subsequence	longest substring	20 hours ago
Longest-increasing-substring	longest substring	20 hours ago
Sieve-of-Eratosthenes	longest substring	20 hours ago
Square-roots	fixed formatting	5 days ago
README.md	fixed typo	20 hours ago

README.md

Exercises for “Introduction to Algorithms”

Exercises:

- Below or above
- Finding square roots
- Changing base
- Sieve of Eratosthenes
- Longest increasing substring
- Computing power-sets
- Longest increasing subsequence