01 Physical World and Measurement

6. Least count = $\dfrac{0.01\text{cm}}{100} = 10^{-4}$ cm

8. $\dfrac{\Delta v}{v}\times 100 = \left(\dfrac{\Delta l}{l}+\dfrac{\Delta b}{b}+\dfrac{\Delta h}{h}\right)\times 100\%$

$= \left(\dfrac{0.02}{13.12}+\dfrac{0.01}{7.18}+\dfrac{0.02}{4.16}\right)\times 100\%$

$= 0.77\ \%$

11. Velocity gradient = $\dfrac{v}{x} = \dfrac{[LT^{-1}]}{[L]} = [T^{-1}]$

Potential gradient $= \dfrac{V}{x}$

$= \dfrac{[ML^2T^{-3}A^{-1}]}{[L]}$

$= [MLT^{-3}A^{-1}]$

Energy gradient $= \dfrac{E}{x} = \dfrac{[ML^2T^2]}{[L]}$

$= [MLT^{-2}]$

Latent heat $= \dfrac{\text{heat}(Q)}{\text{mass}(m)} = \dfrac{[ML^2T^{-2}]}{[M]}$

$= [M^0L^2T^{-2}]$

Gravitational potential = $\dfrac{W}{m_0} = \dfrac{[ML^2T^{-2}]}{[M]}$

$= [M^0L^2T^{-2}]$

12. $\dfrac{h}{e^2} = \dfrac{ML^2T^{-1}}{(AT)^2} = [ML^2T^{-3}A^{-2}]$

= Resistance (ohm)

14. The prefix Pico is used for 10^{-12}

17. distance between Earth and Sirius = 8.6 ly

$= 8.6 \times 9.46 \times 10^{15}$

$= 81.3 \times 10^{15}$ m

1 AU = 1.5×10^{11}m

$\therefore$ distance in AU = $\dfrac{81.3\times10^{15}}{1.5\times10^{11}}$

$= 54.2 \times 10^4$

$= 5.4 \times 10^5$ AU

18. From Biot-Savart's law, B = $\dfrac{\mu_0}{4\pi}\dfrac{idl\sin\theta}{r^2}$

$\mu_0 = \dfrac{4\pi Br^2}{idl\sin\theta}$

$\therefore$ Unit of magnetic permeability = $\dfrac{\text{tesla m}^2}{\text{A m}}$

$= \text{Wb A}^{-1}\text{ m}^{-1}$

19. $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\therefore$ $\mu_0\varepsilon_0 = \left(\dfrac{1}{c^2}\right)$

where c = velocity of light.

20. $L \propto v^x a^y F^z \Rightarrow L = kv^x a^y F^z$

Putting the dimensions in the above relation we get,

$[ML^2T^{-1}] = k[LT^{-1}]^x[LT^{-2}]^y[MLT^{-2}]^z$

$\therefore$ $[ML^2T^{-1}] = k[M^zL^{x+y+z}T^{-x-2y-2z}]$

Comparing the powers of M, L and T, we get

$z = 1$ ....(i)

$x + y + z = 2$ ....(ii)

$-x - 2y - 2z = -1$ ....(iii)

On solving (i), (ii) and (iii), we get

$x = 3,\ y = -2,\ z = 1$

So dimensions of L in terms of v, A and F is $[L] = [Fv^3a^{-2}]$

02 Motion in One Dimension

1. Displacement may be positive, negative or zero.

6. At a particular instant, the particle will not have different positions.

7. The relative speed of policeman w.r.t. thief $= 10 - 9 = 1 \text{m/s}$.

$\therefore$ Time taken by police to catch the thief $= \frac{100}{1}$ $= 100$ s

11. For vertically upward motion,

$h_1 = v_0 t - \frac{1}{2} g t^2$ and for vertically downward motion,

$h_2 = v_0 t + \frac{1}{2} g t^2$

$\therefore$ Total distance covered in t s, $h = h_1 + h_2 = 2v_0 t$.

12. The maximum height h attained by the bullet is given by,

$v^2 - u^2 = -2gh$

or $h = \frac{u^2}{2g}$ $\quad (\because v = 0)$

or $h = \frac{50 \times 50}{2 \times 10} = 125$ m. The total time taken by the stone to attain this height is given by,

$t = \frac{u}{g} = \frac{50}{10} = 5$ s.

During the first second (t = 1 s), the stone covers a distance h_1 given by

$h_1 = ut - \frac{1}{2} g t^2 = 50 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 45$ m

During the first four seconds (t = 4 s), the stone covers a height h given by

$h = 50 \times 4 - \frac{1}{2} \times 10 \times (4)^2 = 120$ m

$\therefore$ Distance travelled by the stone during the last (i.e., fifth) second of its upward motion is

$h_2 = 125 - 120 = 5$ m

Hence $h_1/h_2 = 45/5 = 9 : 1$

13. As $v^2 = u^2 + 2as$,

$\therefore$ $(2u)^2 = u^2 + 2as$

$\therefore$ $2as = 3u^2$

Now, after covering an additional distance s, if velocity becomes v, then,

$v^2 = u^2 + 2a(2s) = u^2 + 4as = u^2 + 6u^2 = 7u^2$

$\therefore$ $v = \sqrt{7}u$

14. As $v^2 = u^2 - 2as$,

$\therefore$ $u^2 = 2as$ $\quad (\because v = 0)$

$\therefore$ $u^2 \propto s$

$\therefore$ $\frac{u_2}{u_1} = \left(\frac{s_2}{s_1}\right)^{1/2}$

$\therefore$ $u_2 = \left(\frac{9}{4}\right)^{1/2} u_1 = \frac{3}{2} u_1 = 300$ cm/s

16. If v_1 is the speed of swimmer in still water and v_2 is the speed of flow of river, then relative speed of swimmer in the direction of flow is $v_1 + v_2 = 16$ km/h ....(i)

Relative speed in opposite direction is

$v_1 - v_2 = 8$ km/h ....(ii)

On solving (i) and (ii) we get,

$v_1 = 12$ km/h, $v_2 = 4$ km/h

18. Since direction of v is opposite to the direction of g and h, from equation of motion we get,

$h = -vt + \frac{1}{2} g t^2$

$\therefore$ $gt^2 - 2vt - 2h = 0$ which is a quadratic in 't'.

$\therefore$ $t = \frac{2v \pm \sqrt{4v^2 + 8gh}}{2g}$

$\therefore$ $t = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$

19. Let height of tower be h and let the body take t s to reach to ground when it falls freely.

$\therefore$ $h = \frac{1}{2} g t^2$ ....(i)

In last second i.e. t^{th} s, body travels = 0.36 h

It means, in rest of the time i.e. in (t – 1) s it travels,

$h - 0.36h = 0.64h$

Now, applying equation of motion for (t – 1) s we get,

$0.64h = \frac{1}{2} g (t-1)^2$ ....(ii)

From (i) and (ii) we get, t = 5 s and h = 125 m

20. Let time taken by first stone to reach the water surface from the bridge be t. Then

$h = ut + \frac{1}{2} g t^2$

$\therefore$ $44.1 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$

$\therefore$ $t = \sqrt{\frac{2 \times 44.1}{9.8}} = 3$ s

Second stone is thrown 1 s later and both strike the water simultaneously. This means that the time left for second stone = 3 – 1 = 2 s

Hence, $44.1 = u \times 2 + 9.8(2)^2$

$\therefore$ $44.1 - 19.6 = 2u$

$\therefore$ $u = 12.25$ m/s

03 Motion in two Dimensions

1. Magnitude of vector = 1

$\therefore$ $\sqrt{a_x^2 + a_y^2 + a_z^2} = 1$

$\therefore$ $\sqrt{0.5^2 + 0.8^2 + c^2} = 1$

$\therefore$ $c = \sqrt{0.11}$

6. $R_{net} = R + \sqrt{R^2 + R^2} = R + \sqrt{2}R = R(\sqrt{2} + 1)$

7. Direction of velocity is always tangential to the path. At the top of trajectory, velocity is in horizontal direction and acceleration due to gravity is always in vertically downward direction. It means $\vec{v}$ and $\vec{g}$ are perpendicular to each other.

8. $y = (\tan\theta)\, x - \left(\dfrac{g}{2u^2\cos^2\theta}\right)x^2$

Here, $\theta = 45°$, $u = 20$ m/s

$\therefore$ $y = (\tan 45°)\, x - \left(\dfrac{g}{2(20)^2\cos^2 45°}\right)x^2$

$\therefore$ $y = x - \dfrac{gx^2}{400}$

$y = x\left(1 - \dfrac{gx}{400}\right)$

9. Horizontal velocity, $v_x = 20$ m/s

Vertical velocity,

$v_y = u + gt = 0 + 10 \times 5 = 50$ m/s

Net velocity,

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (50)^2} \approx 54$ m/s.

10. $\dfrac{\delta R}{R} = \dfrac{2\delta v_0}{v_0} = 2 \times 0.05 = 0.1$

Therefore R will increase by 10%.

11. Since range is given to be the same, therefore the other angle is $(90° - 60°) = 30°$

$\therefore$ $H = \dfrac{u^2\sin^2 60°}{2g} = \dfrac{3}{4}\left(\dfrac{u^2}{2g}\right)$ and

$H' = \dfrac{u^2\sin^2 30°}{2g} = \dfrac{1}{4}\left(\dfrac{u^2}{2g}\right)$

$\therefore$ $\dfrac{H'}{H} = \dfrac{1}{4} \times \dfrac{4}{3}$

$\therefore$ $H' = \dfrac{1}{3}H$

12. Here $|\vec{v}_2| = |\vec{v}_1| = v$

$|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1|$

$= \sqrt{v_2^2 + v_1^2 - 2v_1v_2\cos\theta}$

$= \sqrt{v^2 + v^2 - 2v^2\cos\theta}$

$= \sqrt{2v^2(1 - \cos\theta)}$

$= \sqrt{2v^2 2\sin^2(\theta/2)}$

$= 2v\sin(\theta/2)$

As $\theta = 40°$,

$\therefore$ $|\Delta\vec{v}| = 2v\sin(20°)$

13. $T = 2$ s

In 2 s, it covers a distance = $2\pi r$

In 60 s, it covers a distance = $[30(2\pi r)]$

Let $60\pi r = 62.8$

$\therefore$ $r = \dfrac{62.8}{60 \times 3.14} = \dfrac{1}{3} = 0.33$ m

14. $a = r\omega^2$;

$\therefore$ $5g = 10\,\omega^2$

$\therefore$ $\omega = \sqrt{\dfrac{5g}{10}} = 2.21$ rad/s

15. Centripetal force $F_{cp} = \dfrac{mv^2}{r}$

Changed centripetal force $F_{cp} = \dfrac{m(v/2)^2}{4r}$

$= \dfrac{mv^2}{16r}$

i.e., decrease in $F_{cp} = \left(1 - \dfrac{1}{16}\right) = \dfrac{15}{16}$

16. The frictional force between tyres and road provides necessary c.p.f., but this force also has a moment about centre of mass of cyclist, which causes the cyclist to topple.

18. $r = 400$ m, $v = 72$ km/hr $= 72 \times \dfrac{5}{18} = 20$ m/s

$l = 1$ m

$\therefore$ $\dfrac{v^2}{rg} = \dfrac{h}{l}$

$\therefore$ $h = \dfrac{v^2 l}{rg} = \dfrac{20 \times 20 \times 1}{400 \times 10} = 0.1$ m = 10 cm

19. $mg = 2 \times 10$ N = 20 N

$\dfrac{mv^2}{r} = \dfrac{2 \times 30 \times 30}{5}$ N = 360 N

$\therefore$ $\dfrac{T_t}{T_b} = \dfrac{\left(\dfrac{mv^2}{r} - mg\right)}{\left(\dfrac{mv^2}{r} + mg\right)} = \dfrac{360 - 20}{360 + 20} = \dfrac{340}{380} = \dfrac{17}{19}$

20. We have, $h = \dfrac{v_0^2 \sin^2\theta}{2g}$

The increase of δh in h when v_0 changes by δv_0 can be obtained by partial differentiation. Thus,

$$\delta h = \frac{2v_0 \delta v_0 \sin^2\theta}{2g}$$

$\therefore \quad \dfrac{\delta h}{h} = \dfrac{2\delta v_0}{v_0}$

Since h is increased by 10%, $\dfrac{\delta h}{h} = 0.1$

Now, $R = \dfrac{v_0^2 \sin 2\theta}{g}$,

$\therefore \quad \delta R = \dfrac{2v_0 \delta v_0 \sin 2\theta}{g}$

$\therefore \quad \dfrac{\delta R}{R} = \dfrac{2\delta v_0}{v_0} = 0.1 = 10\%$

$\therefore$ R also increases by 10%.

21. $\mu = \dfrac{\omega^2 r}{g} = \dfrac{4\pi^2 f^2 r}{g}$

$= \dfrac{4 \times \pi^2 \times (3)^2 \times 2 \times 10^{-2}}{\pi^2}$

$= 72 \times 10^{-2} = 0.72$

22. Angular speed of second hand,

$\omega_s = \dfrac{2\pi}{T_s} = \dfrac{2\pi}{60}$

Angular speed of minute hand,

$\omega_m = \dfrac{2\pi}{T_m} = \dfrac{2\pi}{60 \times 60}$

$\therefore \quad \dfrac{\omega_s}{\omega_m} = \dfrac{2\pi}{60} \times \dfrac{60 \times 60}{2\pi} = 60 : 1$

23. $T = \dfrac{2u \sin\theta}{g} = \dfrac{2 \times 98 \times \sin 60^\circ}{9.8} = 17.32$ s

04 Laws of Motion

3. $p = mv = 3t^2 + 4$. Since $m = 2kg$, $v = \frac{3}{2}t^2 + 2$.

The acceleration is

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{3}{2}t^2 + 2\right) = 3t$$

Thus, the acceleration of the body is increasing with time.

4. Total mass of bullets = Nm, time $t = \frac{N}{n}$

Momentum of the bullets striking the wall
= Nmv
Rate of change of momentum (Force)

$$= \frac{Nmv}{t} = nmv$$

(Use Note no. 10)

5. $p = at^3 + bt$

Force, $F = \frac{dp}{dt} = 3at^2 + b$

$\therefore$ $F \propto t^2$

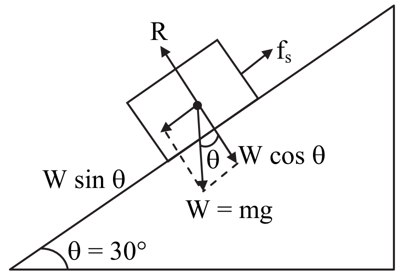
6. For upward acceleration, apparent weight $= m(g + a)$
If lift suddenly stops during upward motion, then apparent weight $= m(g - a)$ because instead of acceleration, we will consider retardation.
In the problem, it is given that scale reading initially was 60 kg and due to sudden jerk reading decreases and finally comes back to the original mark i.e., 60 kg.
So, we can conclude that lift was moving upward with constant speed and stops suddenly.

7. Since the person is rising upwards with the helicopter with acceleration a, the force exerted by him on the floor will be
$F = m(g + a)$ vertically downwards.

11. The weight W = mg of the block can be resolved into two rectangular components; $W \sin\theta$ along the plane and $W \cos\theta$ perpendicular to the plane. Let R be the magnitude of the normal reaction and f_s be the force of sliding friction (see figure). When these forces are in equilibrium, the block just begins to slide, i.e.,
$f_s = W \sin\theta$
Also, $R = W \cos\theta$

$\therefore$ Coefficient of sliding friction is

$$\mu_s = \frac{f_s}{R} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta = \tan 30°$$



12. $\mu_k = \frac{F}{R} = \frac{19.6}{5 \times 9.8} = \frac{2}{5} = 0.4$

13. Mass of the ball = 0.15 kg. Initial momentum of the ball = 0.15 kg × 12 ms^{-1} = 1.8 kg ms^{-1}.
Final momentum of the ball
= 0.15 kg × (– 20 ms^{-1}) = – 3 kg ms^{-1}.
Change in momentum = 1.8 – (– 3)
= 4.8 kg ms^{-1}
This is the impulse of the force exerted by the bat.
Now, impulse = average force × time of impact.

$\therefore$ Average force $= \frac{\text{impulse}}{\text{time}} = \frac{4.8}{0.1} = 48$ N.

14. Mass of each coin (m) = 10 g = 0.01 kg. The 7^{th} coin from the bottom has 3 coins above it. Hence, the force on the 7^{th} coin = weight of 3 coins = 3 mg = 3 × 0.01 × 10 = 0.3 N, vertically downwards.

15. $T = \frac{2m_1m_2}{(m_1 + m_2)}(g + a) = \frac{2m_1m_2(g + g)}{m_1 + m_2}$

$\therefore$ $T = \frac{4m_1m_2}{m_1 + m_2} g = \frac{4w_1w_2}{w_1 + w_2}$

16. $T = M \times a = M \times \left(\frac{F}{m + M}\right)$

18. Let 'a' be the maximum acceleration with which the boy should climb the rope. Then, the tension in the rope will be
$T_{max} = m(g + a)$

$\therefore$ $500 = 40 \times (10 + a)$

$\therefore$ $a = 2.5\ ms^{-2}$.

19. Since the bomb is at rest, its initial momentum is zero. From the principle of conservation of momentum it follows that, after explosion, the total momentum of all the fragments must be zero. Hence the correct choice is (A).

20. The law of conservation of momentum gives

$$m_1v = m_1\frac{v}{2} + m_2v'$$

where v′ is the speed of a ball B after collision. Thus, $v' = \frac{m_1v}{2m_2}$

05 Work, Energy and Power

1. As surface is smooth, work done against friction is zero. Also the displacement and force of gravity are perpendicular. So work done against gravity is zero.

2. Displacement along the Z-axis is

$s = 4\hat{k}$ meters. Therefore, work done is

$W = \vec{F}.\vec{s}$

$= \left(-\hat{i}+2\hat{j}+3\hat{k}\right)\cdot\left(4\hat{k}\right)$

$= 0 + 0 + 12$

$\therefore$ W = 12 J.

3. $W = Fs = F\times\frac{1}{2}at^2$ $\left[\text{from } s = ut + \frac{1}{2}at^2\right]$

$\therefore$ $W = F\left[\frac{1}{2}\left(\frac{F}{m}\right)t^2\right]$

$= \frac{F^2t^2}{2m} = \frac{25\times(1)^2}{2\times15}$

$= \frac{25}{30} = \frac{5}{6}$ J

5. PE = mgh

$\therefore$ $h = \frac{1}{1\times10} = 0.1$ m

6. Kinetic energy acquired by the body = Force applied on it × Distance covered by the body

K.E. = F × d

Thus, K.E. is independent of m.

7. $U = \frac{1}{2}kx^2$. If x becomes 5 times, then energy will become 25 times i.e 4 × 25 = 100 J

8. When body moves under action of constant force, then kinetic energy acquired by the body

K.E. = F × s

$\therefore$ KE $\propto$ s (If F = constant)

So the graph will be a straight line.

9. Work done = change in kinetic energy

$W = \frac{1}{2}mv^2$

$\therefore$ $W \propto v^2 \Rightarrow$ graph will be parabolic in nature.

10. In a perfectly inelastic collision, velocity of separation is zero. Hence, coefficient of restitution is zero.

11. For $m_1 = 0.05$ kg

$u_1 = 6$ m/s

and $m_2 = 0.05$ kg

$u_2 = -6$ m/s

After collision, the balls rebound

$\therefore$ $v_1 = -6$ m/s

$v_2 = 6$ m/s

$\therefore$ change in momentum for first ball A,

$\Delta p_1 = m_1v_1 - m_1u_1 = m_1(v_1 - u_1)$

$= 0.05(-6 - 6)$

$= -0.6$ kgm/s

To conserve the linear momentum,

Δp_2 has to be 0.6 m/s

12. Loss of K.E. $= \frac{1}{2}\times 0.02\times(250)^2 = 625$ J

Loss of K.E. = W = F × 0.12

$\therefore$ 625 = 0.12 F

$\therefore$ $F = \frac{625}{0.12} = 5.2\times10^3$ N

13. Kinetic energy for first condition

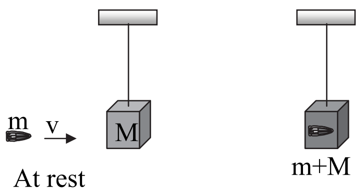
$= \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m(20^2 - 10^2) = 150$ m J

K.E. for second condition $= \frac{1}{2}m(10^2 - 0^2)$

$= 50$ m J

$\therefore$ $\frac{(K.E.)_I}{(K.E.)_{II}} = \frac{150\,m}{50\,m} = 3$

14.



Initial kinetic energy of bullet $= \frac{1}{2}mv^2$

After inelastic collision, system moves with velocity V

By the conservation of momentum,

mv + 0 = (m + M)V

$\therefore$ $V = \frac{mv}{m+M}$

Kinetic energy of the system

$= \frac{1}{2}(m+M)V^2$

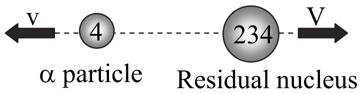
$= \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$

Loss of kinetic energy

$= \frac{1}{2}mv^2 - \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$

$= \frac{1}{2}mv^2\left(\frac{M}{m+M}\right)$

15. Initially, ^{238}U nucleus was at rest and after decay its part moves in opposite direction.



According to conservation of momentum,
$4v + 234V = 238 \times 0$

$\therefore \quad V = -\dfrac{4v}{234}$

16. Here, $\dfrac{mv^2}{r} = \dfrac{K}{r^2}$

$\therefore \quad K.E. = \dfrac{1}{2}mv^2 = \dfrac{K}{2r}$

$U = -\int_{\infty}^{r} F.dr = -\int_{\infty}^{r}\left(-\dfrac{K}{r^2}\right)dr = -\dfrac{K}{r}$

Total energy, E = K.E. + P.E.

$$= \frac{K}{2r} - \frac{K}{r} = -\frac{K}{2r}$$

17. After explosion, mass m come to rest and mass (M − m) move with velocity v. By the law of conservation of momentum,
$MV = (M - m)v$

$\therefore \quad v = \dfrac{MV}{M-m}$

18. Energy stored = mgh = $2 \times 10 \times 10 = 200$ J
Work done = 300 J
$\therefore$ Work done against friction
= 300 J − 200 J = 100 J.

19. Let initial kinetic energy, $E_1 = E$
Final kinetic energy, $E_2 = E + 300\%$ of $E = 4E$
As $P \propto \sqrt{E}$

$\therefore \quad \dfrac{P_2}{P_1} = \sqrt{\dfrac{E_2}{E_1}} = \sqrt{\dfrac{4E}{E}} = 2$

$\therefore \quad P_2 = 2P_1$

$\therefore \quad P_2 = P_1 + 100\%$ of P_1

i.e. Momentum will increase by 100%.

20. $E = \dfrac{P^2}{2m}$.

If m is constant, then $E \propto P^2$

$\therefore \quad \dfrac{E_2}{E_1} = \left(\dfrac{P_2}{P_1}\right)^2 = \left(\dfrac{1.2P}{P}\right)^2 = 1.44$

$\therefore \quad E_2 = 1.44E_1 = E_1 + 0.44E_1$

$\therefore \quad E_2 = E_1 + 44\%$ of E_1

i.e. the kinetic energy will increase by 44%.

06 System of particles and Rotational Motion

1. Force $\vec{F}$ lies in the x-y plane so a vector along Z-axis will be perpendicular to $\vec{F}$.

2. Given

$\vec{A} = 2\hat{i} + 2\hat{j} - x\hat{k}$; $\vec{B} = 2\hat{i} - \hat{j} - 3\hat{k}$

For $\vec{A}$ to be perpendicular to $\vec{B}$ we must have;

$\vec{A} \cdot \vec{B} = 0$

$(2\hat{i} + 2\hat{j} - x\hat{k}) \cdot (2\hat{i} - \hat{j} - 3\hat{k}) = 0$

$4 - 2 + 3x = 0$

$\therefore$ $2 + 3x = 0$

$\therefore$ $x = \dfrac{-2}{3}$

3. $|\vec{A}.\vec{B}| = 18 + 32 = 50$

$\dfrac{|\vec{A}|}{|\vec{B}|} = \dfrac{5}{10} = \dfrac{1}{2}$,

$|\vec{A}| = \sqrt{a_x^2 + a_y^2} = \sqrt{3^2 + 4^2} = 5$

But $|\vec{A} \times \vec{B}| = 0$

5. Given, $|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A} \cdot \vec{B})$

$\therefore$ AB sin θ = $\sqrt{3}$ AB cos θ

$\therefore$ tan θ = $\sqrt{3}$, i.e. θ = 60°

6. $\vec{A}$ is along + X-axis, so $\vec{B}$ should be either along the + X-axis or –X-axis.

[As $\vec{A} \times \vec{B} = 0$]

8. The position of centre of mass of the system,

$x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$= \dfrac{3\times0 + 8\times1 + 4\times2}{3+8+4} = \dfrac{16}{15}$

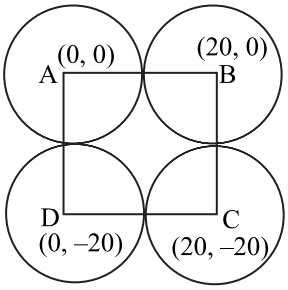
≈ 1.1 m

$y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$= \dfrac{3\times0 + 8\times2 + 4\times1}{3+8+4} = \dfrac{20}{15}$

≈ 1.3 m

9.



Taking centre of sphere A as the origin we have,

A ≡ (0, 0); B ≡ (20, 0) C ≡ (20, – 20);

D ≡ (0, –20)

Co-ordinates of centre of mass of the system are

$x_{cm} = \dfrac{mx_1 + mx_2 + mx_3 + mx_4}{4m}$ and;

$y_{cm} = \dfrac{my_1 + my_2 + my_3 + my_4}{4m}$

$\therefore$ $x_{cm} = \dfrac{x_1 + x_2 + x_3 + x_4}{4}$ and;

$y_{cm} = \dfrac{y_1 + y_2 + y_3 + y_4}{4}$

$\therefore$ $x_{cm} = \dfrac{0 + 20 + 20 + 0}{4}$ and;

$y_{cm} = \dfrac{0 + 0 - 20 - 20}{4}$

$\therefore$ $x_{cm} = 10$ cm;

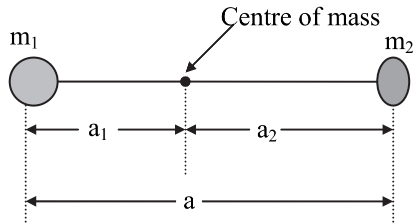
$\therefore$ $y_{cm} = 10$ cm

Let O(x_{cm}, y_{cm}) be centre of mass of system

$\therefore$ $AO = \sqrt{(10-0)^2 + (10-0)^2}$

$AO = 10\sqrt{2}$ cm

10. Referring to figure, the distances of the centre of mass from masses m_1 and m_2 respectively are



$a_1 = \dfrac{m_2a}{(m_1 + m_2)}$ and $a_2 = \dfrac{m_1a}{(m_1 + m_2)}$

$\therefore$ $\dfrac{a_1}{a_2} = \dfrac{m_2}{m_1}$

13. K.E. in first case = $\dfrac{1}{2}$ $I\omega^2 = E$

In second case,

$I' = 3I$

According to conservation of angular momentum,

$I\omega = I'\omega'$

$\therefore$ $\omega' = \dfrac{I\omega}{I'} = \dfrac{I\omega}{3I} = \dfrac{\omega}{3}$

Now, K. E. in second case = $\dfrac{1}{2}$ $I'\,\omega'^2$

$= \dfrac{1}{2} \times 3I \times \dfrac{\omega^2}{9} = \dfrac{1}{3}\left(\dfrac{1}{2}I\omega^2\right) = \dfrac{1}{3}E$

$\therefore \quad \frac{E_1 - E_2}{E_1} = \frac{\left(E - \frac{1}{3}E\right)}{E} = \frac{2}{3}$

14. $E_1 = \frac{1}{2}mv^2$

$E_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}(m r^2)\omega^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

$\therefore \quad \frac{E_1}{E_2} = \frac{\left(\frac{1}{2}mv^2\right)}{mv^2} = \frac{1}{2}$

15. $K = \frac{1}{2}I\omega^2$

$\therefore \quad \frac{K_1}{K_2} = \frac{\left(\frac{1}{2}I_1\omega_1^2\right)}{\left(\frac{1}{2}I_2\omega_2^2\right)}$

$\therefore \quad \frac{K_1}{K_2} = \left(\frac{\omega_1}{\omega_2}\right)^2$ ….[$\because I_1 = I_2$]

$= \left(\frac{\omega_1}{2\omega_1}\right)^2 = \frac{1}{4}$

$\therefore \quad K_2 = 4K_1$

16. $\tau = I\alpha$; $\therefore \ \alpha = \frac{\tau}{I}$

17. $K = \frac{1}{2}I\omega^2$

$\therefore \quad \omega^2 = \frac{2K}{I} = \frac{2\times600}{3} = 400$

or $\quad \omega = 20$

$\therefore \quad \frac{2\pi}{T} = 20$

$\therefore \quad T = \frac{\pi}{10} = \frac{3.14}{10} = 0.31$ s

19. $\omega = \tau\theta = I\,\alpha\,\theta = mK^2\,\alpha\,\theta$

20. K.E. of Rolling body $= \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

K.E. $= \frac{1}{2}mv^2\left(1+\frac{2}{5}\right)$

$= \frac{7}{10}mv^2$

According to law of conservation of energy,

$mgh = \frac{7}{10}mv^2$

$gh = \frac{7}{10}v^2$

or $v = \sqrt{\frac{10gh}{7}}$

$= \sqrt{\frac{10\times9.8\times0.6}{7}}$

$= \sqrt{8.4} \approx 2.9$ m/s

21. For rolling: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}Mv^2 + \frac{1}{2}\times\left(\frac{2}{5}MR^2\right)\times\frac{v^2}{R^2}$

[$\because I = \frac{2}{5}MR^2$ and $\omega = \frac{v}{R}$]

$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$

$= \frac{7}{10}Mv^2$

For sliding: $Mgh = \frac{1}{2}Mv'^2$.

Therefore, $\frac{1}{2}M(v')^2 = \frac{7}{10}Mv^2$

$\therefore \quad \frac{v'}{v} = \sqrt{\frac{7}{5}}$

22. K.E. = Rotational K.E. + Translational K.E.

23. By symmetry,
$I_1 = I_2$ and $I_3 = I_4$
By perpendicular axes theorem,
$I_o = I_1 + I_2$

24. $E = \frac{1}{2}I\omega^2 = \frac{J^2}{2I}$

$\therefore$ log E = 2logJ – log2 – logI

$\therefore$ log E = 2logJ – (log2 + logI)

Comparing with y = mx + c option (D) best represents logJ – log E graph.

07 Gravitation

1. $\frac{T'}{T} = \left[\frac{R'}{R}\right]^{3/2} = \left[\frac{4R}{R}\right]^{3/2}$

T is increased by a factor of $(4)^{3/2}$ i.e. 8 times.
$T' = 8 \times 83$ minutes = 664 minutes.

2. $g_e = \frac{GM_e}{R_e^2}$, $g_m = \frac{GM_m}{R_m^2}$

$\therefore \quad \frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2$

$\therefore \quad mg_m = mg_e \times \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2$

$= 10 \times \frac{1}{80} \times (4)^2 = 2$ kg-wt.

3. $\frac{g_1}{g_2} = \frac{GM_1/R_1^2}{GM_2/R_2^2} = \frac{M_1}{M_2} \times \left(\frac{R_2}{R_1}\right)^2$

$= \frac{2}{3} \times \left(\frac{2}{3}\right)^2 = \frac{8}{27}$

5. $U_1 = -\frac{GMm}{r_1}$

$W_2 = \frac{GMm}{r_2^2} = \frac{GMm}{r_1} \times \frac{r_1}{r_2^2} = |U_1| \times \frac{r_1}{r_2^2}$

$= 4 \times 10^9 \times \frac{10^7}{(10^9)^2} = 4 \times 10^{-2}$ N.

7. $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$

$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \approx 0.35$yr

8. $F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2} = \frac{4}{9}\pi^2\rho^2R^4 \propto R^4$

9. Gravitational force does not depend on the medium.

10. $g = \frac{GM}{r^2}$. Since M and r are constant,
Hence, $g = 9.8$ m/s^2

11. $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

$\therefore \quad \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

$\frac{r_1}{r_2} = \frac{1}{4}$

$\therefore \quad r_2 = 4\,r_1 = 4 \times 10^4$ km

$v_1 = \frac{2\pi r_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4$ km/hr and

$v_2 = \frac{2\pi r_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4$ km/hr

Speed of S_2 relative to S_1, $v_{21} = |v_2 - v_1|$
$= \pi \times 10^4$ km/hr.

12. Increase in potential energy

$= -\frac{GmM}{2R} - \left(-\frac{GmM}{R}\right) = \frac{GmM}{2R}$

$= \frac{GM}{R^2} \times \frac{mR}{2} = g \times \frac{1}{2}mR = \frac{1}{2}mgR$

13. $v = \sqrt{\frac{GM}{R+h}}$

For first satellite, h = 0, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite, $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v$

14. $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}} = \sqrt{2 \times \frac{1}{3}} = \sqrt{\frac{2}{3}}$

$\therefore \quad v_p = \sqrt{\frac{2}{3}}v_e$

15. $g' = g\left(1 - \frac{d}{R}\right)$

$\therefore \quad \frac{g}{n} = g\left(1 - \frac{d}{R}\right)$

$\therefore \quad d = \left(\frac{n-1}{n}\right)R$

17. $T_2 = 24\left(\frac{6400}{36000}\right)^{3/2} \approx 2$ hours

18. Given that, $T_1 = 1$ day and $T_2 = 8$ days

$\therefore \quad \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$

$\therefore \quad \frac{r_2}{r_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{8}{1}\right)^{2/3} = 4$

$\therefore \quad r_2 = 4r_1 = 4R$

19. $-\frac{GMm}{R}+\frac{1}{2}m(4v_e)^2 = \frac{1}{2}\ mv^2$

and $\frac{GMm}{R} = \frac{1}{2}mv_e^2$

$\therefore$ $-\frac{1}{2}mv_e^2 + 16 \times \frac{1}{2}\ mv_e^2 = \frac{1}{2}mv^2$

Solving it, we get $v = \sqrt{15}\ v_e$

$= \sqrt{15} \times 11.2$ km/s.

20. Gravitational Potential,

$V = -\frac{GM}{1}+\left(\frac{-GM}{2}\right) + \left(\frac{-GM}{4}\right) + \ldots$

$= -\ GM \left[1+\frac{1}{2}+\frac{1}{4}+\ldots\right]$

$= -\ GM \left[\frac{1}{1-\frac{1}{2}}\right] = -\ 2\ GM$

$= -\ 2\ G \times 1 = -\ 2\ G$

08 Mechanical properties of solids: Elasticity

1. According to Hooke's law, $F \propto \text{Strain} \propto l$

3. As stress is shown on X-axis and strain on Y-axis,

   so we can say that, $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$

   So, elasticity of wire P is minimum and of wire R is maximum

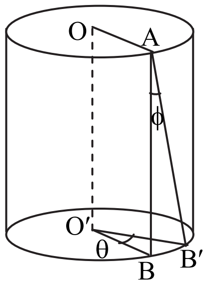
4. $\frac{(l_1 / L_1)}{(l_2 / L_2)} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

7. $Y = \frac{Mg(L/2)}{Al} = \frac{AL\rho gL}{2Al} = \frac{L^2\rho g}{2l}$

   $\therefore \quad l = \frac{L^2\rho g}{2Y} = \frac{(8)^2 \times (1.5\times10^3)\times(10)}{5\times10^6\times2}$

   $= \frac{64\times1.5\times10^4}{10^7} = 96 \times 10^{-3} = 9.6 \times 10^{-2}$ m

8. In figure, θ is the angle of twist and ϕ is the angle of shear.

   $BB' = O'B \times \theta = AB \times \phi$

   or $\quad \phi = O'B \times \frac{\theta}{AB}$

   $= r \times \frac{\theta}{l}$

   $= \frac{2\times10^{-3}\times45^\circ}{1} = 0.09^\circ$

9. Breaking strength = stress × area = stress × πr^2

   $\therefore \quad F \propto r^2$

   $\therefore \quad \frac{F_1}{F_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{d_1/2}{d_2/2}\right)^2 = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{2}{1}\right)^2$

   $= 4$

   $\therefore \quad F_2 = \frac{F_1}{4} = \frac{2\times10^5}{4} = 0.5 \times 10^5$ N

10. For isothermal process,

    PV = constant

    $\therefore \quad PdV + VdP = 0$

    or $PdV = -VdP$

    $\therefore \quad -\frac{dP}{dV}V = P$ = isothermal bulk modulus.

11. Energy $= \frac{1}{2}$ × maximum stretching force × extension

    $= \frac{1}{2} \times 40 \times 2 \times 10^{-3}$

    $= 40 \times 10^{-3} = 0.04$ J

12. $\eta = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{Ax} = \frac{0.36\times10\times10^{-2}}{100\times10^{-4}\times x} = 600$

    $\therefore \quad x = \frac{0.36\times10\times10^{-2}}{100\times10^{-4}\times600} = 0.6$ cm

13. $Y = \frac{FL}{Al}$ where length, cross section, load is same.

    $\therefore \quad Y \propto \frac{1}{l}$

    $\therefore \quad \frac{l_c}{l_s} = \frac{Y_s}{Y_c} = \frac{2\times10^{11}}{1.2\times10^{11}} = \frac{5}{3}$

    $\therefore \quad l_s = \frac{3}{5}\, l_c$

    Given, $4 = l_c + \frac{3}{5}\, l_c$

    $\therefore \quad 4 = \frac{8}{5} l_c$

    $\therefore \quad l_c = \frac{4\times5}{8} = \frac{5}{2}$ mm = 2.5 mm

14. $l_1 = \frac{FL_1}{A_1Y_1}, \; l_2 = \frac{FL_2}{A_2Y_2}$

    $\therefore \quad \frac{l_1}{l_2} = \frac{L_1}{A_1Y_1} \times \frac{A_2Y_2}{L_2}$

    $= \frac{4}{3} \times \frac{3}{5} \times \left(\frac{1}{2}\right)^2 = \frac{1}{5}$

15. $K = \frac{VdP}{dV}$

    Now, $K = 6 \times 10^9$ N/m^2 and

    $dP = 6 \times 10^8$ N/m^2

    $\therefore \quad \frac{dV}{V} = \frac{dP}{K} = \frac{6\times10^8}{6\times10^9} = \frac{1}{10}$

    $\therefore \quad \frac{dV}{V} \times 100 = \frac{1}{10} \times 100 = 10\%$

17. Breaking force = breaking stress × area of cross-section

    = wt. of wire

    $\therefore \quad S \times A = A\, l \times \rho\, g$

    $\therefore \quad l = \frac{S}{\rho g} = \frac{10^6}{3\times10^3\times9.8} \approx 34$ m

18. K.E of missile = elastic P.E.

    $\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}\, F \times l$

    or $\quad v = \sqrt{\frac{Fl}{m}}$ ....(i)

Let $Y = \frac{FL}{Al}$

$\therefore \quad F = \frac{YAl}{L} = \frac{5\times10^{8}\times10^{-6}\times2\times10^{-2}}{10\times10^{-2}} = 100\text{ N}$

From (i), $v = \sqrt{\frac{100\times0.02}{5\times10^{-3}}} = \sqrt{400} = 20\text{ m/s}$

19. As $Y = \frac{FL}{A\Delta L} = \frac{FL}{\pi r^2\Delta L}$,

$\therefore \quad r^2\Delta L = \frac{FL}{\pi Y} = \text{constant}$

$\therefore \quad r_1^2\,\Delta L_1 = r_2^2\,\Delta L_2$

or $\quad \Delta L_2 = \left(\frac{r}{2r}\right)^2 \Delta L_1 = \frac{\Delta L_1}{4}$

Now, $u = \text{Energy/Volume} = \frac{1}{2}Y\,(\text{strain})^2$

or $u \propto (\Delta L)^2$ or $\frac{u_2}{u_1} = \left(\frac{\Delta L_2}{\Delta L_1}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\therefore \quad u_1 : u_2 = 16 : 1.$

20. Energy per unit volume $= \frac{(\text{stress})^2}{2Y}$

$\therefore \quad \frac{E_1}{E_2} = \frac{Y_2}{Y_1}$ (Stress is constant)

$\therefore \quad \frac{E_1}{E_2} = \frac{3}{2}$

09 Mechanical properties of fluids: Viscosity

3. $v = \dfrac{2r^2(\rho-\sigma)g}{9\eta}$

$= \dfrac{2\times(2\times10^{-3})^2\times(8-1.3)\times10^3\times9.8}{9\times0.83}$

$= 0.07$ m/s

6. Av = constant

$\therefore$ $A_1v_1 = A_2v_2$

$\therefore$ $10 \times 2 = 12 \times v_2$

$\therefore$ $v_2 \approx 1.67$ cm/s

7. velocity of efflux, $v = \sqrt{2gh}$. It is independent of area of orifice and density of liquids.

8. $6\pi\eta rv = \dfrac{4}{3}\pi r^3\rho g$

$\therefore$ $v \propto r^2$

10. Pressure is independent of area.

11. Pressure is same on both sides

Hence $F_1/A_1 = F_2/A_2$.

$\therefore$ $F_2 = F_1A_2/A_1 = 10 \times 4/0.08 = 500$ N

13. $V = \dfrac{\pi pr^4}{8\eta l}$

$\therefore$ $V \propto \dfrac{r^4}{l}$ and $V_1 \propto \dfrac{(r/2)^4}{(2l)}$

$\therefore$ $\dfrac{V_1}{V} = \dfrac{(r/2)^4}{r^4}\dfrac{l}{2l} = \dfrac{1}{32}$

$\therefore$ $V_1 = \dfrac{V}{32}$

16. $a_1v_1 = a_2v_2$

$\therefore$ $\pi r_1^2 v_1 = \pi r_2^2 v_2$

$\therefore$ $\left(\dfrac{4}{2}\right)^2 \times 5 = v_2$

i.e., $v_2 = 20$ m/s

19. Hydraulic brakes work as per Pascal's law. Hence change in liquid pressure is transmitted equally to wheels.

20. Velocity head $h = \dfrac{v^2}{2g} = \dfrac{(5)^2}{2\times10} = 1.25$ m

22. $d_A = 2$ cm and $d_B = 4$ cm

$\therefore$ $r_A = 1$ cm and $r_B = 2$ cm

From equation of continuity, av = constant

$\therefore$ $\dfrac{v_A}{v_B} = \dfrac{a_B}{a_A} = \dfrac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\dfrac{2}{1}\right)^2$

$\therefore$ $v_A = 4v_B$

10 Mechanical properties of fluids: Surface Tension

5. $\frac{r_1}{r_2} = \frac{P_2}{P_1} = \frac{1}{3}$

$\therefore \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \frac{1}{27}$

6. Excess pressure $= \frac{4T}{R} = \frac{4\times30}{4\times10^{-1}}$
$= 300 \text{ dyne/cm}^2$

7. $h = \frac{2T}{r\rho g} = \frac{2\times75}{(0.005)\times1\times1000} = 30 \text{ cm}$

10. $l = \frac{h}{\sin\alpha} = \frac{24}{\sin60°} = 27.7 \text{ cm}$

where, α = angle of inclination

11. $h = \frac{2T\cos\theta}{r\rho g}$
$= \frac{2\times0.070\times\cos0}{0.04\times10^{-2}\times1000\times10}$
$= 0.035$ m
$= 3.5$ cm

12. In a gravity free space, the liquid in a capillary tube will rise to the full height of tube i.e., upto infinite height.

13. $2\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 2^{1/3}r$

Final surface area $= 4\pi R^2 = 4\pi 2^{2/3} r^2$
Initial surface area $= 2\times4\pi r^2$

$\therefore$ Energy released $= [8\pi r^2 - 4\times2^{2/3}\pi r^2]\,\sigma$

14. $h \propto \frac{1}{r}$ or $\frac{h_2}{h_1} = \frac{r_1}{r_2}$

Now, $r \propto \sqrt{A}$

$\therefore \frac{h_2}{h_1} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{1}{1/2}} = \sqrt{2}$

$\therefore h_2 = \sqrt{2}\,h_1 = 4\sqrt{2}$ cm

15. Net force $= (\sigma - \sigma')l = (0.072 - 0.038)\times1$
$= 0.034$ N

16. $\frac{R_1}{R_2} = \frac{2}{3}$

Let P_1 = Excess pressure inside the bubble of radius R_1

P_2 = Excess pressure inside the bubble of radius R_2

$\therefore P_1 = \frac{4T}{R_1}$ and $P_2\ \frac{4T}{R_2}$

$\therefore \frac{P_1}{P_2} = \frac{4T}{R_1}\times\frac{R_2}{4T} = \frac{R_2}{R_1} = \frac{3}{2}$

17. $g_m = \frac{g_e}{6}$

Now, $h = \frac{2T}{r\rho g}$

$\therefore \frac{h_m}{h_e} = \frac{g_e}{g_m} = 6 \Rightarrow h_m = 6\,h_e = 6\,h = 6\times6 = 36\text{cm}$

18. $\left[P_0 + \frac{4\sigma}{R_2}\right] - \left[P_0 + \frac{4\sigma}{R_1}\right] = \frac{4\sigma}{R}$

or $\frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$

or $R = \frac{R_1R_2}{R_1 - R_2} = \frac{20\times30}{10}$ mm

$= \frac{600}{10}$ mm $= 60$ mm $= 0.06$ m

19. Work done = change in area $\times$ T
$= (80 - 48)\times 2\times 30$ erg
$= 1920$ erg

20. Let thickness of layer be t

$V = At \Rightarrow t = \frac{V}{A}$

$2r = \frac{V}{A} \Rightarrow r = \frac{V}{2A}$

$\Delta P = \frac{T}{r}$

$\therefore F = \Delta P\times A = \frac{T}{r}\times A = \frac{T}{\left(\frac{V}{2A}\right)}A = \frac{2TA^2}{V}$

$\therefore T = \frac{FV}{2A^2} = \frac{14.5\times10^5\times0.06}{2\times(18)^2}$
$= 134.26$ dyne/cm

11 Thermal properties of Matter: Heat

7. $L = L_0(1 + \alpha t)$

$\therefore$ $50 = L_0 (1 + 16 \times 10^{-6} \times 65)$

$\therefore$ $50 = L_0 (1 + 1040 \times 10^{-6}) = L_0(1.001)$

$\therefore$ $L_0 = 49.95$ cm

8. $L_2 - L_1 = L_1 \alpha (t_2 - t_1)$

$\therefore$ $0.5 \times 10^{-2} = 12 \times 11 \times 10^{-6} \times (t_2 - 10)$

$\therefore$ $t_2 = 47.8$ °C

10. $$\gamma = \frac{\text{change in volume}}{\text{original volume} \times \text{change in temperature}}$$

$$= \frac{0.84}{100 \times 200}$$

$\therefore$ $\gamma = 42 \times 10^{-6} /$ °C

11. Heat lost by hot ball = Heat gained by water

$\therefore$ $m_1 \times c_1 (t_2 - t_0) = m_2 \times c_2 (t_0 - t_1)$

$\therefore$ $200 \times 0.08 \times (t - 22.8) = 500 \times 1 \times (22.8 - 10)$

$\therefore$ $t = 422.\,8$ °C

12. $$c = \frac{Q}{m\theta}$$

There is no change of temperature.

$\therefore$ $\theta = 0$ and $c = \infty$

13. In parallel combination, equivalent conductivity,

$$K = \frac{K_1A_1 + K_2A_2}{A_1 + A_2} = \frac{K_1 + K_2}{2} \quad [\because A_1 = A_2]$$

14. $a + r + t = 1$

$\therefore$ $0.70 + 0.25 + t = 1$

or $t = 1 - 0.95 = 0.05$

$\therefore$ $t = \dfrac{Q_t}{Q}$ or $Q_t = t \times Q = 0.05 \times 200 = 10$ cal

15. Temperature of interface,

$$\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1} = \frac{K \times 0 \times 2 + 3K \times 100 \times 1}{K \times 2 + 3K \times 1}$$

$\therefore$ $$\theta = \frac{300K}{5K} = 60 \text{ °C}$$

17. $$\frac{Q}{t} = \sigma A(T^4 - T_0^4)$$

$$\frac{Q}{t} \propto R^2 T^4$$

$\therefore$ $$\frac{\left(\frac{Q}{t}\right)_1}{\left(\frac{Q}{t}\right)_2} = \left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{T_1}{T_2}\right)^4 = 1$$

18. $\lambda_m T = b$

$\therefore$ $$T = \frac{b}{\lambda_m} = \frac{3 \times 10^{-3}}{48 \times 10^{-8}} = 6250\text{K}$$

20. $$\frac{61 - 59}{4} = K\left[\frac{61 + 59}{2} - 30\right] \quad \text{and}$$

$$\frac{51 - 49}{t} = K\left[\frac{51 + 49}{2} - 30\right]$$

$\therefore$ $\dfrac{t}{4} = \dfrac{30}{20}$ or $t = 6$ min

21. $R \propto (\theta - \theta_0)$

$\therefore$ $R_1 \propto (50 - 20)$ and $R_2 \propto (40 - 20)$

$\therefore$ $R_1 / R_2 = 3/2$

Since same amount of heat is lost in both the cases, therefore $R_1t_1 = R_2t_2$

This gives $t_2 = 3/2 \times 8 = 12$ minutes

22. $$\frac{(d\theta_1 / dt_1)}{(d\theta_2 / dt_2)} = \frac{(\theta_1' - \theta_0)}{(\theta_2' - \theta_0)}$$

$$\frac{0.75}{(d\theta_2 / dt_2)} = \frac{50}{30}$$

$\therefore$ $$\left(\frac{d\theta_2}{dt_2}\right) = \frac{0.75 \times 30}{50} = 0.45\text{°C/s}$$

23. $$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

$\therefore$ $$\frac{90 - 80}{10} = K[85 - 25] \quad \text{....(i) and}$$

$$\frac{80 - 70}{t} = K[75 - 25] \quad \text{....(ii)}$$

$\therefore$ From (i) and (ii),

$$\frac{10/10}{10/t} = \frac{60K}{50K} = \frac{6}{5}$$

$\therefore$ $\dfrac{t}{10} = \dfrac{6}{5}$ or $t = \dfrac{60}{5} = 12$ min.

12 Thermodynamics

14. $TV^{\gamma-1}$ = constant

$\therefore \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ or $\left(\frac{1}{2}\right)^{\gamma-1} = \sqrt{\frac{1}{2}}$

$\therefore \quad \gamma - 1 = \frac{1}{2}$ or $\gamma = \frac{3}{2}$

$\therefore \quad PV^{3/2}$ = constant

15. Amount of heat given = 540 cal
Change in volume, ΔV = 1670 c.c
Atmospheric pressure,
$P = 1.01 \times 10^6$ dyne /cm^2

$\therefore$ Work done against atmospheric pressure,

$W = P\Delta V = \frac{1.01 \times 10^6 \times 1670}{4.2 \times 10^7} \approx 40$ cal

17. $\Delta Q = \Delta U + \Delta W$

$\therefore \quad \Delta U = \Delta Q - \Delta W$

i.e $\frac{\Delta U}{\Delta Q} = 1 - \frac{\Delta W}{\Delta Q}$

$$= 1 - \frac{P\Delta V}{nC_p\Delta T}$$

$$= 1 - \frac{nR\Delta T}{nC_p\Delta T}$$

$$= \frac{C_p - R}{C_p}$$

$$\frac{\Delta U}{\Delta Q} = \frac{C_p - (C_p - C_V)}{C_P} = \frac{C_V}{C_P} = \frac{1}{\gamma} = \frac{5}{7}$$

$\therefore$ % of energy utilized in increasing internal energy is = 71.43%

18. According to first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
As the process is isothermal and gas is compressed
$\Delta U = 0$, $\Delta W = -$ve

$\therefore \quad \Delta Q = -\Delta W$

$\therefore \quad \Delta W = -\Delta Q$
$= -650 \times 4.18$ J
$= -2717$ J

19. n = 2, W = 5000 J, T = 300 K ,
R = 8.2 J/mol K
Using,

$W = 2.303nRT \log \left[\frac{P_1}{P_2}\right]$

$\therefore \quad 5000 = 2.303 \times 2 \times 8.2 \times 300 \times \log_{10}\left[\frac{P_1}{P_2}\right]$

$\log_{10}\left[\frac{P_1}{P_2}\right] = \frac{5000}{11330.76} = 0.44$

$\therefore \quad \frac{P_1}{P_2} = (10)^{0.44} = 2.75 \approx \frac{11}{4}$

$\therefore \quad \frac{P_2}{P_1} = \frac{4}{11}$

20. He is monoatomic

$\therefore \quad \gamma_{He} = \frac{5}{3}$

Also, For adiabatic change,
Bulk modulus (K_{adi}) = γP

And, Compressibility (B) = $\frac{1}{K}$

$\Rightarrow P = \frac{K_{adi}}{\gamma} = \frac{1}{B_{adi}\gamma} = \frac{1 \times 3}{3.73 \times 10^{-8} \times 5}$

$= 1.62 \times 10^7$ Pa

21. W_{ABCD} = Area under ABCD
$W_{AB} = 0$ …..(V = Constant)

$\therefore \quad W_{ABCD} = W_{BC} + W_{CD}$

$= 2V_0P_0 + \frac{1}{2}P_0V_0 + P_0V_0$

$= \frac{7}{2}P_0V_0$

13 Kinetic Theory of Gases

1. As a molecular weight of oxygen is 32 kg/mole, 8 kg of oxygen has

$$n = \frac{8\,\text{kg}}{32\,\text{kg/mole}} = \frac{1}{4} \text{ mole}$$

The equation of state is PV = nRT

$\therefore$ $PV = \frac{1}{4} RT$ or $PV = \frac{RT}{4}$

2. According to the gas equation, PV = nkT

For the gas A, we have $PV = n_1kT$

For the gas B, we have $(2P)(V/4) = n_2k(2T)$

$PV = 4n_2 kT$

$\therefore$ $n_1 = 4 n_2$

$\therefore$ $\frac{n_1}{n_2} = 4$

5. $P = \frac{1}{3}\rho c^2$

Since mass and volume are same, the density is constant. $P \propto c^2$

But $c^2 \propto 1/M$

$\therefore$ $P \propto \frac{1}{M}$

$\therefore$ $\frac{P_O}{P_H} = \frac{M_H}{M_O} = \frac{2}{32} = \frac{1}{16}$

$\therefore$ $P_O = 0.25$ atm

6. $PV = \frac{m}{M} RT$

$\therefore$ $P = \frac{4.032}{2} \times \frac{8.3 \times 273}{16 \times 10^{-3}} = 2.8$ atm

8. K.E / mole $= \frac{3}{2} RT$

$= 1.5 \times 8.3 \times 273$

$= 3.40 \times 10^3$ J/mole

9. Average kinetic energy is independent of the volume of the gases.

10. $E_{av} = \frac{3}{2} kT$ or $E_{av} \propto T$

$\therefore$ $\frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$

$\therefore$ $E_2 = 2E_1 = 2E$

12. $\frac{R}{C_v} = 0.4 = \frac{2}{5}$

$\therefore$ $C_v = \frac{5}{2} R$, which holds true for diatomic gases.

13. Specific heat for monatomic gas, $C_V = \frac{3}{2} R$

$(\Delta Q)_V = nC_V\Delta T$

$\therefore$ $\Delta Q = 2 \times \frac{3}{2} \times 100 \times R$

$= 300$ R

14. For diatomic gas like CO,

$C_p = \frac{7}{2} R$ and $C_v = \frac{5}{2} R$

$\therefore$ $\gamma_{di} = \frac{C_P}{C_V} = \frac{\left(\frac{7R}{2}\right)}{\left(\frac{5R}{2}\right)} = 1.40$

18. Average K.E. of molecule $= \frac{3}{2} kT$

k = Boltzmann's constant

Energy gain by electron $= eV = 1.6 \times 10^{-19} \times 1$

$\therefore$ $\frac{3}{2} \times 1.38 \times 10^{-23} \times T = 1.6 \times 10^{-19} \times 1$

$\therefore$ $T = \frac{3.2 \times 10^4}{1.38 \times 3} = \frac{32}{4.14} \times 10^3 = 7.7 \times 10^3$ K

19. Specific heat for monatomic gas $= \frac{3}{2} R$

$\therefore$ $\frac{3}{2} R = \frac{\Delta Q}{2 \times (473 - 273)} = \frac{\Delta Q}{2 \times 200}$

$\therefore$ $\Delta Q = \frac{3}{2} \times 2 \times 200 \times R = 600$ R.

14 Oscillations

1. $v = \omega(\sqrt{A^2 - x^2})$

$= \omega\sqrt{A^2 - \frac{A^2}{4}}$

$= \omega\sqrt{\frac{3A^2}{4}} = \frac{\omega\sqrt{3}A}{2}$

$= \frac{\sqrt{3}}{2}\omega A = \frac{\sqrt{3}}{2}v_0$ ....($\because v_{max} = A\omega = v_0$)

2. Pendulum clock has period T. When this clock is placed on moon where acceleration due to gravity is one-sixth that of earth, the period will increase. Hence more time will be required to complete one rotation. Hence it goes slower.

3. Total energy = $\frac{1}{2}m\omega^2a^2$

Kinetic energy = $\frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$= \frac{1}{2}m\omega^2A^2\left(1 - \frac{1}{4}\right)$

$= \frac{3}{4} \times \left(\frac{1}{2}m\omega^2A^2\right)$

$= \frac{3}{4}$ (T.E.)

$\therefore \frac{\text{K.E.}}{\text{T.E.}} = \frac{3}{4}$

4. A = 5 m, T = 0.02 s

$\therefore \omega = \frac{\pi}{6} = \frac{2\pi}{0.02} = 100\,\pi$ rad/s

Equation of particle performing S.H.M is given by,

$x = A\sin(\omega t + \theta)$

$\therefore 2.5 = 5\sin(100\pi \times 0 + \theta)$

$\therefore \frac{2.5}{5} = \sin\theta = \frac{1}{2}$

$\therefore \theta = 30°$ or $\frac{y^2}{A^2} + \frac{v^2}{A^2\omega^2}$

$\therefore x = 5\sin\left(100\pi t + \frac{\pi}{6}\right)$

5. Let n_1 and n_2 be the number of oscillations by the two pendulums with length $l_1 = 1$ m and $l_2 = 4$ m respectively.

$\therefore \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore T_2 = 2T_1$

They are in phase again, when

$(\omega_1 - \omega_2)t = 2\pi$

$\therefore \left(\frac{2\pi}{T_1} - \frac{2\pi}{T_2}\right)t = 2\pi$

$\therefore \left(\frac{2\pi}{T_1} - \frac{2\pi}{2T_1}\right)t = 2\,\pi$

$\therefore \frac{2\pi}{T_1}\left(\frac{1}{2}\right)t = 2\pi$ or $t = 2T_1$

Number of oscillations of pendulum having time period T in t second is $n = \frac{t}{T}$

$\therefore n_1 = \frac{2T_1}{T_1} = 2$ and

$n_2 = \frac{2T_1}{T_2} = \frac{2T_1}{2T_1} = 1$

$\therefore$ When shorter pendulum completes 2 oscillations, they will be in phase.

6. For second's pendulum,

T = 2 s

$T = 2\pi\sqrt{\frac{l}{g}}$ or $2 = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \frac{1}{\pi^2} = \frac{l}{g}$ or $l = \frac{g}{\pi^2}$

7. $v = \omega\sqrt{A^2 - x^2}$

$\therefore 10 = \omega(A^2 - 16)^{1/2}$

or $100 = \omega^2A^2 - 16\omega^2$ ....(i)

Also, $8 = \omega(A^2 - 25)^{1/2}$

or $64 = \omega^2A^2 - 25\omega^2$ ....(ii)

Equation (i) – Equation (ii) gives,

$100 - 64 = \omega^2A^2 - 16\omega^2 - \omega^2A^2 + 25\omega^2$

$\therefore 36 = 9\omega^2$

$\therefore \omega^2 = 4$ or $\omega = 2$

$\therefore \frac{2\pi}{T} = 2$ or $T = \pi$ s

8. $v_{max} = A\omega = 0.4 \times \frac{2\pi \times 180}{60} \approx 7.6$ m/s^2

9. Equation of S.H.M is,

$x = A\sin\omega t$

for $x = \frac{A}{2}$ we have,

$\frac{A}{2} = A\sin\omega t$

$\therefore \omega t = \sin^{-1}\left(\frac{1}{2}\right)$

$\therefore \frac{2\pi t}{T} = \frac{\pi}{6}$ $\therefore t = \frac{T}{12}$ s

10. $T = 2\pi\sqrt{\frac{l}{g}}$ or $T^2 \propto l$

$\therefore$ $2\left(\frac{dT}{T}\times 100\right) = \left(\frac{dl}{l}\times 100\right)$

$\therefore$ $\frac{dT}{T} \times 100 = \frac{1}{2}(0.1) = 0.05\%$

$\therefore$ For 100 s, it loses 0.05 s
That means for a day of 86400 s, it loses
$864 \times 0.05 = 43.2$ second

12. Comparing with differential equation of S.H.M. we get,
$\omega^2 = 100 \Rightarrow \omega = 10$

$\therefore$ $2\pi n = 10$ or $n = \frac{10}{2\pi}$ Hz

13. Acceleration $= -\omega^2 x$

$$= -\left(\frac{2\pi}{T}\right)^2 \times \frac{A}{2}$$

$$= \frac{-2\pi^2 A}{T^2}$$

14. $x = A\sin(\omega t + \alpha)$

$\therefore$ $4 = 8\sin(\omega t + \alpha)$
where $t = 0$, $4 = 8\sin\alpha$
$\sin\alpha = 0.5$

$\therefore$ $\alpha = \pi/6$ or $30°$

15. $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore$ $T \propto \sqrt{l}$

$\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$

But $l_2 = 0.98\, l_1$

$\therefore$ $\frac{T_2}{T_1} = \sqrt{\frac{0.98\, l_1}{l_1}}$

$\therefore$ $T_2 = T_1\sqrt{0.98}$
$= 0.99\, T_1$

$\therefore$ Time period decreases by 1%

16. $T = 2\pi\sqrt{\frac{m}{k}}$ and $mg = kx_0$

or $\frac{m}{k} = \frac{x_0}{g} = \frac{10\times 10^{-2}}{10} = \frac{1}{100}$

$\therefore$ $T = 2\pi\sqrt{\frac{1}{100}} = \frac{\pi}{5}$ s

17. $\frac{1}{2}m\omega^2(A^2 - x^2) = 3\left(\frac{1}{2}m\omega^2 x^2\right)$

$\therefore$ $A^2 = 4x^2$ or $A = 2x$

$\therefore$ $x = \frac{A}{2} = \frac{8}{2} = 4$ mm

18. $T = 2\pi\sqrt{\frac{l}{g}}$

$$T' = 2\pi\sqrt{\frac{l}{\left(g + \frac{g}{2}\right)}} = 2\pi\sqrt{\frac{l}{\left(\frac{3g}{2}\right)}}$$

$$= 2\pi\sqrt{\frac{2}{3}}\sqrt{\frac{l}{g}} = \sqrt{\frac{2}{3}}\,T$$

19. Given differential equation is,

$$\frac{d^2x}{dt^2} + 2x = 0$$

Comparing this with equation of linear S.H.M,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \text{ we get,}$$

$\omega^2 = 2 \Rightarrow \omega = \sqrt{2}$ rad/s

$\therefore$ $\omega = 2\pi n$ or $\sqrt{2} = 2\pi n$

$\therefore$ $n = \frac{\sqrt{2}}{2\pi} = \frac{1}{\sqrt{2}\pi}\, s^{-1}$

20. $\omega = \frac{2\pi}{T} = \frac{2\times 3.14}{1.57} = 4$ rad/s

Now, K.E $= \frac{1}{2} \times m\omega^2 \times (A^2 - x^2)$

$$= \frac{1}{2} \times 10 \times (4)^2 \times (100 - 36)$$

$$= \frac{1}{2} \times 10 \times 16 \times 64 = 5120 \text{ erg}$$

15 Wave Mechanics

1. Comparing the given equation with standard wave equation,

$\omega = 2\pi$ or $2\pi n = 2\pi$

$\therefore$ $n = 1\ s^{-1}$

4. $y_1 = 5 \sin 516\, t$, $y_2 = 5 \sin 524\, t$

Comparing with equation $y = A \sin \omega t$

$\omega_1 t = 516\, t$ or $\omega_1 = 516$

$\therefore$ $\frac{2\pi}{T_1} = 516$ or $\frac{1}{T_1} = \frac{516}{2\pi}$

$\therefore$ $n_1 = \frac{516}{2\pi}$

Similarly, $n_2 = \frac{524}{2\pi}$

$\therefore$ Beat frequency, $n_2 - n_1 = \frac{524}{2\pi} - \frac{516}{2\pi} = \frac{8}{2\pi} = \frac{4}{\pi}$

6. Here, $\frac{v}{4l} = 412$ Hz

For one part closed at one end,

$n_1 = \frac{v}{4(l/2)} = 2\left(\frac{v}{4l}\right)$

$= 2 \times 412 = 824$ Hz

For second part, open at both ends,

$n_2 = \frac{v}{2(l/2)}$

$= \frac{v}{l} = 4 \times \frac{v}{4l}$

$= 4 \times 412 = 1648$ Hz

8. As source approaches the observer,

$n' = n\left(\frac{v}{v - v_s}\right) = \frac{1200 \times 400}{400 - 100} = 1600$ Hz.

10. $n \propto \sqrt{T}$

$\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$n_2 = 3n_1$ or $\frac{n_1}{n_2} = \frac{1}{3}$

$\therefore$ $\frac{1}{3} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{T_1}{T_1 + 8}}$

$\therefore$ $\frac{T_1}{T_1 + 8} = \frac{1}{9}$

$\therefore$ $9T_1 = T_1 + 8$

$\therefore$ $8T_1 = 8$

$\therefore$ $T_1 = 1$ kg-wt

12. $y = A \sin(100t) \cos(0.01x)$

Comparing it with standard wave equation

$y = 2A \sin\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi x}{\lambda}\right)$, we get

$\frac{2\pi t}{T} = 100\, t$ or $T = \frac{2\pi}{100}$

$\therefore$ $n = \frac{1}{T} = \frac{100}{2\pi}$

Also, $\frac{2\pi x}{\lambda} = 0.01\, x$

$\therefore$ $\lambda = \frac{2\pi}{0.01}$

Velocity of wave, $v = n\lambda$

$= \frac{100}{2\pi} \times \frac{2\pi}{0.01}$

$= 10^4$ mm/s

$= 10 \times 10^3$ mm/s

$= 10$ m/s

13. n_A = Known frequency = 256 Hz

x = 4 bps, which is decreasing after loading (i.e. x$\downarrow$) Also given that, tuning fork is loaded. So $n_A\downarrow$

Hence, $n_A\downarrow - n_B = x\downarrow$ .... (i) $\longrightarrow$ Correct

$\therefore$ $n_B - n_A\downarrow = x\downarrow$ ....(ii) $\longrightarrow$ Wrong

$\therefore$ $n_B = n_A - x = 256 - 4 = 252$ Hz.

14. $v \propto \sqrt{T}$, $v_1 \propto \sqrt{T_1}$ and $v_2 \propto \sqrt{T_2}$

Also, $\frac{v_2}{v_1} = 2$

$\therefore$ $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{T_2}{273 + 27}} = \sqrt{\frac{T_2}{300}}$

$\therefore$ $2 = \sqrt{\frac{T_2}{300}}$

$\therefore$ $4 = \frac{T_2}{300}$

$\therefore$ $T_2 = 4 \times 300 = 1200$ K

$\therefore$ $T_2 = 1200 - 273$

$\therefore$ $T_2 = 927$ °C

16. $v = \sqrt{\frac{T}{m}}$

Now, $m = \frac{M}{L} = \frac{V\rho}{L} = \frac{AL\rho}{L} = A\rho$

$\therefore$ $v = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{10^3}{10 \times 10^{-6} \times 100}}$

$= \sqrt{10^6} = 1000$ m/s.

19. Fundamental frequency of closed pipe,

$n' = \frac{v}{4l}$

$\therefore \quad l = \frac{v}{4n'}$

Fundamental frequency of open pipe,

$n = \frac{v}{2l}$

$\therefore \quad l = \frac{v}{2n}$

$\therefore \quad \frac{v}{4n'} = \frac{v}{2n}$

$\therefore \quad n' = \frac{n}{2}$

22. velocity of sound in gas

$v = \sqrt{\frac{\gamma RT}{M}}$ Here $v \propto \frac{1}{\sqrt{M}}$

($\because$ γ and T are constant)

$\therefore \quad \frac{v_{H_2}}{v_{O_2}} = \frac{M_{O_2}}{M_{H_2}}$

$\frac{1248}{v_{O_2}} = \sqrt{\frac{32}{2}}$

$\therefore \quad v_{O_2} = \frac{1248}{4} = 312$ m/s

23. Distance between crest and nearest trough

$= \frac{\lambda}{2} = 2.5$ or $\lambda = 5$ cm

$n = 4$ /s ....[Given]

$\therefore \quad v = n\lambda = 4 \times 5 = 20$ cm/s

24. Let x be the number of tuning forks

$n_x = n + (x - 1)\,4$

Given that, $n_x = 2n$

$\therefore \quad n + (x - 1)\,4 = 2n$

$\therefore \quad x - 1 = \frac{n}{4} = \frac{256}{4} = 64$

$\therefore \quad x = 65.$

25. $n \propto \frac{1}{l}$

$\therefore \quad \frac{n_2}{n_1} = \frac{l_1}{l_2} = \frac{36}{48} = \frac{3}{4}$

$\therefore \quad n_2 = \frac{3}{4} \times n_1 = \frac{3}{4} \times 256 = 3 \times 64 = 192$ Hz.

27. $\frac{A_1}{A_2} = \frac{5}{3}$ or $A_1 = \frac{5}{3} A_2$

$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\frac{5}{3}A_2 + A_2\right)^2}{\left(\frac{5}{3}A_2 - A_2\right)^2}$

$= \left(\frac{\frac{8A_2}{3}}{\frac{2A_2}{3}}\right)^2 = \left(\frac{4}{1}\right)^2$

$= \frac{16}{1}$

$\therefore \quad I_{max} : I_{min} = 16 : 1$

28. $n \propto \sqrt{T}$

$\therefore \quad \frac{n_2}{n_1} = \sqrt{\frac{4T_1}{T_1}} = 2$

29. $y_1 = A \sin (1000\pi t)$

$\therefore \quad \omega_1 = 2\pi n_1 = 1000\,\pi \quad \Rightarrow n_1 = 500$ Hz

$\omega_2 = y_2 = A \sin (1008\pi t)$

$\therefore \quad 2\pi n_2 = 1008\,\pi \Rightarrow n_2 = 504$ Hz

$\therefore \quad n_2 - n_1 = 504 - 500 = 4$ Hz.