Performance evaluation of multi-passive solar applications of a non air-conditioned building

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Abstract: This paper describes the performance evaluation of solar passive cooling techniques such as solar shading insulation of building components and air exchange rate. Thermal performance of the non air-conditioned building is predicted by analysing a one-dimensional numerical model. Numerical calculation has been made corresponding to three different climates, viz. composite, hot and dry and warm and humid.

A decrease in the indoor temperature by about 2.5°C to 4.5°C is noticed for solar shading. Results modified with insulation and controlled air exchange rate show a further decrease of 4.4°C to 6.8°C in room temperature. For a shaded and selectively ventilated building, 2 cm of thermal insulation gives the same result as 8-10 cm of insulation for a conventional building, which is non-shaded and not ventilated. The analysis suggests that solar shading along with other two options is quite useful to development of passive cooling system to maintain indoor room air temperature lower than the conventional building.

Keywords: solar shading; insulation; ACH; composite; hot and dry; warm and humid.

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1 Introduction

Solar passive cooling techniques are significantly relevant to thermal cooling of buildings especially in Indian scenario owing to their cost effectiveness and natural aesthetic appearance. This can be explained reasonably considering the degree of 'coolness'. During night, the ambient temperature is low and if enough ventilation is carried out during these hours, the components of building will store a lot of 'coolness'. During daytime, although the building is shaded from direct radiation, still it is exposed to higher ambient temperature and diffuse radiation. Hence, the 'coolness' of the building will escape into the outside environment. Thus for checking it, different passive techniques should be considered. The discussion regarding such techniques has been widely addressed in a number of experimental and numerical studies (Duffie and Beckman, 1991; Sodha et al., 1986; Sodha, Garg and Sharma, 1989; Sodha, Kaur and Sawhney, 1992; Srivastava et al., 1984). The key passive techniques used for thermal cooling, such as shading, evaporative cooling, movable and immovable insulation, vegetation etc. are becoming increasingly important. However, passive cooling techniques are not as standardised as solar heating methods and are more or less applied independently in the building.

Thermal passive cooling may be treated as an application of multi-cooling techniques rather than single passive cooling technique. The state of art of solar passive cooling techniques has been given in Givoni (1991). Some of the known techniques for passive cooling, viz. Sky-therm (Prasad et al., 1979; Yellot and Hay, 1969), insulated roof and wall (Kumar et al. 1989; Shariah et al. 1997; Asan, 1998), roof pond (Sodha et al., 1978; Sodha et al., 1980), earth–air tunnel (Sodha et al., 1989; Mihalakakou et al., 1995) and ventilation (Hamdy and Firky, 1998) are applicable in all the three climates mentioned. These studies tend to emphasise on one passive cooling option. This however limits the comparative evaluation of several passive-cooling techniques. In this study, multi-passive cooling techniques are implemented and evaluated in residential buildings in three ways, viz. solar shading of building block, insulation of building components and ventilation. It is seen logistically that the application of solar shading combined with a proper insulation and window infiltration rate reduces the inside room temperature of the building to a significant degree.

Various techniques of solar passive cooling have been mentioned in Passive Building Design Handbook (Bansal et al., 1994) and most recently by Salaini (1998). Bansal et al. (1994) proposed that Hot and Dry climatic zone requires solar shading throughout the year and composite climatic zone requires shading from February to October. However, Warm and Humid climate (Chennai, India) requires complete year of solar shading with necessary ventilation. According to Bansal et al. (1994), shading with tree reduces ambient temperature near outer wall by 2°C to 2.5°C. An important extension of this study is to estimate insulation thickness required for building facades in shading. The supporting wall consists of four layers namely 15mm plaster, 23 cm brickwork, 15 mm of plaster and expanded polystyrene insulation on the external surface. In the simulation program, different thickness of insulations, 2-10cm, has been studied. The roof is four layered, 15mm plaster on the innermost side, 22cm of RCC, 8 cm of mudphaska (a mixture of soil and rice husk/wheat husk and acts as a Thermal insulator) and 2.5 cm of broken tiles on the outer most surface. Thermo physical properties of the materials used in simulation are given in Table 1. A room 10m × 5m × 3m with window of size 1m × 1m × 1m on south wall is considered for the analysis. Window has hangover of appropriate size so that no summer radiation can penetrate through it.

Computed results are obtained for different climatic conditions corresponding to three different cities of India, namely New Delhi, Jodhpur, and Chennai. Three climatic zones viz. Composite, Hot and Dry, and Hot and Humid give a wide overview of overall climate of India. Months of June and May are considered as the hottest months according to meteorological data. Therefore, analysis is done for the months of June (New Delhi) and May (Jodhpur and Chennai) only. This paper presents a performance analysis of single story building incorporating all three passive cooling techniques viz. shading, window-controlled ventilation and insulation.

Table 1 Thermophysical properties of building materials

Components	*Thickness* (cm)	*Density* (kg/m^3)	*Conductivity* (W/m-K)	*Specific Heat* (J/kg-K))
Walls (bricks)	23	1820	0.81	880
Roof (R.C.C.)	22	2280	1.58	880
Plaster	1.5	1762	0.72	840
Mud phuska	8	1622	0.52	880
Broken tiles	2.5	1820	0.81	880
Floor (soil)		1958	1.21	840
Expanded polystyrene	–	34	0.035	1340

2 Methodology

The analysis is based on the time-dependent analysis of heat flow through various building components. The heat flux associated with each mode is computed for all possible building components. The associated heat fluxes are determined from the nature of heat transfer. Thus, convective heat transfer through ventilation, conduction for walls and roof is precisely specified. The governing heat fluxes are programmed on an hourly heat balance basis for each surface of different zones. However, natural solar shading is considered within the purview of diffused radiation intercepting building facades. The major envelope characteristics that are studied through parametric variations in the simulation are air exchange rate between internal and external climates.

3 Numerical analysis

Fourier series is used to transform the periodic nature of ambient temperature and solar radiation into algebraic form. The individual heat flux components are computed and net summation for heat fluxes equals the rate of increase in internal energy of the room air. The energy balance equation obtained consists of Fourier series of harmonics for each variable.

All time-varying variables, such as solar radiation and ambient temperature, can be written in terms of Fourier series, e.g.

$$f(t)=\sum_{n=-\infty}^{n+\infty} f_n \,\mathrm{e}^{(in\omega t)}. \tag{1}$$

Thus, we can represent individual heat fluxes numerically as described below.

- Heat flow transmitted through the walls and roof may be expressed as

$$\dot{Q}_{wr}=A\sum_{y=-\infty}^{y=+\infty}\frac{S_y\left(T_{x,y}+\dfrac{\alpha_i I(t)_{iwn}}{h_i}\right)-\left(T_{ay}+\dfrac{\alpha_o I(t)_{oy}}{h_o}\right)}{Q_y}e^{(iy\omega t)}+A\sum_{y=-\infty}^{y=+\infty}\left(\alpha_i I(t)_{i\omega n}\right)e^{(iy\omega t)} \tag{2}$$

  where, S_y and Q_y are complex variables and depend upon the thermo-physical properties of walls and roof material. The calculations can be seen in the Appendix.

- Heat flux through the floor is given as

$$\dot{Q}_f=A_F\sum_{y=-\infty}^{y=+\infty}\frac{S_y\left(T_{x,y}+\dfrac{\alpha_i I(t)_{ifn}}{h_i}\right)}{Q_y}e^{(iy\omega t)}+A_F\sum_{y=-\infty}^{y=+\infty}\left(\alpha_i I_{ifn}\right)e^{(iy\omega t)}. \tag{3}$$

- The heat gain due to infiltration into room is calculated as:

$$\dot{Q}_I=C_{inf}\sum_{y=-\infty}^{y=+\infty}\left(T_{ay}-T_{x,y}\right)\mathrm{e}^{(iy\omega t)}. \tag{4}$$

- And finally, the equation for ventilation is given as

$$\begin{aligned}\dot{Q}_V&=\sum_{y=-\infty}^{y=+\infty}C_v\left(T_{ay}-T_{x,y}\right)\mathrm{e}^{(iy\omega t)}\\&=\sum_{z=-\infty}^{z=+\infty}\sum_{y=-\infty}^{y=+\infty}C_{vy}\left(T_{ay}-T_{x,y}\right)\mathrm{e}^{\left[i(y+z)\omega t\right]},\end{aligned} \tag{5}$$

  where we have assumed that the ventilation term C_v is also time dependent and the same is expressed in terms of Fourier coefficients as

$$C_v=\sum_{z=-\infty}^{z=+\infty}C_{vy}\mathrm{e}^{(iz\omega t)}.$$

These sets of algebraic equations are solved for the unknown room air temperature. The combined equation requires the summation of net heat fluxes as given below in the form of an energy balance equation:

$$M_x \frac{\mathrm{d}}{\mathrm{d}t}\left[\sum_{y=-\infty}^{y=+\infty} T_{x,y} \mathrm{e}^{(iy\omega t)}\right] = \sum_{j=2}^{5} \dot{Q}_j \ , \tag{6}$$

where, M_x is the thermal mass of room air and j corresponds to equations (2)–(5). In these equations, only harmonics from $y = -6$ to $y = +6$ are considered because even after neglecting higher order harmonics, it ensures the minimisation of error to fairly accurate (Sodha et al., 1986). Once we compare coefficients of different harmonic frequencies, equation (6) yields 13 equations that can be represented in the matrix form. The matrix representation for the same is as follows:

$$[X]_{13\times13}\ [T_{x,y}]_{13\times1} = [Y]_{13\times1}$$

or,

$$[T_{x,y}]_{13\times1} = [X]^{-1}{}_{13\times13}\ [Y]_{13\times1.} \tag{7}$$

On solving the matrix, equation (7), the amplitude of temperature $[T_{x,y}]$ determines the different harmonic components of the indoor air temperature. Once amplitude of temperature is computed, hourly variation of indoor air temperature is determined accurately from equation (8). This is written as

$$T\,(t) = T_{\mathrm{o}} + \sum_{y=-6}^{y=+6} T_{x,y}\ \mathrm{e}^{\,(iywt)} \ . \tag{8}$$

It is worth mentioning the numerical values for the parameters used in calculations. It primarily depends on heat transfer coefficient, absorption coefficient for wall and roof. Heat transfer coefficient (W/m^2K) takes following values for different building facades.

1. Wall: Outer surface h_o: 20.00

   Inner surface h_i: 11.00

   Roof: Outer surface h_o: 23.00

   Inner surface hi: 6.13

   Floor: Inside Surface hi: 9.26

2. Absorption Coefficients α (Dimensionless) takes following values:

   Wall: Outer surface α_o: 0.60

   Inner surface α_i: 0.08

   Roof: Outer surface α_o: 0.70

   Inner surface α_i: 0.08

   Floor: Inside Surface α_i: 0.60

4 Presentation and discussion of results

Results are shown considering numerical calculations, which involve the combined effect of shading, thickness of insulation and window ventilation. A typical set of climates is accounted for.

Figure 1 shows the effect of shading for composite climate. The relevant data is obtained from New Delhi climatic conditions. In Figure 1(a) the variations in shaded and unshaded indoor air temperature is shown, while the ventilation effects are not considered. Ventilation rate as mentioned in this manuscript needs some clarification: It is given for two periods. First period is for six hours, from midnight to early morning i. e. 0:00 to 6:00 A.M. when ambient temperature is low and ventilation stores 'coldness' in the building material. The next period is for 18 hours, from 6:00A.M. to midnight, the period in which ambient temperature is comparatively high and ventilation is kept low. The statement (ACH = 1.0, 0.5) means that during first period of 6 hours, the ventilation rate is 1.0 exchanges per hour and during and during next period of 18 hours, it is 0.5 exchanges per hour.

A typical Delhi summer day temperature is varying over a range of 9°C, i.e. (29.6°C – 38.5°C). The corresponding variation in indoor air temperature for unshaded building is 3°C, fluctuating from 34°C to 37°C. Results depict the effect of shading in reducing the fluctuation in indoor air temperature by 1.8°C i.e. from 32.6°C to 34.4°C.

Figure 1 **(a)** Hourly room air temperature variation of shaded and unshaded building for ACH = (1.0, 1.0) and no insulation, at New Delhi for the month of June

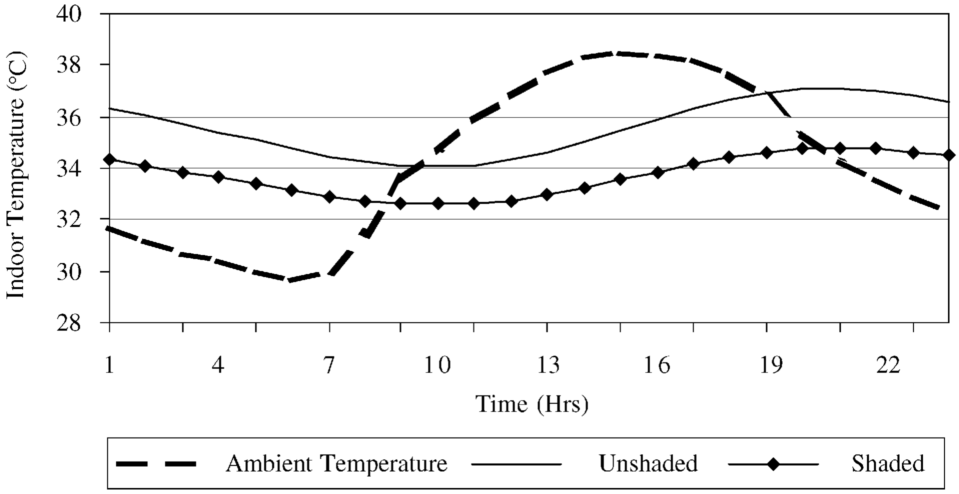


Figure 1(b) shows the effect of variation of air exchange rate on indoor air temperature. With increased degree of coolness by increasing ACH, indoor air temperature decreased within 1°C to1.8°C for shaded and unshaded building envelope. However, insulation minimizes heat losses and thus captures indoor coolness, which is effected by increased air exchange rate. The results to this effect are shown in Figure 1(c). A layer of 10-cm insulation is necessary for unshaded case to minimize the indoor temperature. On the contrary, we achieved almost same indoor temperature just by providing a 2-cm of insulation for shaded building. This also adds to cost effectiveness of the building.

Finally, applying the combined effect of passive applications, the overall indoor temperature is decreased very significantly as shown in Figure 1(d). A comparison shows that maximum drop in indoor air temperature is about 4.3°C. Results also show a comparative fluctuation of 1.0°C (31.9°C to 32.9°C) temperature in relation to 3°C in unshaded indoor temperature. It can be seen clearly that the passive room temperature was nearly constant at about 32°C for the shaded building even when the outdoor temperature went up to 38.5°C.

Figure 1 **(b)** Variation of room air temperature for different ACH and no insulation for shaded and unshaded building, at New Delhi

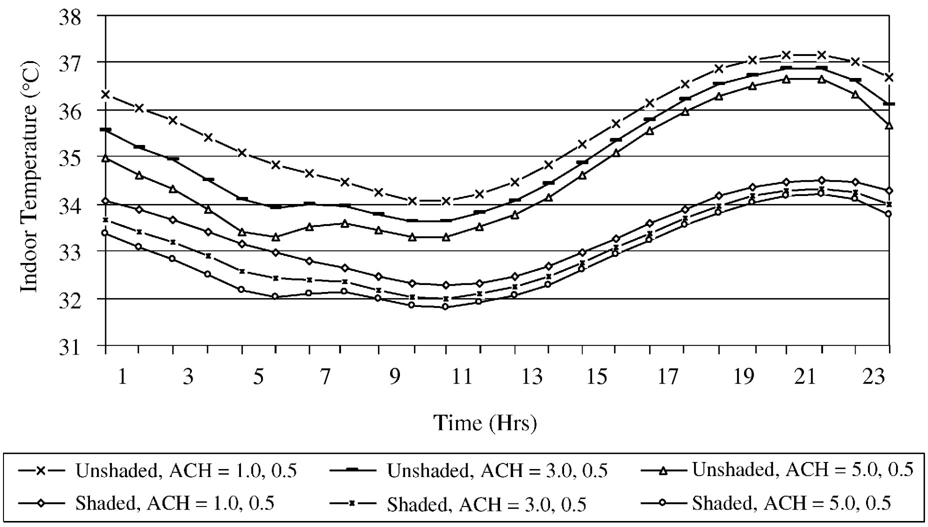


Figure 1 **(c)** Variation of room air temperature for different thicknesses of insulation and fixed ACH (5.0, 0.5) for shaded and unshaded building, at New Delhi

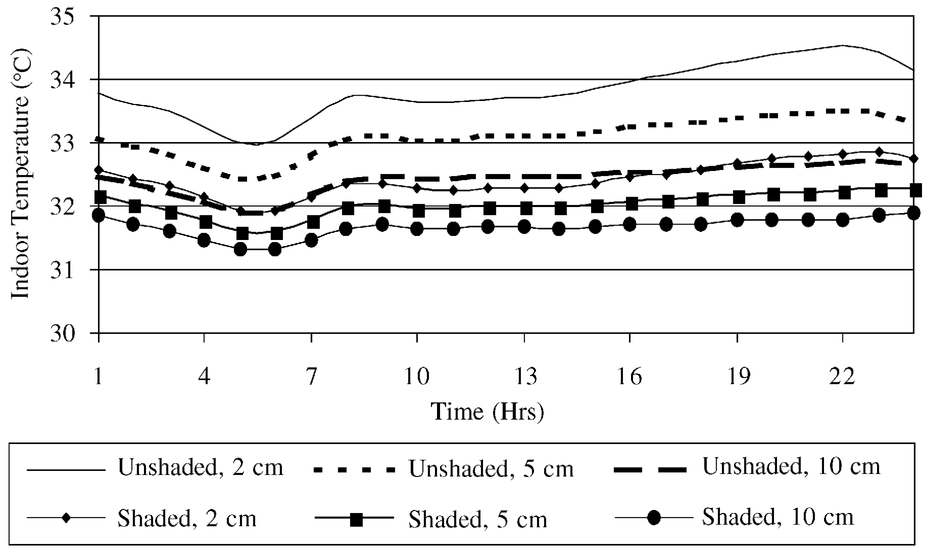


Figure 1 **(d)** Comparison of a Passive building, (implemented with all the three passive techniques: shaded, ACH 5.0, 0.5 and 2cm of insulation) with a conventional building, at New Delhi, for the month of June

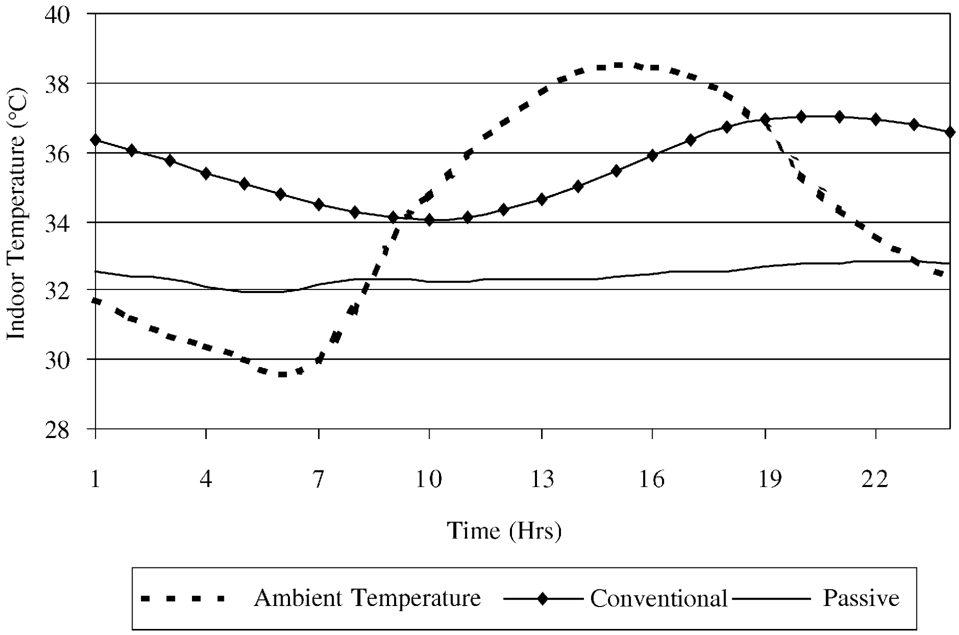


Figure 2 illustrates the effect of passive cooling applications for Jodhpur. The results show similar trend as observed for Delhi conditions. However, a greater drop in ambient temperature of about 4.5°C is noticed due to shading as shown in Figure 2(a). The indoor air temperature is varying by only 2.8°C for shaded building facades. Jodhpur is a desert station where ambient temperature varies from 27.1°C to 40.4°C *i.e.* a change of about 13.3°C. This can be checked by the application of thermal insulation, the results of which are shown in Figure 2(b). It is observed that insulation thickness of 15-cm was inappropriate for unshaded building whereas, a mere 2-cm of insulation thickness with solar shading could effectively minimize the passive room temperature. Room temperature decreased with increasing insulation thickness. Multi-passive techniques gave the most effective results as shown in Figure 2(c), where the room temperature decreased by about 6.8°C due to solar shading combined with insulation. A steady temperature varying from 30.5°C to 31.9°C is being achieved. However, unshaded building indoor air temperature showed a significant fluctuation varying from 34.89°C to 38.41°C.

Figure 2 **(a)** Hourly room air temperature variation at Jodhpur with ACH = (1.0, 1.0) without any wall/roof insulation, for the month of May

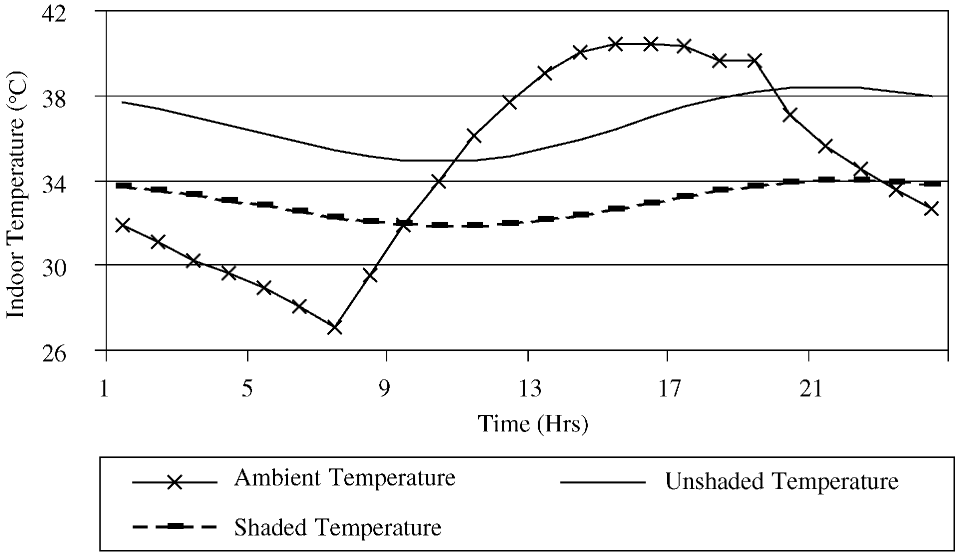


Figure 2 **(b)** Hourly room air temperature variation at Jodhpur with fixed ACH = (5.0, 0.5) and varying insulation on walls/roof

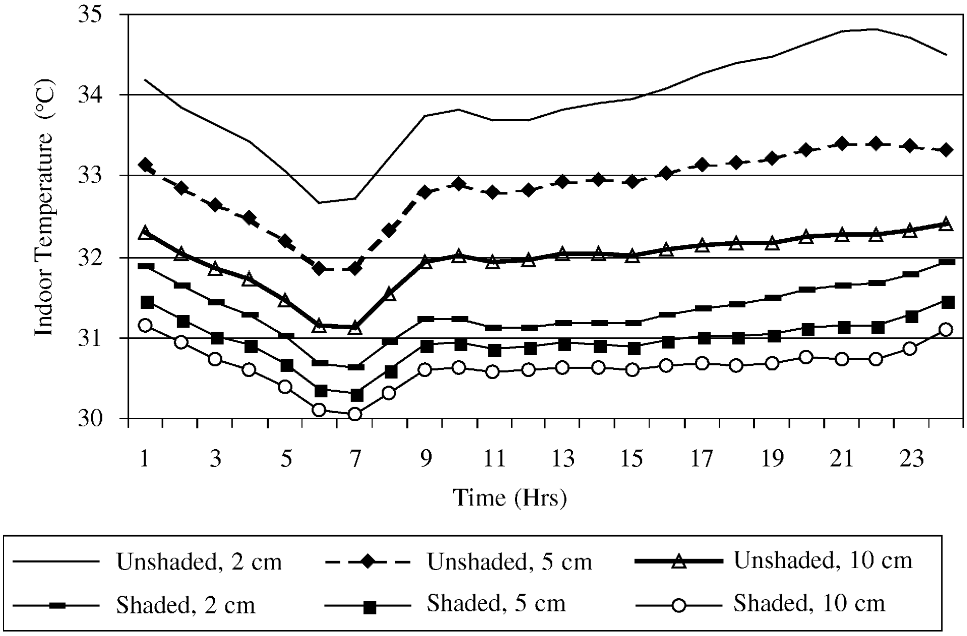
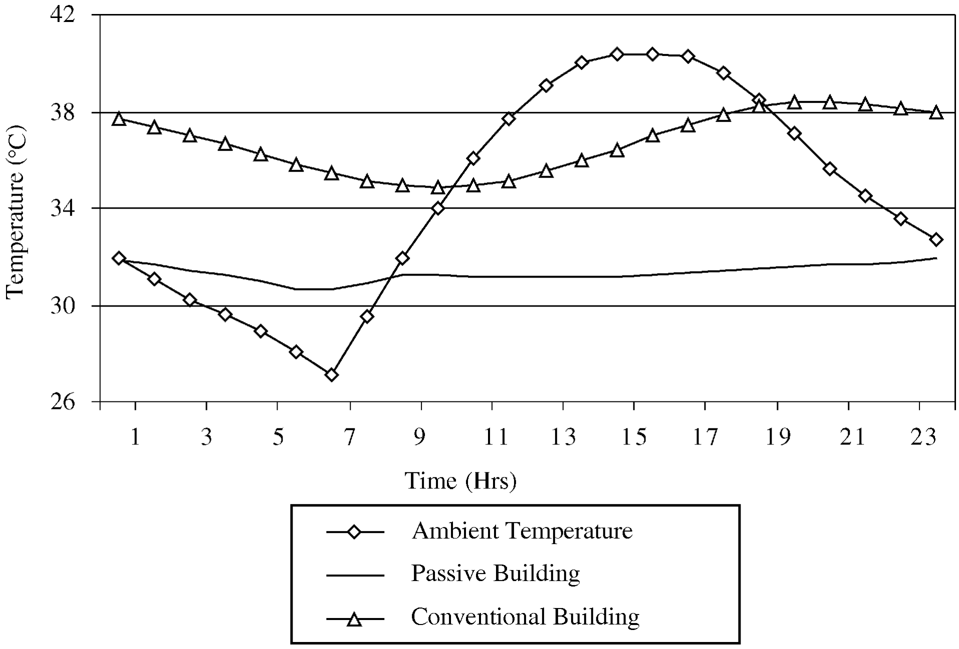


Figure 2 **(c)** Comparison of a Passive building (implemented with all the three passive techniques: shaded, ACH 5.0:0.5, 2cm of insulation) with a Conventional building, at Jodhpur, for the month of May



Finally, Figure 3 shows the effect of shading and other multi-passive cooling techniques for Chennai region which is representing warm and humid climate with much lower passive room temperature swings compared to ambient condition. A very important observation is made from Figure 3(a), where passive room temperature for shaded building varies from 28.5°C to 30.2°C. This is remarkable since insulation and increased air exchange are not required for establishing the required room temperature. However, providing 2-cm insulation in walls and roof can lower temperature swings. This can be easily comprehended from Figure 3(b), where the fluctuations are reduced for shaded building with room air temperature varying from 28.55°C to 29.07°C. It offers added advantage of holding energy inside the room for winter conditions.

Hence, the study clearly illustrates that application of shading and low level of thermal insulation is satisfactory for maintaining a steady and reasonable indoor air temperature in such regions.

Table 2 provides the required value of insulation thickness of different building facades for the Indian climatic zones cited. The performance of multi-application passive techniques consisting of solar shading, controlled window ventilation and insulation described in this paper seems to have a good potential of achieving thermal comfort in composite, hot and dry and warm and humid climates experienced over a large part of India. These techniques can be easily retrofitted to the existing single floor building.

Figure 3 **(a)** Hourly variation of indoor air temperature at Chennai, for a shaded building and an unshaded building for the month of May. ACH is fixed at 1.0, 1.0 and there is no insulation on walls/roof

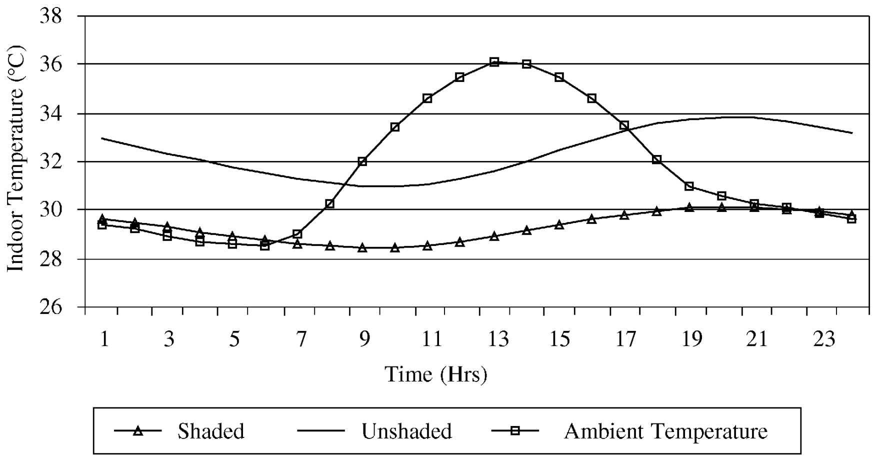


Figure 3 **(b)** Comparison of a Passive building (implemented with all the three passive techniques: shaded, ACH 5.0:0.5, no insulation on walls/roof) with a Conventional building, at Chennai, for the month of May

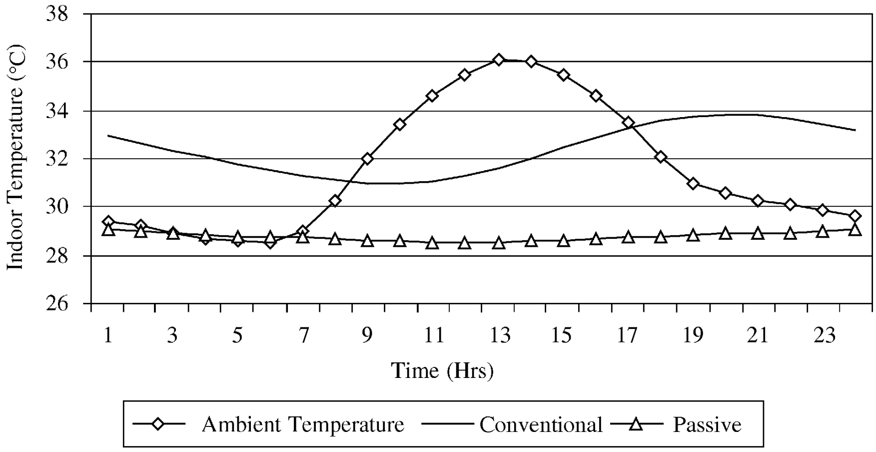


Table 2 Effective thickness of insulation for a shaded building

Name of Climatic Zones	*All Insulated* (cm)	*Only Roof Insulated* (cm)	*Only Wall Insulated* (cm)
Hot and cold (New Delhi)	2.00	3.00–4.00	1.00–2.00
Hot and dry (Jodhpur)	2.00	3.00	2.00
Warm and humid (Chennai)	0.00	0.00	0.00

Performance analysis of passive application in this paper is performed for an empty room. In summer, building thermal load will increase due to increasing use of appliances, metabolism, etc. The indoor temperature will be higher than that indicated by an empty room. However, an actual building outdoor temperature will be lower than that indicated in analysis due to natural solar shading. Also actual building will have lesser indirect gain than a single room. Overall, it should be expected that the achieved thermal comfort level would be little higher.

5 Concluding remarks

- The passive model, based on transient one-dimensional periodic heat conduction equation, seems to perform well in predicting the temperature variations of a room with the effects of multi-passive solar cooling techniques. The results indicate that the solar shading can decrease indoor temperature substantially for all three climate zones. The present study also lends confidence in the efficient use of shading and insulation for cost-effective design of the buildings.
- Multi-passive cooling techniques reduced indoor temperature to a great degree. A significant decrease of about 4.5ºC in the indoor air temperature is seen for Jodhpur zone by only shading in comparison to unshaded building. With the application of controlled window ventilation and insulation, shaded room temperature decreased by 6.8°C. Similarly, a decrease in room temperature of 2.4°C is observed in New Delhi due to shading alone and 4.4°C with other applications in comparison to unshaded building.
- The increase of ACH (1.0–5.0) independent of other passive cooling techniques is not found beneficial for unshaded/shaded buildings even during the night. Application of thermal insulation minimises the fluctuations in indoor air temperature to a great extent.
- The performance of the multi-application passive techniques is dependent on climatic conditions. In the present study, their use was observed much better in Chennai than in Jodhpur and Delhi. For Chennai region, shading of building causes a decrease in room temperature by 3.7°C, while application of other passive cooling techniques decreases it further by 4.9°C.
- An interesting and useful contribution of this study is reducing the passive room temperature and its fluctuations, which carries a great potential for industrial applications such as silkworm rearing. In particular, this conclusion is likely to be useful in determining the level of ACH and insulation for shaded as well as unshaded buildings. The results of this study will certainly be helpful in designing more energy-efficient buildings using passive techniques.

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Nomenclature

A	area of the fabric (m^2)
ACH	number of air exchanges per hour
C	heat transfer coefficient for air exchange (W/K)
C_{inf}	infiltration coefficient
C_{v}	ventilation coefficient
C	specific heat (J/kg/K)
H	convective heat transfer coefficient ($W/m^2/K$)

$I(t)$	radiation flux (W/m^2)
K	thermal conductivity (W/m/K)
M	thermal mass (J/K)
q	rate of heat flow across any surface per unit area (w/m^2)
Q	rate of heat flow across any fabric surface (W)
ΔR	longwave radiative loss to the sky (W/m^2)
T	temperature (°C)
t	time (sec.)
U	conduction transmittance ($W/m^2/K$)
V	volume of room (m^3)
X	coefficient of matrix of order 13×13
Y	column matrix of order 13×1
y	*y*th harmonic

Greek letter

ω	angular frequency ($2\pi/24\times3600\ s^{-1}$)
ρ	density (kg/m^3)
τ	transmitivity
α	absorptivity
β	$(in\omega\rho C/k)^{0.5}$

Superscript

i	inner surface
o	outer surface
y	*y*th harmonic
z	*z*th harmonic

Subscript

ay	ambient temperature of yth harmonic
c	convective
F	floor
g	ground
i	inner
inf	infiltration
j	dummy index

o outer
r roof
v ventilation
w wall
wn window
x room
y yth harmonic
z zth harmonic

Appendix

The heat conduction equation for composite slab is given as:

$$k\frac{\partial^2 T}{\partial x^2} = \rho\, c\frac{\partial T}{\partial t} \quad \text{(A1)}$$

To determine the temperature of a room, the ambient temperature and solar radiation input data vary periodically and are expressible in terms of Fourier coefficient. Therefore, room temperature will also be varying periodically with the same frequency. Equation A1 can be written in matrix form with proper boundary condition as :

$$\begin{bmatrix} T^i_{x,y} \\ q^i_{x,y} \end{bmatrix} = \begin{bmatrix} A_{x,y} & B_{x,y} \\ C_{x,y} & D_{x,y} \end{bmatrix} \begin{bmatrix} T^{\,o}_{x,y} \\ q^o_{x,y} \end{bmatrix}.$$

Here $T^i_{x,y}$ and $T^o_{x,y}$ are the inner and outer surface temperatures of the fabric, respectively, corresponding to the yth harmonic and $\dot{q}^i_{x,y}$ and $\dot{q}^o_{x,y}$ are the nth harmonics of the heat flux per unit area. The matrix elements $A_{x,y}$, $B_{x,y}$ and $D_{x,y}$ depend upon the thermo-physical properties of the building component and are defined as:

$$A_{x,y} = \cosh(\propto_y L)$$

$$B_{x,y}\, \frac{-\sinh(\alpha_y L)}{k\alpha_y}$$

$$D_{x,y} = -k\propto_y \sinh(\propto_y L)$$

where

$$\alpha_y = \sqrt{\frac{iy\omega\rho c}{k}}\,.$$

If a composite slab is considered having an inner and an outer layer of fluid then equation (A1) gets modified to

$$\begin{bmatrix} T^i_{x,y} \\ q^i_{x,y} \end{bmatrix} = \begin{bmatrix} P_{x,y} & Q_{x,y} \\ R_{x,y} & S_{x,y} \end{bmatrix} \begin{bmatrix} T^o_{x,y} \\ \dot{q}^o_{x,y} \end{bmatrix} \tag{A2}$$

where

$$\begin{bmatrix} P_{x,y} & Q_{x,y} \\ R_{x,y} & S_{x,y} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{1}{h_i} \\ 0 \quad 1 \end{bmatrix} \begin{bmatrix} A_{x,y} & B_{x,y} \\ D_{x,y} & A_{x,y} \end{bmatrix} \begin{bmatrix} 1 - \dfrac{1}{h_o} \\ 0 \quad 1 \end{bmatrix}$$

$$\begin{bmatrix} A^2_{x,y} & B^2_{x,y} \\ D^2_{x,y} & A^2_{x,y} \end{bmatrix} \begin{bmatrix} A^3_{x,y} & B^3_{x,y} \\ D^3_{x,y} & A^3_{x,y} \end{bmatrix} \tag{A3}$$

Solving equation (A2) for $\dot{q}^i_{x,y}$, one gets

$$\dot{Q}^i_{x,y} = A \frac{S_y T^i_{x,y} - T^o_{x,y}}{\dot{Q}^o_{x,y}}, \tag{A4}$$

when solar radiation intercepts both the inner and outer surfaces of the building components, then (A1) gets modified and one gets the expression for $\dot{q}^i_{x,y}$, which is similar to (A4), except that the respective temperatures are replaced by their corresponding solar values, together with an additional term, i.e.

$$\dot{Q}^i_{x,y} = A \frac{S_y (T^i_{x,y} + \alpha_i I(t)_{x,y} / h_i) - (T^o_{x,y} + \alpha_0 I(t)_{x,y} / h_o)}{Q_y} + A\alpha_i I(t)_{x,y}. \tag{A5}$$

Similarly, the heat conduction equation will change accordingly for floor

$$\dot{Q}^i_{x,y} = A_F \frac{S_y \left(T^i_{x,y} + \left(\alpha_i I(t)_{x,y} / h_i \right) \right)}{Q_y} + A_{F\alpha_I} I(t)_{iy} \tag{A6}$$

where, for $y \neq 0$, $S_y = 1.0$, $Q_{y} = -1/h_i + 1/k\alpha_{in}$ and for $y = 0$, i.e. steady conditon, $S_y/Q_y = 0$. Here k is the thermal conductivity of the semi-infinite medium and y is the yth harmonic.