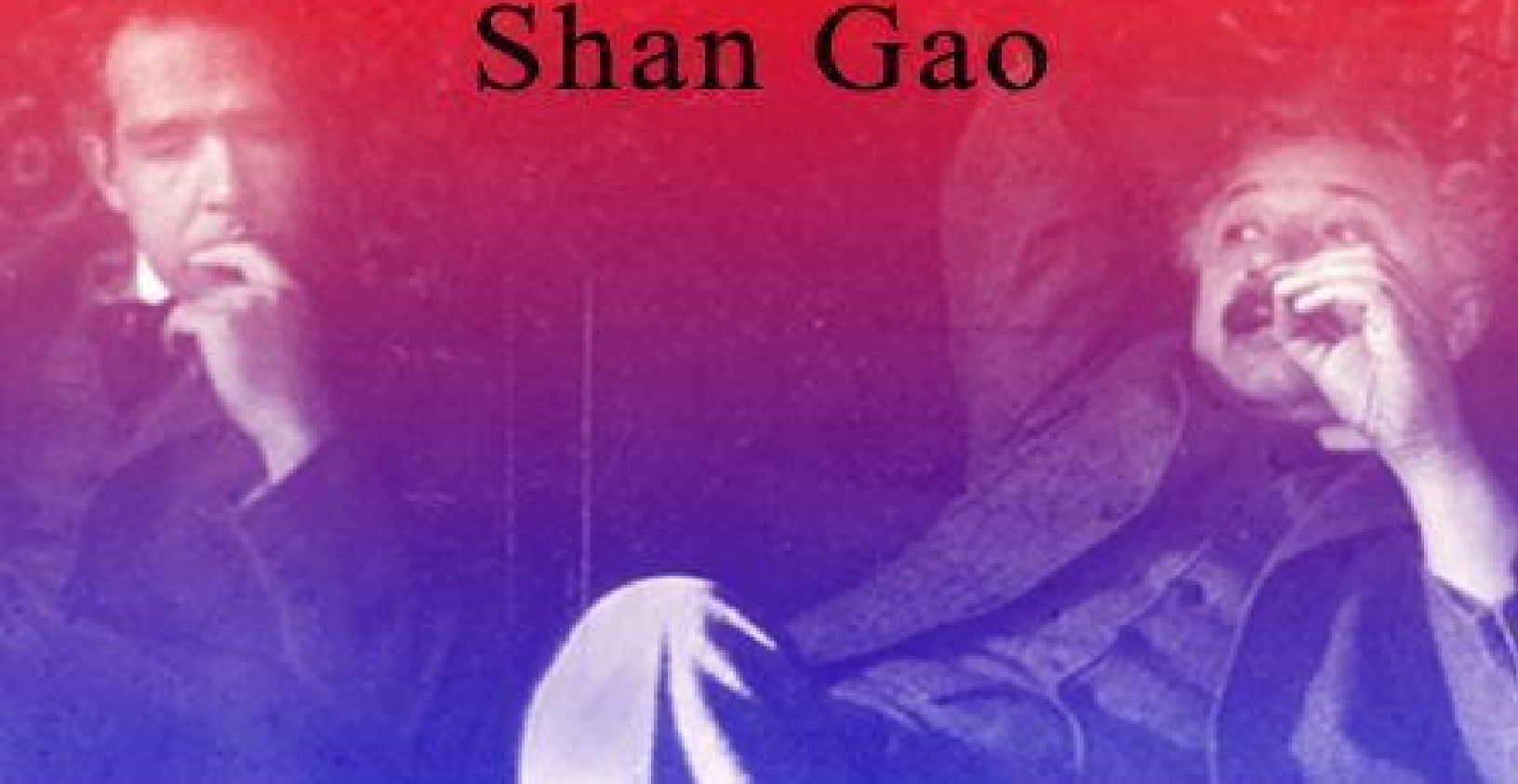
Understanding Quantum Physics

An Advanced Guide for the Perplexed

Shan Gao



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This book is dedicated to Erwin Schrödinger, who introduced the wave function, discovered its equation named after him, and argued that quantum mechanics is incomplete by his famous cat paradox.

It has even been doubted whether what goes on in an atom can be described within a scheme of space and time. From a philosophical standpoint, I should consider a conclusive decision in this sense as equivalent to a complete surrender. For we cannot really avoid our thinking in terms of space and time, and what we cannot comprehend within it, we cannot comprehend at all.

—Erwin Schrödinger

Someday we'll understand the whole thing as one single marvelous vision that will seem so overwhelmingly simple and beautiful that we may say to each other; "Oh, how could we have been so stupid for so long? How could it have been otherwise!"

—John Archibald Wheeler

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Acknowledgements

The idea of random discontinuous motion of particles came to my mind when I was a postgraduate at the Institute of Electronics, Chinese Academy of Sciences in 1993. I am happy that finally it has a more logical and satisfying formulation in this book. During the past twenty or so years, I have benefited from discussions with many physicists and philosophers of physics who care about the way the world really is. They are: Stephen Adler, Jacob Bekenstein, Samuel Braunstein, Jeremy Butterfield, Tian Yu Cao, Zexian Cao, Philippe Eberhard, Bernard d' Espagnat, Shelly Goldstein, Basil Hiley, Gerard 't Hooft, Sabine Hossenfelder, Chris Isham, Frederick M. Kronz, Gui-Lu Long, Alwyn van der Merwe, David Miller, Philip Pearle, Roger Penrose, Huw Price, Alastair Rae, Erasmo Recami, Dean Rickles, Abner Shimony, Henry P. Stapp, George Svetlichny, Antoine Suarez, Antony Valentini, Hans Westman, Christian Wüthrich, Heinz-Dieter Zeh, and Anton Zeilinger, among others. I thank them all deeply.

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Shan Gao

Sydney

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A True Story of Quantum Exploration

—An Excerpt from *God Does Play Dice With the Universe*

During my childhood, it had been a wonder for me that the twinkling stars strewed in the night sky don't fall to the Earth. I had a strong desire to know the whys and wherefores. Later I found the answer in textbooks. It changed my picture of the universe. When I was an undergraduate, I began to be entranced by the deep mysteries of the atomic world. I was especially stunned by the fact that the commonsense planetary picture of atoms turns out to be utterly false; the electron in an atom cannot rotate round the atomic nucleus as the Earth rotates round the sun, or else it would soon radiate its energy and fall into the nucleus, and as a result, my body composed of atoms would collapse in a blink. How does the electron move then? It must exist in the atom. It must move in some way there. But more surprisingly, textbooks provided no picture of the motion of the electron. On 22 August 1987, I wrote in my diary: "Is it really true that we have no way to describe the atomic processes as processes happening in space and time?" I could only search for the answer by myself. Then I started on a lonely journey to "trace" the elusive electron at the age of 16.

In order to find how the electron moves in an atom, I went to the Institute of Electronics, Chinese Academy of Sciences to pursue my graduate study. But it was according to expectation that nobody there could give me any tips either. I then spent nearly every day in musing on the seemingly indescribable motion of electrons. If a ball indeed moves in a continuous way, then it seems that an electron or an atom should also move in the same way. The ball is composed of atoms after all. But, on the other hand, if an electron moves continuously in an atom, it will soon fall into the nucleus, while the tragedy does not happen in reality. This is a great dilemma. I found some possible solutions, but they shortly proved to be wrong.

The puzzle had been plaguing me. Day after day, I gradually doubted the reality of continuous motion. But I still felt in my bones that the particles must move in some way. Finally, in the early morning of 12 October 1993, I experienced a sudden enlightenment. At that moment, I felt that my body permeated the whole universe and I was united with it. I "disappeared". A clear picture then appeared: a particle is jumping in a random and discontinuous way. It is not inert but active; it moves purely by its own "free will". Maybe God does play dice in the atomic world. I finally broke loose the tightest shackles of continuous motion with the help of inspiration. After this event, the outcome seems very natural from a logical point of view. If a particle cannot move continuously, it must move in a discontinuous way. How deep-rooted the prejudice of the uniqueness

of continuous motion is!

If an atom moves in a random and discontinuous way, then it can easily pass through two slits at the same time. But why does a ball appear to move in a contrary way? Moreover, why on earth does God play dice? These puzzles further haunted me. After graduated from the Institute of Electronics, I decided to be an independent theoretical physicist, or more accurately, a natural philosopher who aims at understanding the mysterious universe. Life was not easy. But I never gave up my research, and I never stop thinking. It had become the theme of my life.

As time went on, the picture of random discontinuous motion became clearer and clearer in my mind. When I took a walk one afternoon in June 2001, I suddenly had another inspiration after long reflection in solitude and meditation. I realized that motion has no cause in reality, and thus it must be essentially random, i.e., God must play dice. Moreover, the familiar phenomenon of inertia has already revealed that a ball also jumps in a random and discontinuous way just like an atom. This is another new idea. Maybe the path to truth is always devious in order that surprise can hide at the turn waiting for persevering seekers. God also plays dice in our everyday world. He actually plays dice with the whole universe. What a harmonic world!

I simply want to know the answer of a naive question. I simply think on it continually. But the exploration has completely changed my life. It shapes my way through the world and finally leads me to God, the ultimate reality. As Trinity said in *The Matrix*, "It’s the question that brought you here… The answer is out there, Neo, and it’s looking for you, and it will find you if you want it to."

Chapter 1
How Can We Understand Quantum Mechanics?

I think I can safely say that nobody understands quantum mechanics... Do not keep saying to yourself, if you can possible avoid it, "But how can it be like that?" because you will get 'down the drain', into a blind alley from which nobody has escaped. Nobody knows how it can be like that.

—Richard Feynman

Quantum mechanics, according to its Schrödinger picture, is a nonrelativistic theory about the wave function and its evolution. There are two main problems in the conceptual foundations of quantum mechanics. The first one concerns the physical meaning of the wave function in the theory. It has been widely argued that the probability interpretation is not wholly satisfactory because of resorting to the vague concept of measurement - though it is still the standard interpretation in textbooks nowadays. On the other hand, the meaning of the wave function is also in dispute in the alternative formulations of quantum mechanics such as the de Broglie-Bohm theory and the many-worlds interpretation. Exactly what does the wave function describe then?

The second problem concerns the evolution of the wave function. It includes two parts. One part concerns the linear Schrödinger evolution. Why does the linear nonrelativistic evolution of the wave function satisfy the Schrödinger equation? It seems that a satisfactory derivation of the equation is still missing. The other part concerns the collapse of the wave function during a measurement, which is usually called the measurement problem. The collapse postulate in quantum mechanics is ad hoc, and the theory does not tell us how a definite measurement result emerges. Although the alternatives to quantum mechanics already give their respective solutions to this problem, it has been a hot topic of debate which solution is right or in the right direction. In the final analysis, it is still unknown whether the wavefunction collapse is real or not. Even if the wave function does collapse under some circumstances, it remains unclear exactly why and how the wave function collapses. The measurement problem has been widely acknowledged as one of the hardest and most important problems in the foundations of quantum mechanics.

Let's illustrate these problems with a typical double-slit experiment with single electrons. In the experiment, the single electron is emitted from a source one after the other, and then passes through two slits to arrive at the detecting screen. Each electron is detected only as a random spot on the screen. But when a large number of electrons with the same energy arrive at the screen, these spots collectively form an undulant double-slit pattern. The ridges in the pattern are

formed in the positions where more electrons reach, and the valleys in the pattern are formed in the positions where nearly no electrons reach. In particular, the double-slit interference pattern is significantly different from the direct mixture of two one-slit patterns, each of which is formed by opening each of the two slits independently. It is well known that classical mechanics cannot provide a satisfactory explanation of the double-slit experiment. Unfortunately, quantum mechanics cannot either.

The quantum mechanical "explanation" of the double-slit experiment with electrons can be formulated as follows. A wave function is prepared and emitted from the source of electrons. This mathematical wave function then passes through two physical slits, and its evolution follows the linear Schrödinger equation. At last, the superposed wave function reaches the detecting screen and is measured there. By the collapse postulate, it instantaneously and randomly collapses to a local wave function, which corresponds to a determinate, random measurement result, a spot on the screen. Moreover, according to the Born rule, the probability density of the appearance of the spot is given by the modulus square of the wave function (immediately before the measurement) there. Although the predictions of quantum mechanics for the probability distribution of measurement results agree with the double-slit interference pattern to astonishing precision, it keeps silent as to what physical process happens from the preparation to the measurement of a single electron; there is only a mathematical wave function that spreads, superposes and collapses during the whole process.

As Feynman once claimed, the double-slit experiment contains the only mystery of quantum mechanics. In fact, there are two mysteries, corresponding to the above two fundamental problems of quantum mechanics. First of all, it is unknown what physical state the mathematical wave function describes. Exactly what is an electron? Is it a localized particle or a spreading wave or both or neither? How does it pass through the two slits? Note that the wave function lives not in the real three-dimensional space but in the multi-dimensional configuration space for a many-body system. Then what does the system described by it really look like in real space? Next, it remains unclear how come the Schrödinger equation and the Born rule. This is the key to account for the double-slit interference pattern and all other quantum phenomena. Why does the wave function of a single electron obey the linear Schrödinger equation when not being measured? Why does it undergo collapse when being measured? Is the collapse of the wave function a real physical process? If the answer is negative, then how to explain the emergence of definite measurement results? If the answer is positive, then why and how does the wave function collapse?

In this book, we will try to solve these problems from a new angle. The key is to realize that the problem of interpreting the wave function may be solved independent of how to solve the measurement problem, and the solution to the first problem can then have important implications for the solution to the second one. Although the meaning of the wave function should be ranked as the first interpretative problem of quantum mechanics, it has been treated as a marginal problem, especially compared with the measurement problem. As noted above, there are already several alternatives to quantum mechanics which give respective solutions to the measurement problem. However, these theories in their present stages are unsatisfactory at least in one aspect; they have not succeeded in making sense of the wave function. Different from them, our strategy is to first find what physical state the wave function describes and then investigate the implications of the answer for the solutions to other fundamental problems of quantum mechanics.

It seems quite reasonable that we had better know what the wave function is before we want to figure out how it evolves, *e.g.* whether it collapses or not during a measurement. However, these problems are generally connected to each other. In particular, in order to know what physical state the wave function of a quantum system describes, we need to measure the system in the first place, while the measuring process and the measurement result are necessarily determined by the evolution law for the wave function. Fortunately, it has been realized that the conventional measurement that leads to the collapse of the wave function is only one kind of quantum measurement, and there also exists another kind of measurement that avoids the collapse of the wave function, namely the protective measurement proposed by Aharonov, Vaidman and Anandan in 1993. Protective measurement is a method to measure the expectation values of observables on a single quantum system, and its mechanism is independent of the controversial process of wavefunction collapse and only depends on the established parts of quantum mechanics. As a result, protective measurement can not only measure the physical state of a quantum system and help to unveil the meaning of the wave function, but also be used to examine the solutions to the measurement problem before experiments give the last verdict. A full exposition of these ideas will be given in the subsequent chapters.

In Chapter 2, we first investigate the physical meaning of the wave function. According to protective measurement, the mass and charge density of a quantum system as one part of its physical state can be measured as expectation values of certain observables, and it turns out that they are proportional to the modulus square of the wave function of the system. The key to unveil the meaning of the wave function is to find the origin of the mass and charge density. It is shown

that the density is not real but effective; it is formed by the time average of the ergodic motion of a localized particle with the total mass and charge of the system. Moreover, it is argued that the ergodic motion is not continuous but discontinuous and random. Based on this result, we suggest that the wave function represents the state of random discontinuous motion of particles, and in particular, the modulus square of the wave function gives the probability density of the particles appearing in certain positions in real space.

In Chapter 3, we further analyze the linear evolution law for the wave function. It is shown that the linear nonrelativistic evolution of the wave function of an isolated system obeys the free Schrödinger equation due to the requirements of spacetime translation invariance and relativistic invariance. Though these requirements are already well known, an explicit and complete derivation of the free Schrödinger equation using them is still missing in the literature. The new integrated analysis, which is consistent with the suggested interpretation of the wave function, may help to understand the physical origin of the Schrödinger equation, as well as the meanings of momentum and energy for the random discontinuous motion of particles. In addition, we also analyze the physical basis and meaning of the principle of conservation of energy and momentum in quantum mechanics.

In Chapter 4, we investigate the implications of protective measurement and the suggested interpretation of the wave function based on it for the solutions to the measurement problem. To begin with, we argue that the two no-collapse quantum theories, namely the de Broglie-Bohm theory and the many-worlds interpretation, are inconsistent with protective measurement and the picture of random discontinuous motion of particles. This result strongly suggests that wavefunction collapse is a real physical process. Secondly, we argue that the random discontinuous motion of particles may provide an appropriate random source to collapse the wave function. The key point is to realize that the instantaneous state of a particle not only includes its wave function but also includes its random position, momentum and energy that undergo the discontinuous motion, and these random variables can have a stochastic influence on the evolution of the wave function and further lead to the collapse of the wave function. Moreover, it is argued that the principle of conservation of energy (for an ensemble of identical systems) requires that the random variable that influences the evolution of the wave function is not position but energy, and due to the discontinuity of motion the influence can accumulate only when time is discrete. As a result, wavefunction collapse will be a discrete process, and the collapse states will be the energy eigenstates of the total Hamiltonian of a given system in general. Thirdly, we propose a discrete model of energy-conserved

wavefunction collapse based on the above analysis. It is shown that the model is consistent with existing experiments and our macroscopic experience. Lastly, we also give some critical comments on other dynamical collapse models, including Penrose's gravity-induced collapse model and the CSL (Continuous Spontaneous Localization) model.

In the last chapter, we give some primary considerations on the unification of quantum mechanics and special relativity in terms of random discontinuous motion of particles. It is argued that a consistent description of random discontinuous motion of particles requires absolute simultaneity, and this leads to the existence of a preferred Lorentz frame when combined with the requirement of the constancy of speed of light. Moreover, it is shown that the collapse dynamics may provide a method to detect the frame according to our energy-conserved collapse model.

Chapter 2
Meaning of the Wave Function

What does the ψ-function mean now, that is, what does the system described by it really look like in three dimensions?

—Erwin Schrödinger

The physical meaning of the wave function is an important interpretative problem of quantum mechanics. Notwithstanding more than eighty years' developments of the theory, however, it is still a debated issue. Besides the standard probability interpretation in textbooks, there are various conflicting views on the wave function in the alternatives to quantum mechanics. In this chapter, we will try to solve this fundamental interpretive problem through a new analysis of protective measurement and the mass and charge density of a single quantum system.

The meaning of the wave function is often analyzed in the context of conventional impulse measurements, for which the coupling interaction between the measured system and measuring device is of short duration and strong. As a result, even though the wave function of a quantum system is in general extended over space, an impulse position measurement will inevitably collapse the wave function and can only detect the system in a random position in space. Then it is unsurprising that the wave function is assumed to be only related to the probability of these random measurement results by the standard probability interpretation. However, it has been known that there also exist other kinds of measurements in quantum mechanics, one of which is the protective measurement. Protective measurement also uses a standard measuring procedure, but with a weak and long duration coupling interaction. Besides, it adds an appropriate procedure to protect the measured wave function from collapsing (in some situations the protection is provided by the measured system itself). These differences permit protective measurement to be able to gain more information about the measured quantum system and its wave function. In particular, it can measure the mass and charge distributions of a quantum system, and it turns out that the mass and charge density in each position is proportional to the modulus square of the wave function of the system there.

The key to unveil the meaning of the wave function is to find the origin of the mass and charge density. Historically, the charge density interpretation for electrons was originally suggested by Schrödinger when he introduced the wave function and founded wave mechanics. Although the existence of the charge density of an electron can provide a classical explanation for some phenomena of radiation, its explanatory power is very limited. In fact, Schrödinger clearly

realized that the charge density cannot be classical because his equation does not include the usual classical interaction between the densities. Presumably since people thought that the charge density could not be measured and also lacked a consistent physical picture, this initial interpretation of the wave function was soon rejected and replaced by Born's probability interpretation. Now protective measurement re-endows the charge density of an electron with reality by a more convincing argument. The question is then how to find a consistent physical explanation for it[1]. Our following analysis can be regarded as a further development of Schrödinger's idea to some extent. The twist is: that the charge density is not classical does not imply its nonexistence; rather, its existence points to a non-classical physical picture of motion hiding behind the mathematical wave function.

The charge distribution of a charged quantum system such as an electron has two possible existent forms: it is either real or effective. The charge distribution is real means that it exists throughout space at the same time, and the charge distribution is effective means that there is only a localized particle with the total charge of the system at every instant, and its motion forms the effective charge distribution. If the charge distribution is effective, then there will exist no electrostatic self-interaction of the charge distribution, as there is only a localized charged particle at every instant. By contrast, if the charge distribution is real, then there will exist electrostatic self-interaction of the charge distribution, as the distribution exists throughout space at the same time. Since the superposition principle of quantum mechanics prohibits the existence of electrostatic self-interaction, and especially, the existence of the electrostatic self-interaction for the charge distribution of an electron already contradicts experimental observations, the charge distribution of a quantum system cannot be real but must be effective. This means that for a quantum system, at every instant there is only a localized particle with the total mass and charge of the system, and during an infinitesimal time interval at a given instant the time average of the motion of the particle forms the effective mass and charge density in every position, which is proportional to the modulus square of the wave function of the system there. Since the integral of the formed mass and charge density in any region is equal to the expectation value of the total mass and charge in the region, the motion of the particle is ergodic.

The next question is which sort of ergodic motion the particle undergoes. It can be argued that the classical ergodic models, which assume continuous motion of particles, are inconsistent with quantum mechanics, and the effective mass and charge density of a quantum system is formed by discontinuous motion of a

localized particle with mass and charge. Moreover, the discontinuous motion is not deterministic but random. Based on this result, we suggest that the wave function in quantum mechanics describes the state of random discontinuous motion of particles, and at a deeper level, it represents the property of the particles that determines their random discontinuous motion. In particular, the modulus square of the wave function (in position space) determines the probability density of the particles appearing in certain positions in real space. In the following, we will give a full exposition of this suggested interpretation of the wave function.

2.1 Standard quantum mechanics and conventional measurements

The standard formulation of quantum mechanics, which was first developed by Dirac and von Neumann, is based on the following four basic principles.

1. Physical states

The state of a physical system is represented by a normalized wave function or unit vector $|\psi(t)\rangle$ in a Hilbert space[2]. The Hilbert space is complete in the sense that every possible physical state can be represented by a state vector in the space.

2. Physical properties

Every measurable property or observable of a physical system is represented by a Hermitian operator on the Hilbert space associated with the system. A physical system has a determinate value for an observable if and only if it is in an eigenstate of the observable (this is often called the eigenvalue-eigenstate link).

3. Composition rule

The Hilbert space associated with a composite system is the tensor product of the Hilbert spaces associated with the systems of which it is composed. Similarly, the Hilbert space associated with independent properties is the tensor product of the Hilbert spaces associated with each property.

4. Evolution law

(1). Linear evolution

The state of a physical system $|\psi(t)\rangle$ obeys the linear Schrödinger equation $i\partial|\psi(t)\rangle/\partial t=H|\psi(t)\rangle$ (when it is not measured), where H is the Hamiltonian operator that depends on the energy properties of the system.

(2). Nonlinear collapse evolution

If a physical system is in a quantum superposition of the eigenstates of an observable A, i.e., $|\psi\rangle=\Sigma_i c_i|a_i\rangle$, then an (impulse) measurement of the observable A will instantaneously, discontinuously, and randomly collapse the state into one of the eigenstates $|a_i\rangle$ with probability $|c_i|^2$.[3] This is usually called the collapse postulate, and the nonlinear stochastic process is called the reduction of the state vector or the collapse of the wave function.

The link between the mathematical formalism and experiment is provided by the Born rule. It says that the probability of the above measurement of the observable A yielding the result a_i is $|c_i|^2$. Note that the Born rule can be derived from the collapse postulate by resorting to the eigenvalue-eigenstate link, but it does not necessarily depend on the postulate. Different from the controversial collapse postulate, the Born rule has been confirmed by precise experiments and is an established part of quantum mechanics.

The conventional impulse measurements can be further formulated as follows. According to the standard von Neumann procedure, measuring an observable A in a quantum state $|\psi\rangle$ involves an interaction Hamiltonian

$H_I = g(t)PA$ (2.1)

coupling the measured system to an appropriate measuring device, where P is the conjugate momentum of the pointer variable. The time-dependent coupling strength g(t) is a smooth function normalized to $\int dt g(t) = 1$ during the interaction interval τ, and $g(0) = g(\tau) = 0$. The initial state of the pointer is supposed to be a Gaussian wave packet of width w_0 centered at initial position 0, denoted by $|\varphi(0)\rangle$.

For an impulse measurement, the interaction H_I is of very short duration and so strong that it dominates the rest of the Hamiltonian (i.e. the effect of the free Hamiltonians of the measuring device and the measured system can be neglected). Then the state of the combined system at the end of the interaction can be written as

$$|t = \tau\rangle = e^{-\frac{i}{\hbar}PA}\,|\psi\rangle\,|\phi(0)\rangle\,. \qquad (2.2)$$

By expanding $|\psi$ in the eigenstates of A, $|a_i$, we obtain

$$|t=\tau\rangle=\sum_i e^{-\frac{i}{\hbar}Pa_i}c_i\,|a_i\rangle\,|\phi(0)\rangle\,, \qquad (2.3)$$

where c_i are the expansion coefficients. The exponential term shifts the center of the pointer by a_i:

$$|t=\tau\rangle=\sum_i c_i\,|a_i\rangle\,|\phi(a_i)\rangle\,. \qquad (2.4)$$

This is an entangled state, where the eigenstates of A with eigenvalues a_i get correlated to measuring device states in which the pointer is shifted by these values a_i (but the width of the pointer wavepacket is not changed). Then by the collapse postulate, the state will instantaneously and randomly collapse into one of its branches $|a_i>|\varphi(a_i)>$ with probability $|c_i|^2$. This means that the measurement result can only be one of the eigenvalues of measured observable A, say a_i, with a certain probability $|c_i|^2$. The expectation value of A is then obtained as the statistical average of eigenvalues for an ensemble of identical systems, namely $<A> =\Sigma_i|c_i|^2a_i$.

2.2 Weak measurements

The conventional impulse measurements are only one kind of quantum measurements, for which the coupling between measured system and measuring device is very strong, and thus the results are the eigenvalues of measured observable. In fact, we can also obtain other kinds of measurements by adjusting the coupling strength. An interesting example is weak measurements (Aharonov, Albert and Vaidman 1988), for which the measurement result is the expectation value of the measured observable. In this section, we will introduce the basic principle of weak measurements.

A weak measurement is a standard measuring procedure with weakened coupling. As in the conventional impulse measurement, the Hamiltonian of the interaction with the measuring device is also given by Eq. (2.1) in a weak measurement. The weakness of the interaction is achieved by preparing the initial state of the measuring device in such a way that the conjugate momentum of the pointer variable is localized around zero with small uncertainty, and thus the interaction Hamiltonian (2.1) is small[4]. The explicit form of the initial state of the pointer in position space is:

$$\langle x\,|\phi(0)\rangle=(w_0^2\pi)^{-1/4}e^{-x^2/2w_0^2}. \qquad (2.5)$$

The corresponding initial probability distribution is

$$\mathrm{P}_i(x) = (w_0^2\pi)^{-1/2} e^{-x^2/w_0^2}. \qquad (2.6)$$

Expanding the initial state of the system $|\psi\rangle$ in the eigenstates $|a_i\rangle$ of the measured observable A, $|\psi\rangle=\Sigma_i c_i|a_i\rangle$, then after the interaction (2.1) the state of the system and the measuring device is:

$$|t=\tau\rangle = (w_0^2\pi)^{-1/4} \sum_i c_i |a_i\rangle e^{-(x-a_i)^2/2w_0^2}. \qquad (2.7)$$

The probability distribution of the pointer variable corresponding to the final state (2.7) is:

$$\mathrm{P}_f(x) = (w_0^2\pi)^{-1/2} \sum_i |c_i|^2 e^{-(x-a_i)^2/w_0^2}. \qquad (2.8)$$

In case of a conventional impulse measurement, this is a weighted sum of the initial probability distribution localized around various eigenvalues a_i. Therefore, the reading of the pointer variable in the end of the measurement always yields the value close to one of the eigenvalues. By contrast, the limit of weak measurement corresponds to $w_0>>a_i$ for all eigenvalues a_i. Then, we can perform the Taylor expansion of the sum (2.8) around x = 0 up to the first order and rewrite the final probability distribution of the pointer variable in the following way:

$$\begin{aligned} \mathrm{P}_f(x) &\approx (w_0^2\pi)^{-1/2} \sum_i |c_i|^2 (1-(x-a_i)^2/w_0^2) \\ &= (w_0^2\pi)^{-1/2} e^{-(x-\sum_i |c_i|^2 a_i)^2/w_0^2} \qquad (2.9) \end{aligned}$$

This is the initial probability distribution shifted by the value $\Sigma_i|c_i|^2a_i$. This indicates that the result of the weak measurement is the expectation value of the measured observable in the measured state:

$$\langle A\rangle \equiv \langle\psi|A|\psi\rangle = \sum_i |c_i|^2 a_i. \qquad (2.10)$$

Certainly, since the width of the pointer wavepacket is much greater than the shift of the center of the pointer, namely $w_0>>A$, the above weak measurement of a single system is very imprecise[5]. However, by performing the weak measurement on an ensemble of N identical systems the precision can be improved by a factor $\sqrt{N}$. This scheme of weak measurement has been realized and proved useful in quantum optical experiments (see, *e.g.* Hosten and Kwiat

2008).

Although weak measurements, like conventional impulse measurements, also need to measure an ensemble of identical quantum systems, they are conceptually different. For conventional impulse measurements, every identical system in the ensemble shifts the pointer of measuring device by one of the eigenvalues of the measured observable, and the expectation value of the observable is then regarded as the property of the whole ensemble. By contrast, for weak measurements, every identical system in the ensemble shifts the pointer of measuring device directly by the expectation value of the measured observable, and thus the expectation value may be regarded as the property of individual systems.

2.3 Protective measurements

Protective measurements are improved methods based on weak measurements, and they can measure the expectation values of observables on a single quantum system without disturbing its state.

As we have seen above, although the measured state is not changed appreciably by a weak measurement, the pointer of the measuring device hardly moves either, and in particular, its shift due to the measurement is much smaller than its position uncertainty, and thus little information can be obtained from individual measurements. A possible way to remedy the weakness of weak measurements is to increase the time of the coupling between the measured system and the measuring device. If the state is almost constant during the measurement, the total shift of the pointer, which is proportional to the duration of the interaction, will be large enough to be identified. However, under normal circumstances the state of the system is not constant during the measurement, and the weak coupling also leads to a small rate of change of the state. As a result, the reading of the measuring device will correspond not to the state which the system had prior to the measurement, but to some time average depending on the evolution of the state influenced by the measuring procedure.

Therefore, in order to be able to measure the state of a single system, we need, in addition to the standard weak and long-duration measuring interaction, a procedure which can protect the state from changing during the measuring interaction. A general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically so that the state of the system neither collapses nor becomes entangled with the measuring device appreciably. In this way, protective measurement can measure the expectation values of observables on a single quantum system. In the following, we will introduce the principle of protective measurement in more detail (Aharonov and

Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996)[6].

2.3.1 Measurements with natural protection

As a typical example, we consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed.

The interaction Hamiltonian for a protective measurement of an observable A in this state involves the same interaction Hamiltonian as the standard measuring procedure:

$H_I = g(t)PA$, (2.11)

where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. Let the initial state of the pointer at t = 0 be $|\varphi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0. The time-dependent coupling strength g(t) is also a smooth function normalized to $\int dt g(t) = 1$. But different from conventional impulse measurements, where the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic, and thus the free Hamiltonians cannot be neglected. Let the Hamiltonian of the combined system be

$H(t) = H_S + H_D + g(t)PA$, (2.12)

where H_S and H_D are the Hamiltonians of the measured system and the measuring device, respectively. The interaction lasts for a long time T , and g(t) is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

The state of the combined system after T is given by

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle . \qquad (2.13)$$

By ignoring the switching on and switching off processes[7], the full Hamiltonian (with g(t) = 1/T) is time-independent

and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle , \tag{2.14}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $\left|E_j^d\right\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j d_j |E_n\rangle \left|E_j^d\right\rangle , \tag{2.15}$$

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k,m)$, we have

$$|t = T\rangle = \sum_j d_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle \Psi_{k,m}|E_n, E_j^d\rangle |\Psi_{k,m}\rangle. \tag{2.16}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(2.12) can be thought of as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle \left|E_m^d\right\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle \left|E_m^d\right\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2) \end{aligned} \tag{2.17}$$

Note that it is a necessary condition for Eq.(2.17) to hold that $|E_k\rangle$ is a nondegenerate eigenstate of H_S. Substituting Eq.(2.17) in Eq.(2.16) and taking the large T limit yields

$$|t = T\rangle \approx \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} d_j \, |E_n\rangle \left|E_j^d\right\rangle . \tag{2.18}$$

For the special case when P commutes with the free Hamiltonian of the device, i.e., $[P, H_D] = 0$, the eigenstates $\left|E_j^d\right\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t = T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} \, |E_n\rangle \, |\phi(x_0)\rangle . \tag{2.19}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t = T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} \, |E_n\rangle \, |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{2.20}$$

This shows that at the end of the interaction, the center of the pointer shifts by the expectation value of the measured observable in the measured state.

For the general case when $[P, H_D] \neq 0$ and $[A, H_S] \neq 0$, we can introduce an operator $Y = \sum_j \langle P \rangle_j \left| E_j^d \right\rangle \langle E_j^d|$ and rewrite Eq.(2.18) as

$$|t = T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n Y} |E_n\rangle |\phi(x_0)\rangle . \quad (2.21)$$

Then by rechosing the state of the device so that it is peaked around a value x_0' of the pointer variable X' conjugate to Y, i.e., $[X', Y] = i\hbar$[8], we can obtain

$$\begin{aligned} |t = T\rangle &\approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n Y} |E_n\rangle \left|\phi(x_0')\right\rangle \\ &= e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} |E_n\rangle |\phi(x_0' + \langle A\rangle_n)\rangle. \quad (2.22) \end{aligned}$$

Thus the center of the pointer also shifts by $\langle A\rangle_n$ at the end of the interaction. This demonstrates the generic possibility of the protective measurement of $\langle A\rangle_n$ without disturbing the measured state $|E_n\rangle$.

It is worth noting that since the position variable of the pointer does not commute with its free Hamiltonian, the pointer wave packet will spread during the long measuring

[8]Note that it may not always be possible to physically realize the operator Y, and an operator canonically conjugate to Y need not always exist either. For further discussions see Dass and Qureshi (1999).

time. For example, the kinematic energy term $P^2/2M$ in the free Hamiltonian of the pointer will spread the wave packet without shifting the center, and the width of the wave packet at the end of interaction will be $w(T) = [1/2(w_0^2 + T^2/M^2 w_0^2)]^{1/2}$ (Dass and Qureshi 1999). However, the spreading of the pointer wave packet can be made as small as possible by increasing the mass M of the pointer, and thus it will not interfere with resolving the shift of the center of the pointer in principle[8].

2.3.2 Measurements with artificial protection

Protective measurements can not only measure the discrete nondegenerate energy eigenstates of a single quantum system, which are naturally protected by energy conservation, but also measure the general quantum states by adding an

artificial protection procedure in principle (Aharonov and Vaidman 1993). For this case, the measured state needs to be known beforehand in order to arrange a proper protection.

For degenerate energy eigenstates, the simplest way is to add a potential (as part of the measuring procedure) to change the energies of the other states and lift the degeneracy. Then the measured state remains unchanged, but is now protected by energy conservation like nondegenerate energy eigenstates. Although this protection does not change the state, it does change the physical situation. This change can be brought to a minimum by adding strong protection potential for a dense set of very short time intervals. Then most of the time the system has not only the same state, but also the original potential.

The superposition of energy eigenstates can be measured by a similar procedure. One can add a dense set of time-dependent potentials acting for very short periods of time such that the state at all these times is the nondegenerate eigenstate of the Hamiltonian together with the additional potential. Then most of the time the system also evolves under the original Hamiltonian. A stronger protection is needed in order to measure all details of the time-dependent state. The simplest way is via the quantum Zeno effect. The frequent impulse measurements can test and protect the time evolution of the quantum state. For measurement of any desired accuracy of the state, there is a density of the impulse measurements which can protect the state from being changed due to the measuring interaction. When the time scale of intervals between consecutive protections is much smaller than the time scale of the original state evolution, the system will evolve according to its original Hamiltonian most of the time, and thus what's measured is still the property of the system and not of the protection procedure (Aharonov and Vaidman 1993).

Lastly, it is worth noting that the scheme of protective measurement can also be extended to a many-particle system (Anandan 1993). If the system is in a product state, then this is easily done by protectively measuring each state of the individual systems. But this is impossible when the system is in an entangled state because neither particle is then in a unique state that can be protected. If a protective measurement is made only on one of the particles, then this would also collapse the entangled state into one of the eigenstates of the protecting Hamiltonian. The right method is by adding appropriate protection procedure to the whole system so that the entangled state is a nondegenerate eigenstate of the total Hamiltonian of the system together with the added potential. Then the entangled state can be protectively measured. Note that the additional protection usually contains a nonlocal interaction for separated particles. However, this measurement may be performed without violating Einstein causality by having

the entangled particles sufficiently close to each other so that they have this protective interaction. Then when the particles are separated they would still be in the same entangled state which has been protectively measured.

2.3.3 Further discussions

According to the standard view, the expectation values of observables are not the physical properties of a single system, but the statistical properties of an ensemble of identical systems. This seems reasonable if there exist only conventional impulse measurements. An impulse measurement can only obtain one of the eigenvalues of the measured observable, and thus the expectation value can only be defined as a statistical average of the eigenvalues for an ensemble of identical systems. However, as we have seen, there exist other kinds of quantum measurements, and in particular, protective measurements can measure the expectation values of observables for a single system, using an adiabatic measuring procedure. Therefore, the expectation values of observables should be considered as the physical properties of a single quantum system, not those of an ensemble (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996)[9].

It is worth pointing out that a realistic protective measurement (where the measuring time T is finite) can never be performed on a single quantum system with absolute certainty because of the tiny unavoidable entanglement in the final state (e.g. Eq.(2.17))[10]. For example, we can only obtain the exact expectation value A with a probability very close to one, and the measurement result may also be the expectation value $A_{\perp}$ with a probability proportional to $1/T^2$, where $\perp$ refers to the normalized state in the subspace normal to the initial state as picked out by the first-order perturbation theory (Dass and Qureshi 1999). Therefore, a small ensemble is still needed for a realistic protective measurement, and the size of the ensemble is in inverse proportion to the duration of measurement. However, the limitation of a realistic protective measurement does not influence the above conclusion. The key point is that a protective measurement can measure the expectation values of observables on a single quantum system with certainty in principle, using an adiabatic measuring procedure, and thus they should be regarded as the physical properties of the system.

In addition, we can also provide an argument against the standard view, independent of our analysis of protective measurement. First of all, although the expectation values of observables can only be obtained by measuring an ensemble of identical systems in the context of conventional impulse measurements, this fact does not necessarily entails that they can only be the

statistical properties of the ensemble. Next, if each system in the ensemble is indeed identical as the standard view holds (this means that the quantum state is a complete description of a single system), then obviously the expectation values of observables will be also the properties of each individual system in the ensemble. Thirdly, even if the quantum state is not a complete description of a single system and hidden variables are added as in the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952), the quantum state of each system in an ensemble of identical systems is still the same, and thus the expectation values of observables, which are calculated in terms of the quantum state, are also the same for every system in the ensemble. As a result, the expectation values of observables can still be regarded as the properties of individual systems.

Lastly, we stress that the expectation values of observables are instantaneous properties of a quantum system (Aharonov, Anandan and Vaidman 1996). Although the measured state may be unchanged during a protective measurement and the duration of measurement may be very long, for an arbitrarily short period of time the measuring device always shifts by an amount proportional to the expectation value of the measured observable in the state. Therefore, the expectation values of observables are not time-averaged properties of a quantum system during a finite period of time, but instantaneous properties of the system.

2.4 How does the mass and charge of a quantum system distribute?

The fundamental assumption is that the space density of electricity is given by the square of the wavefunction. — Erwin Schrödinger, 1926[11]

According to protective measurement, the expectation values of dynamical variables are properties of a single quantum system. Typical examples of such properties are the mass and charge density of a quantum system. In this section, we will present a detailed analysis of this property, as it may have important implications for the physical meaning of the wave function.

2.4.1 A heuristic argument

The mass and charge of a classical system always localize in a definite position in space at each moment. For a charged quantum system described by the wave function $\psi(x, t)$, how do its mass and charge distribute in space then? We can measure the total mass and charge of the quantum system by the gravitational and electromagnetic interactions and find them in some region of space. Thus it seems that the mass and charge of a quantum system must also exist in space with a certain distribution. Before we discuss the answer given by protective measurement, we will first give a heuristic argument.

The Schrödinger equation of a charged quantum system under an external

electromagnetic potential may provide a clue to the answer. The equation is

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}(\nabla - \frac{iQ}{\hbar c}A)^2 + Q\varphi]\psi(x,t), \qquad (2.23)$$

where m and Q are the mass and charge of the system, respectively, ϕ and A are the electromagnetic potential, and c is the speed of light. The electrostatic interaction term $Q\phi\psi(x, t)$ in the equation indicates that the interaction exists in all regions where the wave function of the system, $\psi(x, t)$, is nonzero, and thus it seems to suggest that the charge of the system also distributes throughout these regions. If the charge does not distribute in some regions where the wave function is nonzero, then there will not exist an electrostatic interaction there. Furthermore, since the integral $\int Q|\psi(x, t)|^2 d^3x$ is the total charge of the system, the charge density in space, if indeed exists, will be $Q|\psi(x, t)|^2$. Similarly, the mass density can be obtained from the Schrödinger equation of a quantum system under an external gravitational potential:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + mV_G]\psi(x,t). \qquad (2.24)$$

The gravitational interaction term $mV_G\psi(x, t)$ in the equation also suggests that the (passive gravitational) mass of the quantum system distributes throughout the whole region where its wave function $\psi(x, t)$ is nonzero, and the mass density in space is $m|\psi(x, t)|^2$.

2.4.2 The answer of protective measurement

In the following, we will show that protective measurement provides a more convincing argument for the existence of mass and charge density. The mass and charge density of a single quantum system, as well as its wave function, can be measured by protective measurement as expectation values of certain observables (Aharonov and Vaidman 1993). For example, a protective measurement of the flux of the electric field of a charged quantum system out of a certain region will yield the expectation value of its charge inside this region, namely the integral of its charge density over this region. Similarly, we can also measure the mass density of a quantum system by a protective measurement of the flux of its gravitational field in principle (Anandan 1993).

Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \qquad (2.25)$$

The protective measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \qquad (2.26)$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ everywhere in space.

Since the physical realization of the observable A_n and the corresponding interaction Hamiltonian must always resort to the electromagnetic or gravitational interaction between the measured system and the measuring device, what the above protective measurement measures is in fact the charge or mass density of the quantum system[12], and its result indicates that the mass and charge density is proportional to the modulus square of the wave function of the system, namely the density $\rho(x)$. In the following, we will give a concrete example to illustrate this important result (see also Aharonov, Anandan and Vaidman 1993).

2.4.3 A specific example

Consider the spatial wave function of a single quantum system with negative charge Q (e.g. Q = -e)

$\psi(x, t) = a\psi_1(x, t) + b\psi_2(x, t)$, (2.27)

where $\psi_1(x, t)$ and $\psi_2(x, t)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2 + |b|^2 = 1$. An electron, which initial state is a Gaussian wave packet narrow in both position and momentum, is shot along a straight line near box 1 and perpendicular to the line of separation between the boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system were in box 2, namely $|a|^2 = 0$, the trajectory of the electron wave packet would be a straight line as indicated by position “0” in Fig.1. By contrast, if the system were in box 1, namely $|a|^2 = 1$, the trajectory of the electron wave packet would be deviated by the electric field of the system by a maximum amount as indicated by position “1” in Fig.1.

We first suppose that $\psi(x, t)$ is unprotected, then the wave function of the

combined system after interaction will be

$\psi(x, x , t) = a\phi_1(x , t)\psi_1(x, t) + b\phi_2(x , t)\psi_2(x, t)$, (2.28)

where $\phi_1(x , t)$ and $\phi_2(x , t)$ are the wave functions of the electron influenced by the electric fields of the system in box 1 and box 2, respectively, the trajectory of $\phi_1(x , t)$ is deviated by a maximum amount, and the trajectory of $\phi_2(x , t)$ is not deviated and still a straight line. When the electron is detected on the screen, the above wave function will collapse to $\phi_1(x , t)\psi_1(x, t)$ or $\phi_2(x , t)\psi_2(x, t)$. As a result, the detected position of the electron will be either "1" or "0" in Fig.1, indicating that the system is in box 1 or 2 after the detection. This is a conventional impulse measurement of the projection operator on the spatial region of box 1, denoted by A_1. A_1 has two eigenstates corresponding to the system being in box 1 and 2, respectively, and the corresponding eigenvalues are 1 and 0, respectively. Since the measurement is accomplished through the electrostatic interaction between two charges, the measured observable A_1, when multiplied by the charge Q, is actually the observable for the charge of the system in box 1, and its eigenvalues are Q and 0, corresponding to the charge Q being in boxes 1 and 2, respectively. Such a measurement cannot tell us the charge distribution of the system in each box before the measurement.

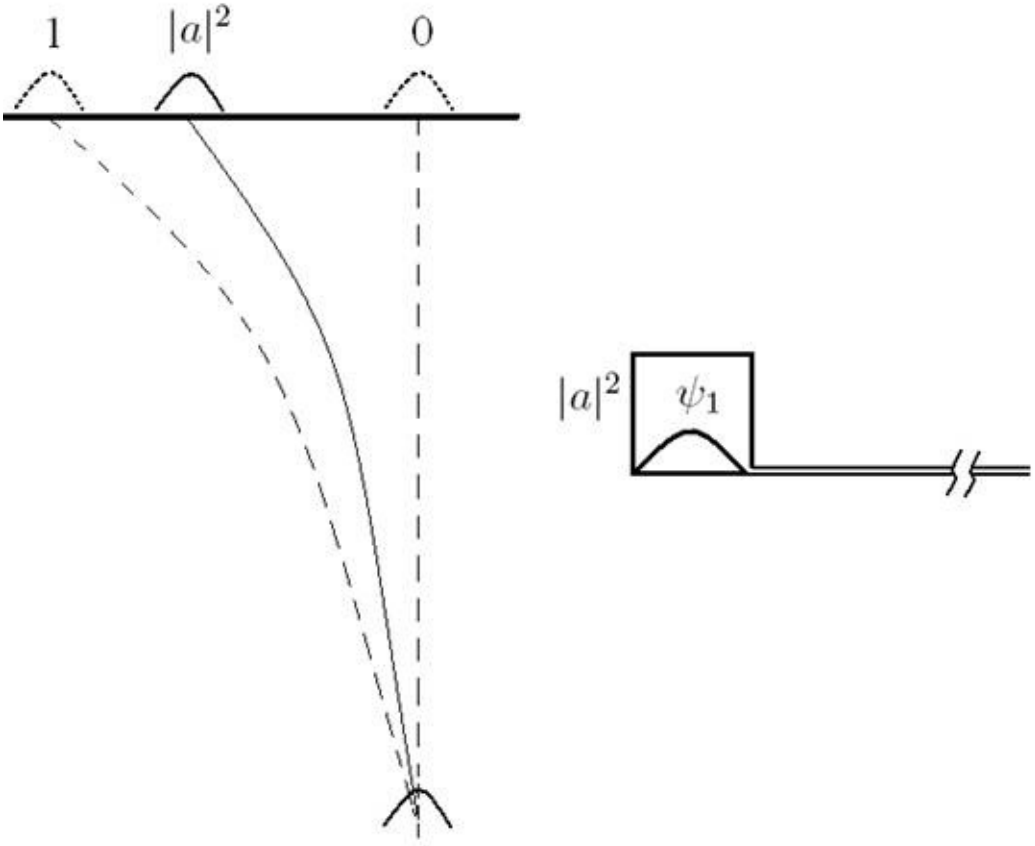

Fig.1 Scheme of a protective measurement of the charge density of a quantum system

Now let's make a protective measurement of A_1. Since $\psi(x, t)$ is degenerate with its orthogonal state $\psi(x, t) = b^*\psi_1(x, t) - a^*\psi_2(x, t)$, we need an artificial protection procedure to remove the degeneracy, *e.g.* joining the two boxes with a long tube whose diameter is small compared to the size of the box[13]. By this protection $\psi(x, t)$ will be a nondegenerate energy eigenstate. The adiabaticity condition and the weakly interacting condition, which are required for a protective measurement, can be further satisfied when assuming that (1) the measuring time of the electron is long compared to /ΔE, where ΔE is the smallest of the energy differences between $\psi(x, t)$ and the other energy eigenstates, and (2) at all times the potential energy of interaction between the electron and the system is small compared to ΔE. Then the measurement of A_1 by means of the electron trajectory is a protective measurement, and the trajectory of the electron is only influenced by the expectation value of the charge of the system in box 1. In particular, when the size of box 1 can be ignored compared with the separation between it and the electron wave packet, the wave function of the electron will obey the following Schrödinger equation:

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m_e}\nabla^2\psi(\vec{r},t) - k\frac{e\cdot|a|^2Q}{|\vec{r}-\vec{r_1}|}\psi(\vec{r},t), \quad (2.29)$$

where m_e is the mass of electron, k is the Coulomb constant, r_1 is the position of the center of box 1, and $|a|^2Q$ is the expectation value of the charge Q in box 1. Correspondingly, the trajectory of the center of the electron wave packet, $r_c(t)$, will satisfy the following equation by Ehrenfest's theorem:

$$m_e\frac{d^2\vec{r_c}}{dt^2} = -k\frac{e\cdot|a|^2Q}{|\vec{r_c}-\vec{r_1}|(\vec{r_c}-\vec{r_1})}. \quad (2.30)$$

Then the electron wave packet will reach the position " $|a|^2$" between "0" and "1" on the screen as denoted in Fig.1. This shows that the result of the protective measurement is the expectation value of the projection operator A_1, namely the integral of the density $|\psi(x)|^2$ in the region of box 1. When multiplied by Q, it is the expectation value of the charge Q in the state $\psi_1(x, t)$ in box 1, namely the integral of the charge density $Q|\psi(x)|^2$ in the region of box 1. In fact, as Eq. (2.29) and Eq. (2.30) clearly show, this is what the protective measurement really measures.

As we have argued in the last section, the result of a protective measurement reflects an objective property of the measured system. Thus the result of the above protective measurement, namely the expectation value of the charge Q in the state $\psi_1(x, t)$, $|a|^2Q$, will reflect the actual charge distribution of the system in box 1. In other words, the result indicates that there exists a charge $|a|^2Q$ in box 1.[14] In the following, we will give another two arguments for this conclusion.

First of all, let's analyze the result of the protective measurement. Suppose we can continuously change the measured state from $|a|^2 = 0$ to $|a|^2 = 1$. When $|a|^2 = 0$, the single electron will reach the position "0" of the screen one by one, and it is incontrovertible that no charge is in box 1. When $|a|^2 = 1$, the single electron will reach the position "1" of the screen one by one, and it is also incontrovertible that there is a charge Q in box 1. Then when $|a|^2$ assumes a numerical value between 0 and 1 and the single electron reaches the position "$|a|^2$" between "0" and "1" on the screen one by one, the results should similarly indicate that there is a charge $|a|^2Q$ in the box by continuity. The point is that the definite deviation of the trajectory of the electron will reflect that there exists a definite amount of charge in box 1.[15] Next, let's analyze the equation that determines the result of the protective measurement, namely Eq. (2.30). It gives a more direct support for the existence of a charge $|a|^2Q$ in box 1. The r.h.s of Eq. (2.30) is the formula of the electric force between two charges located in different spatial regions. It is incontrovertible that e is the charge of the electron, and it exists in the position r. Then $|a|^2Q$ should be the other charge that exists in the position r_1. In other words, there exists a charge $|a|^2Q$ in box 1.

In conclusion, protective measurement shows that a quantum system with mass m and charge Q, which is described by the wave function $\psi(x, t)$, has a mass density $m|\psi(x, t)|^2$ and a charge density $Q|\psi(x, t)|^2$, respectively[16].

2.5 The origin of mass and charge density

We have argued that a charged quantum system has mass and charge density proportional to the modulus square of its wave function. In this section, we will further investigate the physical origin of the mass and charge density. Is it real or only effective? As we will see, the answer may provide an important clue to the physical meaning of the wave function.

2.5.1 The mass and charge density is not real

If the mass and charge density of a charged quantum system is real, that is, if the densities at different locations exist at the same time, then there will exist gravitational and electrostatic self-interactions of the density[17].

Interestingly, the Schrödinger-Newton equation, which was proposed by Diosi (1984) and Penrose (1998), just describes the gravitational self-interaction of the mass density. The equation for a single quantum system can be written as

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t)-Gm^2\int\frac{|\psi(\mathbf{x}',t)|^2}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'\psi(\mathbf{x},t)+V\psi(\mathbf{x},t), \tag{2.31}$$

where m is the mass of the quantum system, V is an external potential, G is Newton's gravitational constant. Much work has been done to study the mathematical properties of this equation (Moroz, Penrose and Tod 1998; Moroz and Tod 1999; Harrison, Moroz and Tod 2003; Salzman 2005). Several experimental schemes have been also proposed to test its physical validity (Salzman and Carlip 2006). As we will see below, although such gravitational self-interactions cannot yet be excluded by experiments[18], the existence of the electrostatic self-interaction for a charged quantum system already contradicts experimental observations.

If there is also an electrostatic self-interaction, then the equation for a free quantum system with mass m and charge Q will be

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x},t)+(kQ^2-Gm^2)\int\frac{|\psi(\mathbf{x}',t)|^2}{|\mathbf{x}-\mathbf{x}'|}d^3\mathbf{x}'\psi(\mathbf{x},t). \tag{2.32}$$

Note that the gravitational self-interaction is attractive, while the electrostatic self-interaction is repulsive. It has been shown that the measure of the potential strength of the gravitational self-interaction is $\varepsilon^2 = (4Gm2/hc)^2$ for a free system with mass m (Salzman 2005). This quantity represents the strength of the influence of the self-interaction on the normal evolution of the wave function; when $\varepsilon^2 \approx 1$ the influence is significant. Similarly, for a free charged system with charge Q, the measure of the potential strength of the electrostatic self-interaction is $\varepsilon^2 = (4kQ2/hc)^2$. As a typical example, for a free electron the potential strength of the electrostatic self-interaction will be $\varepsilon^2 = (4ke2/hc)^2 \approx 1 \times 10^{-3}$. This indicates that the electrostatic self-interaction will have a remarkable influence on the evolution of the wave function of a free electron[19]. If such an interaction indeed exists, it should have been detected by precise interference experiments on electrons. On the other hand, the superposition principle of quantum mechanics, which denies the existence of the observable electrostatic self-interaction, has been verified for microscopic particles with astonishing precision. As another example, consider the electron in

the hydrogen atom. Since the potential of the electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms will be remarkably different from those predicted by quantum mechanics and confirmed by experiments. Therefore, the electrostatic self-interaction cannot exist for a charged quantum system.

In conclusion, although the gravitational self-interaction is too weak to be detected presently, the existence of the electrostatic self-interaction for a charged quantum system such as an electron already contradicts experimental observations. Accordingly, the mass and charge density of a quantum system cannot be real but be effective[20]. This means that at every instant there is only a localized particle with the total mass and charge of the system, and during a time interval the time average of the ergodic motion of the particle forms the effective mass and charge density[21]. There exist no gravitational and electrostatic self-interactions of the density in this case.

2.5.2 The ergodic motion of a particle is discontinuous

Which sort of ergodic motion then? If the ergodic motion of the particle is continuous, then it can only form the effective mass and charge density during a finite time interval. However, the mass and charge density of a particle, which is proportional to the modulus square of its wave function, is an instantaneous property of the particle. In other words, the ergodic motion of the particle must form the effective mass and charge density during an infinitesimal time interval (not during a finite time interval) at a given instant. Thus it seems that the ergodic motion of the particle cannot be continuous. This is at least what the existing quantum mechanics says. However, there may exist a possible loophole here. Although the classical ergodic models that assume continuous motion are inconsistent with quantum mechanics due to the existence of a finite ergodic time, they may be not completely precluded by experiments if only the ergodic time is extremely short. After all quantum mechanics is only an approximation of a more fundamental theory of quantum gravity, in which there may exist a minimum time scale such as the Planck time. Therefore, we need to investigate the classical ergodic models more thoroughly.

Consider an electron in a one-dimensional box in the first excited state $\psi(x)$ (Aharonov and Vaidman 1993). Its wave function has a node at the center of the box, where its charge density is zero. Assume the electron performs a very fast continuous motion in the box, and during a very short time interval its motion generates an effective charge density distribution. Let’s see whether this density can assume the same form as $e|\psi(x)|^2$, which is required by protective measurement[22]. Since the effective charge density is proportional to the

amount of time the electron spends in a given position, the electron must be in the left half of the box half of the time and in the right half of the box half of the time. But it can spend no time at the center of the box where the effective charge density is zero; in other words, it must move at infinite velocity at the center. Certainly, the appearance of velocity faster than light or even infinite velocity may be not a fatal problem, as our discussion is entirely in the context of nonrelativistic quantum mechanics, and especially the infinite potential in the example is also an ideal situation. However, it seems difficult to explain why the electron speeds up at the node and where the infinite energy required for the acceleration comes from. Moreover, the sudden acceleration of the electron near the node may also result in large radiation (Aharonov, Anandan and Vaidman 1993), which is inconsistent with the predictions of quantum mechanics. Again, it seems very difficult to explain why the accelerating electron does not radiate here.

Let's further consider an electron in a superposition of two energy eigenstates in two boxes $\psi_1(x) + \psi_2(x)$. In this example, even if one assumes that the electron can move with infinite velocity (e.g. at the nodes), it cannot continuously move from one box to another due to the restriction of box walls. Therefore, any sort of continuous motion cannot generate the effective charge density $e|\psi_1(x) + \psi_2(x)|^2$. One may still object that this is merely an artifact of the idealization of infinite potential. However, even in this ideal situation, the model should also be able to generate the effective charge density by means of some sort of ergodic motion of the electron; otherwise it will be inconsistent with quantum mechanics. On the other hand, it is very common in quantum optics experiments that a single-photon wave packet is split into two branches moving along two well separated paths in space. The wave function of the photon disappears outside the two paths for all practical purposes. Moreover, the experimental results are not influenced by the environment and setup between the two paths of the photon. Thus it is very difficult to imagine that the photon performs a continuous ergodic motion back and forth in the space between its two paths.

In view of these serious drawbacks of the classical ergodic models and their inconsistency with quantum mechanics, we conclude that the ergodic motion of particles cannot be continuous. If the motion of a particle is discontinuous, then the particle can readily move throughout all regions where the wave function is nonzero during an arbitrarily short time interval at a given instant. Furthermore, if the probability density of the particle appearing in each position is proportional to the modulus square of its wave function there at every instant, the discontinuous motion can also generate the right effective mass and charge

density. This will solve the above problems plagued by the classical ergodic models. The discontinuous ergodic motion requires no existence of a finite ergodic time. Moreover, a particle undergoing discontinuous motion can also move from one region to another spatially separated region, no matter whether there is an infinite potential wall between them, and such discontinuous motion is not influenced by the environment and setup between these regions either. Besides, discontinuous motion can also solve the problems of infinite velocity and accelerating radiation. The reason is that no classical velocity and acceleration can be defined for discontinuous motion, and energy and momentum will require new definitions and understandings as in quantum mechanics.

In conclusion, we have argued that the mass and charge density of a quantum system, which can be measured by protective measurement, is not real but effective. Moreover, the effective mass and charge density is formed by the discontinuous motion of a localized particle, and the probability density of the particle appearing in each position is proportional to the modulus square of its wave function there.

2.5.3 An argument for random discontinuous motion

Although the above analysis demonstrates that the ergodic motion of a particle is discontinuous, it doesn't say that the discontinuous motion must be random. In particular, the randomness of the result of a quantum measurement may be only apparent. In order to know whether the motion of particles is random or not, we need to analyze the cause of motion. For example, if motion has no deterministic cause, then it will be random, only determined by a probabilistic cause. This may also be the right way to find how particles move. Since motion involves change in position, if we can find the cause or instantaneous condition determining the change[23], we will be able to find how particles move in reality.

Let's consider the simplest states of motion of a free particle, for which the instantaneous condition determining the change of its position is a constant during the motion. In logic the instantaneous condition can only be deterministic or indeterministic. That the instantaneous condition is deterministic means that it leads to a deterministic change of the position of a particle at a given instant. That the instantaneous condition is indeterministic means that it only determines the probability of the particle appearing in each position in space at a given instant. If the instantaneous condition is deterministic, then the simplest states of motion of the free particle will have two possible forms. The first one is continuous motion with constant velocity, and the equation of motion of the particle is $x(t + dt) = x(t) + vdt$, where the deterministic instantaneous condition

v is a constant[24]. The second one is discontinuous motion with infinite average velocity; the particle performs a finite jump along a fixed direction at every instant, where the jump distance is a constant, determined by the constant instantaneous condition[25]. On the other hand, if the instantaneous condition is indeterministic, then the simplest states of motion of the free particle will be random discontinuous motion with even position probability density. At each instant the probability density of the particle appearing in every position is the same.

In order to know whether the instantaneous condition is deterministic or not, we need to determine which sort of simplest states of motion are the solutions of the equation of free motion in quantum mechanics (i.e. the free Schrödinger equation)[26]. According to the analysis in the last subsection, the momentum eigenstates of a free particle, which are the solutions of the free Schrödinger equation, describe the ergodic motion of the particle with even position probability density in space. Therefore, the simplest states of motion with a constant probabilistic instantaneous condition are the solutions of the equation of free motion, while the simplest states of motion with a constant deterministic instantaneous condition are not.

When assuming that (1) the simplest states of motion of a free particle are the solutions of the equation of free motion; and (2) the instantaneous condition determining the position change of a particle is always deterministic or indeterministic for any state of motion, the above result then implies that motion, no matter whether it is free or forced, has no deterministic cause, and thus it is random and discontinuous, only determined by a probabilistic cause. The argument may be improved by further analyzing these two seemingly reasonable assumptions, but we will leave this for future work.

2.6 The wave function represents the state of random discontinuous motion of particles

The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what exactly? Not of the electron being there, but of the electron being found there, if its position is measured. Why this aversion to being and insistence on finding? The founding fathers were unable to form a clear picture of things on the remote atomic scale. — John Bell, 1990

In classical mechanics, we have a clear physical picture of motion. It is well understood that the trajectory function x(t) in classical mechanics describes the continuous motion of a particle. In quantum mechanics, the trajectory function x(t) is replaced by a wave function ψ(x, t). If the particle ontology is still viable

in the quantum domain, then it seems natural that the wave function should describe some sort of more fundamental motion of particles, of which continuous motion is only an approximation in the classical domain, as quantum mechanics is a more fundamental theory of the physical world, of which classical mechanics is an approximation. The analysis in the last section provides a strong support for this conjecture, and it suggests that what the wave function describes is the more fundamental motion of particles, which is essentially discontinuous and random. In this section, we will give a more detailed analysis of this suggested interpretation of the wave function (Gao 1993, 1999, 2000, 2003, 2006b, 2008, 2011a, 2011b).

2.6.1 An analysis of random discontinuous motion

Let's first make clearer what we mean when we say a quantum system such as an electron is a particle. The picture of particle appears from our analysis of the mass and charge density of a quantum system. As we have shown in the last section, the mass and charge density of an electron, which is measurable by protective measurement and proportional to the modulus square of its wave function, is not real but effective; it is formed by the ergodic motion of a localized particle with the total mass and charge of the electron. If the mass and charge density is real, i.e., if the mass and charge distributions at different locations exist at the same time, then there will exist gravitational and electrostatic self-interactions of the density, the existence of which not only contradicts experiments but also violates the superposition principle of quantum mechanics. It is this analysis that reveals the basic existent form of a quantum system such as an electron in space and time. An electron is a particle[27]. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge, and it is only in one position in space at an instant. However, as we have argued above, the motion of an electron described by its wave function is not continuous but discontinuous and random in nature. We may say that an electron is a quantum particle in the sense that its motion is not continuous motion described by classical mechanics, but random discontinuous motion described by quantum mechanics.

Next, let's analyze the random discontinuous motion of particles. From a logical point of view, for the random discontinuous motion of a particle, there should exist a probabilistic instantaneous condition that determines the probability density of the particle appearing in every position in space, otherwise it would not "know" how frequently they should appear in every position in space. This condition cannot come from otherwhere but must come from the particle itself. In other words, the particle must have an instantaneous property that determines

its motion in a probabilistic way. This property is usually called indeterministic disposition or propensity in the literature[29]. In a word, a particle has a propensity to be in a particular position in space, and the propensity as a probabilistic instantaneous condition determines the probability density of the particle appearing in every position in space. This can be regarded as the physical basis of random discontinuous motion of particles. As a result, the position of the particle at every instant is random, and its trajectory formed by the random position series is not continuous at every instant[28]. In short, the motion of the particle is essentially random and discontinuous[29].

Unlike the deterministic continuous motion, the trajectory function x(t) no longer provides a useful description for random discontinuous motion. In the following, we will give a strict description of random discontinuous motion of particles based on measure theory. For simplicity but without losing generality, we will mainly analyze the one-dimensional motion that corresponds to the point set in two-dimensional space and time. The results can be readily extended to the three-dimensional situation.

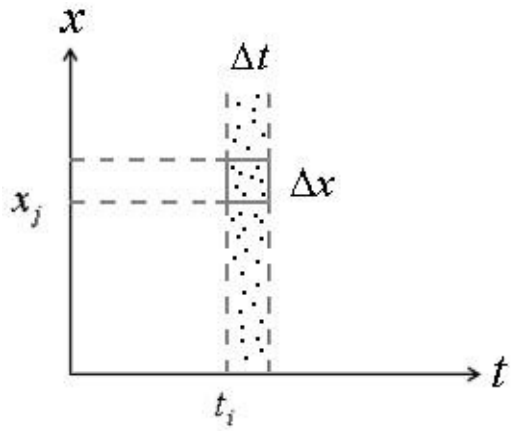

Fig.2 The description of random discontinuous motion of a single particle

We first analyze the random discontinuous motion of a single particle. Consider the state of motion of the particle in finite intervals Δt and Δx near a space-time point (t_i,x_j) as shown in Fig. 2. The positions of the particle form a random, discontinuous trajectory in this square region. We study the projection of this trajectory in the t-axis, which is a dense instant set in the time interval Δt. Let W be the discontinuous trajectory of the particle and Q be the square region $[x_j, x_j + \Delta x]\times [t_i, t_i+ \Delta t]$. The dense instant set can be denoted by $\pi_t(W \cap Q) \in R$, where π_t is the projection on the t-axis. According to the measure theory, we can define the Lebesgue measure:

$$M_{\Delta x,\Delta t}(x_j,t_i)=\int_{\pi_t(W\cap Q)\in\Re} dt. \qquad (2.33)$$

Since the sum of the measures of all such dense instant sets in the time interval Δt is equal to the length of the continuous time interval Δt, we have:

$$\sum_j M_{\Delta x,\Delta t}(x_j,t_i)=\Delta t. \qquad (2.34)$$

Then we can define the measure density as follows[30]:

$$\rho(x,t)=\lim_{\Delta x,\Delta t\to 0} M_{\Delta x,\Delta t}(x,t)/(\Delta x\cdot\Delta t). \qquad (2.35)$$

We call it position measure density or position density in brief. This quantity provides a strict description of the position distribution of the particle or the relative frequency of the particle appearing in an infinitesimal space interval dx near position x during an infinitesimal interval dt near instant t. In other words, ρ(x, t) provides a strict description of the state of random discontinuous motion of the particle at instant t. From Eq. (2.34) we can see that ρ(x, t) satisfies the normalization relation, namely ∫ρ(x, t)dx = 1.

Since the position density will change with time in general, we can further define the position flux density j(x, t) through the relation j(x, t) = ρ(x, t)v(x, t), where v(x, t) is the velocity of the local position density. It describes the change rate of the position density. Due to the conservation of measure, ρ(x, t) and j(x, t) satisfy the continuity equation:

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial j(x,t)}{\partial x}=0. \qquad (2.36)$$

The position density ρ(x, t) and position flux density j(x, t) provide a complete description of the state of random discontinuous motion of a single particle.

The description of the motion of a single particle can be extended to the motion of many particles. For the random discontinuous motion of N particles, we can define joint position density $\rho(x_1, x_2, ...x_N, t)$ and joint position flux density $j(x_1, x_2, ...x_N, t) = \rho(x_1, x_2, ...x_N, t)\, v(x_1, x_2, ...x_N, t)$. They also satisfy the continuity equation:

$$\frac{\partial \rho(x_1, x_2, ...x_N, t)}{\partial t} + \sum_{i=1}^{N} \frac{\partial j(x_1, x_2, ...x_N, t)}{\partial x_i} = 0. \quad (2.37)$$

When these N particles are independent, the joint position density can be reduced to the direct product of the position density for each particle. Note that the joint position density $\rho(x_1, x_2, ...x_N, t)$ and joint position flux density $j(x_1, x_2, ...x_N, t)$ are not defined in the real three-dimensional space, but defined in the 3N-dimensional configuration space.

2.6.2 Interpreting the wave function

Although the motion of particles is essentially discontinuous and random, the discontinuity and randomness of motion is absorbed into the state of motion, which is defined during an infinitesimal time interval, by the descriptive quantities of position density $\rho(x, t)$ and position flux density $j(x, t)$. Therefore, the evolution of the state of random discontinuous motion of particles can be described as a deterministic continuous equation. By assuming that the nonrelativistic equation of random discontinuous motion is the Schrödinger equation in quantum mechanics, both $\rho(x, t)$ and $j(x, t)$ can be expressed by the wave function in a unique way[31]:

$$\rho(x,t) = |\psi(x,t)|^2, \quad (2.38)$$

$$j(x,t) = \frac{\hbar}{2mi}[\psi^*(x,t)\frac{\partial \psi(x,t)}{\partial x} - \psi(x,t)\frac{\partial \psi^*(x,t)}{\partial x}]. \quad (2.39)$$

Correspondingly, the wave function $\psi(x, t)$ can be uniquely expressed by $\rho(x, t)$ and $j(x, t)$ (except for a constant phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)}e^{im\int_{-\infty}^{x}\frac{j(x',t)}{\rho(x',t)}dx'/\hbar}. \quad (2.40)$$

In this way, the wave function $\psi(x, t)$ also provides a complete description of the state of random discontinuous motion of particles. For the motion of many particles, the joint position density and joint position flux density are defined in the 3N-dimensional configuration space, and thus the many-particle wave function, which is composed of these two quantities, is also defined in the 3N-dimensional configuration space.

Interestingly, we can reverse the above logic in some sense, namely by assuming the wave function is a complete objective description for the motion of particles, we can also reach the random discontinuous motion of particles, independent of

our previous analysis. If the wave function $\psi(x, t)$ is a description of the state of motion for a single particle, then the quantity $|\psi(x, t)|^2 dx$ not only gives the probability of the particle being found in an infinitesimal space interval dx near position x at instant t (as in standard quantum mechanics), but also gives the objective probability of the particle being there. This accords with the common-sense assumption that the probability distribution of the measurement results of a property is the same as the objective distribution of the property in the measured state. Then at instant t the particle may appear in any location where the probability density $|\psi(x, t)|^2$ is nonzero, and during an infinitesimal time interval near instant t the particle will move throughout the whole region where the wave function $\psi(x, t)$ spreads. Moreover, its position density is equal to the probability density $|\psi(x, t)|^2$. Obviously this kind of motion is essentially random and discontinuous.

One important point needs to be stressed here. Since the wave function in quantum mechanics is defined at an instant, not during an infinitesimal time interval, it should be regarded not simply as a description of the state of random discontinuous motion of particles, but more suitably as a description of the probabilistic instantaneous condition or dispositional property of the particles that determines their random discontinuous motion at a deeper level [32]. In particular, the modulus square of the wave function determines the probability density of the particles appearing in every position in space at a given instant. By contrast, the position density and position flux density, which are defined during an infinitesimal time interval at a given instant, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we may say that the motion of particles is "guided" by their wave function in a probabilistic way.

We have been discussed random discontinuous motion of particles in real space. The picture of random discontinuous motion may exist not only for position but also for other dynamical variables such as momentum and energy, and thus the suggested interpretation of the wave function in position space may also apply to the wave function in momentum space *etc.* Due to the randomness of motion for each variable, the probability distributions of all variables for an arbitrary wave function can be consistent with quantum mechanics[33]. However, it is worth stressing that spin is a distinct property. Since the spin of a particle is always definite along one direction (though the spin state can always be decomposed into two eigenstates of spin along another direction), the spin of the particle, unlike its position, does not undergo random discontinuous motion for any spin

state[34].

Chapter 3
How Come the Schrödinger Equation?

The motion of particles follows probability law but the probability itself propagates according to the law of causality.

—Max Born

After investigating the physical meaning of the wave function, we will further analyze the linear evolution law for the wave function in this chapter. It is demonstrated that the linear nonrelativistic evolution of the wave function of an isolated system obeys the free Schrödinger equation due to the requirements of spacetime translation invariance and relativistic invariance. In addition, we also investigate the meaning and implications of the conservation laws in quantum mechanics.

Many quantum mechanics textbooks provide a heuristic "derivation" of the Schrödinger equation. It begins with the assumption that the state of a free quantum system has the form of a plane wave $e^{i(kx-\omega t)}$. When combining with the de Broglie relations for momentum and energy $p = hk$ and $E = h\omega$, this state becomes $e^{i(px-Et)/h}$. Then it uses the nonrelativistic energy-momentum relation $E = p^2/2m$ to obtain the free particle Schrödinger equation. Lastly, this equation is generalized to include an external potential, and the end result is the Schrödinger equation. In the following sections, we will show that the heuristic "derivation" of the free Schrödinger equation can be turned into a real derivation by resorting to spacetime translation invariance and relativistic invariance. Spacetime translation gives the definitions of momentum and energy, and spacetime translation invariance entails that the state of a free quantum system with definite momentum and energy assumes the plane wave form $e^{i(px-Et)/h}$. Moreover, the relativistic invariance of the free states further determines the relativistic energy-momentum relation, whose nonrelativistic approximation is $E = p^2/2m$. Though the requirements of these invariances are already well known, an explicit and complete derivation of the free Schrödinger equation using them seems still missing in the literature and textbooks. The new integrated analysis may be helpful in understanding the physical origin of the Schrödinger equation, and moreover, it is also helpful for understanding momentum and energy and their conservation for random discontinuous motion of particles.

3.1 Spacetime translation and its invariance

In this section, we will show that the free states of motion for a quantum system can be basically determined by spacetime translation invariance. The spacetime translation invariance of natural laws reflects the homogeneity of space and time.

The homogeneity of space ensures that the same experiment performed at two different places gives the same result, and the homogeneity in time ensures that the same experiment repeated at two different times gives the same result. There are in general two different pictures of translation: active transformation and passive transformation. The active transformation corresponds to displacing the studied system, and the passive transformation corresponds to moving the coordinate system. Physically, the equivalence of the active and passive pictures is due to the fact that moving the system one way is equivalent to moving the coordinate system the other way by an equal amount (see also Shankar 1994). In the following, we will mainly analyze spacetime translations in terms of active transformations.

A space translation operator can be defined as

$$T(a)\psi(x, t) = \psi(x - a, t). \quad (3.1)$$

It means translating rigidly the state of a system, $\psi(x, t)$, by an amount a in the positive x direction. The operator preserves the norm of the state because $\int\psi^*(x, t)\psi(x, t)dx = \int\psi^*(x - a, t)\psi(x - a, t)dx$. This implies that T (a) is unitary, satisfying $T^\dagger(a)T(a) = I$. As a unitary operator, T (a) can be further expressed as

$$T(a) = e^{-iaP}, \quad (3.2)$$

where P is called the generator of space translation, and it is Hermitian and its eigenvalues are real. By expanding $\psi(x - a, t)$ in order of a, we can further get

$$P = -i\partial/\partial x. \quad (3.3)$$

Similarly, a time translation operator can be defined as

$$U(t)\psi(x, 0) = \psi(x, t). \quad (3.4)$$

Let the evolution equation of state be of the following form:

$$i\partial\psi(x, t)/\partial t = H\psi(x, t). \quad (3.5)$$

where H is a to-be-determined operator that depends on the properties of the system. In the following analysis of this section, we assume H is independent of the evolved state, namely the evolution is linear[35]. Then the time translation operator U(t) can be expressed as $U(t) = e^{-itH}$, and H is the generator of time translation. Note that we cannot determine whether U(t) is unitary and H is Hermitian here.

Let's now analyze the implications of spacetime translation invariance for the law of motion of a free system or an isolated system. First, time translational invariance requires that H has no time dependence, namely $dH/dt = 0$. This can be demonstrated as follows (see also Shankar 1994, p.295). Suppose an isolated system is in state ψ_0 at time t_1 and evolves for an infinitesimal time δt. The state of the system at time $t_1 + \delta t$, to first order in δt, will be

$\psi(x, t_1 + \delta t) = [I - i\delta t H(t_1)]\psi_0$ (3.6)

If the evolution is repeated at time t_2, beginning with the same initial state, the state at $t_2 + \delta t$ will be

$\psi(x, t_2 + \delta t) = [I - i\delta t H(t_2)]\psi_0$ (3.7)

Time translational invariance requires the outcome state should be the same:

$\psi(x, t_2 + \delta t) - \psi(x, t_1 + \delta t) = i\delta t[H(t_1) - H(t_2)]\psi_0 = 0$ (3.8)

Since the initial state ψ_0 is arbitrary, it follows that $H(t_1) = H(t_2)$. Moreover, since t_1 and t_2 are also arbitrary, it follows that H is time-independent, namely $dH/dt = 0$. It can be seen that this result relies on the linearity of evolution. If H depends on the state, then obviously we cannot obtain $dH/dt = 0$ because the state is related to time, though we still have $H(t_1, \psi_0) = H(t_2, \psi_0)$, which means that the state-dependent H also satisfies time translational invariance.

Secondly, space translational invariance requires $[T(a), U(t)] = 0$, which further leads to $[P, H] = 0$. This can be demonstrated as follows (see also Shankar 1994, p.293). Suppose at $t = 0$ two observers A and B prepare identical isolated systems at $x = 0$ and $x = a$, respectively. Let $\psi(x, 0)$ be the state of the system prepared by A. Then $T(a)\psi(x, 0)$ is the state of the system prepared by B, which is obtained by translating (without distortion) the state $\psi(x, 0)$ by an amount a to the right. The two systems look identical to the observers who prepared them. After time t, the states evolve into $U(t)\psi(x, 0)$ and $U(t)T(a)\psi(x, 0)$. Since the time evolution of each identical system at different places should appear the same to the local observers, the above two systems, which differed only by a spatial translation at $t = 0$, should differ only by the same spatial translation at future times. Thus the state $U(t)T(a)\psi(x, 0)$ should be the translated version of A's system at time t, namely we have $U(t)T(a)\psi(x, 0) = T(a)U(t)\psi(x, 0)$. This relation holds true for any initial state $\psi(x, 0)$, and thus we have $[T(a), U(t)] = 0$, which says that space translation operator and time translation operator are commutative. Again, we stress that the linearity of evolution is an important presupposition of this result. If U(t) depends on the state, then the space translational invariance will only lead to $U(t, T\psi)T(a)\psi(x, 0) = T(a)U(t, \psi)\psi(x, 0)$, from which we cannot obtain $[T(a), U(t)] = 0$.

When $dH/dt = 0$, the solutions of the evolution equation Eq.(3.5) assume the following form

$\psi(x, t) = \phi_E(x)e^{-iEt}$, (3.9)

where E is a constant, and $\phi_E(x)$ is the eigenstate of H and satisfies the time-independent equation:

$H\phi_E(x) = E\phi_E(x)$. (3.10)

The commutative relation [P, H] = 0 further implies that P and H have common eigenstates. This means that $\phi_E(x)$ is also the eigenstate of P . Since the eigenstate of $P = -i\partial/\partial x$ is e^{ipx}, where p is a real eigenvalue, the solution of the evolution equation Eq.(3.5) for an isolated system will be $e^{i(px-Et)}$. In quantum mechanics, P and H, the generators of space translation and time translation, are also called momentum operator and energy operator, respectively. Correspondingly, $e^{i(px-Et)}$ is the eigenstate of both momentum and energy, and p and E are the corresponding momentum and energy eigenvalues, respectively. In other words, the state $e^{i(px-Et)}$ describes an isolated system (e.g. a free microscopic particle) with definite momentum p and energy E.

3.2 Relativistic invariance

The relation between momentum p and energy E can be determined by the relativistic invariance of the momentum eigenstate $e^{i(px-Et)}$, and it turns out to be $E^2 = p^2c^2 + m^2c^4$, where m is the mass of the system, and c is the speed of light[36]. In the nonrelativistic domain, the energy momentum relation reduces to $E = p^2/2m$.

Now we will derive the relation between momentum p and energy E in the relativistic domain. Consider two inertial frames S_0 and S with coordinates x_0, t_0 and x, t. S_0 is moving with velocity v relative to S. Then x, t and x_0, t_0 satisfy the Lorentz transformations:

$$x_0 = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad (3.11)$$

$$t_0 = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}} \qquad (3.12)$$

Suppose the state of a free particle is $\psi = e^{i(p0x0-E0t0)}$, an eigenstate of P , in S_0, where p_0, E_0 is the momentum and energy of the particle in S_0, respectively. When described in S by coordinates x, t, the state is

$$\psi = e^{i(p_0 \frac{x-vt}{\sqrt{1-v^2/c^2}} - E_0 \frac{t-xv/c^2}{\sqrt{1-v^2/c^2}})} = e^{i(\frac{p_0+E_0v/c^2}{\sqrt{1-v^2/c^2}}x - \frac{E_0+p_0v}{\sqrt{1-v^2/c^2}}t)} \qquad (3.13)$$

This means that in frame S the state is still the eigenstate of P , and the corresponding momentum p and energy E is[37]

$$p = \frac{p_0 + E_0 v/c^2}{\sqrt{1 - v^2/c^2}} \qquad (3.14)$$

$$E = \frac{E_0 + p_0 v}{\sqrt{1 - v^2/c^2}} \qquad (3.15)$$

[4] We can also get this result from the definition Eq. (3.16) by using the above transformations of momentum and energy Eq.(3.14) and Eq.(3.15).

We further suppose that the particle is at rest in frame S_0. Then the velocity of the particle is v in frame S[4]. Considering that the velocity of a particle in the momentum eigenstate $e^{i(px-Et)}$ or a wavepacket superposed by these eigenstates is defined as the group velocity of the wavepacket, namely

u = dE/dp, (3.16)

we have
$dE_0/dp_0 = 0$, (3.17)
$dE/dp = v$. (3.18)
Eq.(3.17) means that E_0 and p_0 are independent. Moreover, since the particle is at rest in S_0, E_0 and p_0 do not depend on v. By differentiating both sides of Eq. (3.14) and Eq.(3.15) relative to v we obtain

$$\frac{dp}{dv} = \frac{v}{c^2}\frac{p_0 + E_0 v/c^2}{(1 - v^2/c^2)^{\frac{3}{2}}} + \frac{E_0/c^2}{(1 - v^2/c^2)^{\frac{1}{2}}} \qquad (3.19)$$

$$\frac{dE}{dv} = \frac{v}{c^2}\frac{E_0 + p_0 v}{(1 - v^2/c^2)^{\frac{3}{2}}} + \frac{p_0}{(1 - v^2/c^2)^{\frac{1}{2}}} \qquad (3.20)$$

Dividing Eq.(3.20) by Eq.(3.19) and using Eq.(3.18) we obtain

$$\frac{p_0}{\sqrt{1 - v^2/c^2}} = 0 \qquad (3.21)$$

This means that $p_0 = 0$. Inputting this important result into Eq.(3.15) and Eq. (3.14), we immediately obtain

$$E = \frac{E_0}{\sqrt{1 - v^2/c^2}}, \qquad (3.22)$$

$$p = \frac{E_0 v/c^2}{\sqrt{1 - v^2/c^2}}. \qquad (3.23)$$

Then the energy-momentum relation is:

$$E^2 = p^2c^2 + E_0^2 \qquad (3.24)$$

where E_0 is the energy of the particle at rest, called rest energy of the particle, and p and E is the momentum and energy of the particle with velocity v. By defining $m = E_0/c^2$ as the (rest) mass of the particle[38], we can further obtain the familiar energy-momentum relation
$E^2 = p^2c^2 + m^2c^4$ (3.25)
In the nonrelativistic domain, this energy-momentum relation reduces to $E = p^2/2m$.

3.3 Derivation of the free Schrödinger equation

The relation between energy E and momentum p for momentum eigenstates in the nonrelativistic domain implies that the operator relation is $H = P^2/2m$ for an isolated system, where H is the free Hamiltonian of the system. Note that since the value of E is real by Eq.(3.24), H is Hermitian and U(t) is unitary for free evolution. By inputting this operator relation into the evolution equation Eq. (3.5), we can obtain the free evolution equation, which assumes the same form as the free particle Schrödinger equation[39]:

$$i\frac{\partial\psi(x,t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} \qquad (3.26)$$

It is worth noting that, unlike the free particle Schrödinger equation, the reduced Planck constant with dimension of action is missing in this equation. However, this is in fact not a problem. The reason is that the dimension of can be absorbed into the dimension of the mass m. For example, we can stipulate the dimensional relations as p = 1/L, E = 1/T and $m = T/L^2$, where L and T represents the dimensions of space and time, respectively (see Duff, Okun and Veneziano 2002 for more discussions). Moreover, the value of can be set to the unit of number 1 in principle. Thus the above equation is essentially the free particle Schrödinger equation in quantum mechanics.

By using the definition of classical potential and requiring an appropriate expectation value correspondence, $d < P > dt =< F >=< \partial V \partial x >$, we can further obtain the Schrödinger equation under an external potential[40]:

$$i\frac{\partial\psi(x,t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t) \qquad (3.27)$$

The general form of a classical potential may be V (x,∂/∂x,t) and its concrete form is determined by the nonrelativistic approximation of the quantum interactions involved, which are described by the relativistic quantum field theory. Since the potential V (x, t) is real-valued, the Hamiltonian $H = P^2/2m + V$ (x, t) is Hermitian, and as a result, the time translation operator or evolution operator U(t) is also unitary.

3.4 Further discussions

We have derived the free Schrödinger equation in quantum mechanics based on spacetime translation invariance and relativistic invariance. The derivation may not only make the equation more logical and understandable, but also shed some new light on the physical meaning of the wave function ψ(x, t) in the equation.

The free Schrödinger equation is usually "derived" in textbooks by analogy and

correspondence with classical physics. There are at least two mysteries in such a heuristic "derivation". First, even if the behavior of microscopic particles is like wave and thus a wave function is needed to describe them, it is unclear why the wave function must assume a complex form. Indeed, when Schrödinger originally invented his equation, he was very puzzled by the inevitable appearance of the imaginary unit "i" in the equation. Next, one doesn't know why there are the de Broglie relations for momentum and energy and why the nonrelativistic energy-momentum relation must be $E = p^2/2m$. Usually one can only resort to experience and classical physics to answer these questions. This is unsatisfactory in logic as quantum mechanics is a more fundamental theory, of which classical mechanics is only an approximation.

As we have argued above, the key to unveil these mysteries is to analyze the origin of momentum and energy. According to the modern understanding, spacetime translation gives the definitions of momentum and energy. The momentum operator P is defined as the generator of space translation, and it is Hermitian and its eigenvalues are real. Moreover, the form of momentum operator can be uniquely determined by its definition. It is $P = -i\partial/\partial x$, and its eigenstate is e^{ipx}, where p is a real eigenvalue. Similarly, the energy operator H is defined as the generator of time translation. But its form is determined by the concrete situation. Fortunately, for an isolated system the form of energy operator, which determines the evolution equation, can be fixed by the requirements of spacetime translation invariance and relativistic invariance (when assuming the evolution is linear). Concretely speaking, time translational invariance requires that $dH/dt = 0$, and the solution of the evolution equation $i\partial\psi(x,t)/\partial t=H\psi(x, t)$ must assume the form $\psi(x, t) = \phi_E(x)e^{-iEt}$. Space translational invariance requires $[P, H] = 0$, and this further determines that $\phi_E(x)$ is the eigenstate of P , namely $\phi_E(x) = e^{ipx}$. Thus spacetime translation invariance entails that the state of an isolated system with definite momentum and energy assumes the plane wave form $e^{i(px-Et)}$. Furthermore, the relation between p and E or the energy-momentum relation can be determined by the relativistic invariance of the momentum eigenstate $e^{i(px-Et)}$, and its nonrelativistic approximation is just $E = p^2/2m$. Then we can obtain the form of energy operator for an isolated system, $H = P^2/2m$, and the free Schrödinger equation, Eq.(3.26). To sum up, this analysis may answer why the wave function must assume a complex form in general and why there are the de Broglie relations and why the nonrelativistic energy-momentum relation is what it is.

So far so good. But how does the wave function $\psi(x, t)$ in the thus-derived free

Schrödinger equation relate to the actual physical state of the system? Without answering this question the above analysis seems vacuous in physics. This leads us to the problem of interpreting the wave function. According to the standard probability interpretation, the wave function in quantum mechanics is a probability amplitude, and its modulus square gives the probability density of finding a particle in certain locations. Notwithstanding the success of the standard interpretation, our derivation of the free Schrödinger equation seems to suggest that the wave function $\psi(x, t)$ is a description of the objective physical state of a quantum system, rather than the probability amplitude relating only to measurement outcomes. In our derivation we never refer to the measurement of the isolated system at all. Moreover, the derivation seems to further suggest that the wave function $\psi(x, t)$ is a complete description of the physical state of the system. As we have argued in the last chapter, $\psi(x, t)$ can be regarded as an objective description of the state of random discontinuous motion of a particle, and $|\psi(x, t)|^2dx$ gives the objective probability of the particle being in an infinitesimal space interval dx near position x at instant t. This objective interpretation of the wave function is quite consistent with the above derivation of the free Schrödinger equation.

On the other hand, the derivation may provide a further argument for the non-existence of continuous motion from the aspect of the laws of motion. Continuous motion can be regarded as a very special form of discontinuous motion, for which the position density of a particle is $\rho(x, t) = \delta^2(x - x(t))$ and its velocity is $v(t) = dx(t)/dt$, where $x(t)$ is the continuous trajectory of the particle. However, such states are not solutions of the free Schrödinger equation, though they do satisfy the continuity equation. According to the free Schrödinger equation, an initial local state like $\delta(x - x_0)$ cannot sustain its locality during the evolution, and it will immediately spread throughout the whole space. Thus the law of free motion, which is derived based on the requirements of spacetime translation invariance etc, seems to imply that the motion of a particle cannot be continuous but be essentially discontinuous. Note that our derivation of the free Schrödinger equation does not depend on the picture of discontinuous motion, and thus this argument for the non-existence of continuous motion is not a vicious circle.

As noted above, our derivation of the free Schrödinger equation relies on the presupposition that the Hamiltonian H is independent of the evolved state, i.e., that the evolution is linear. It can be reasonably assumed that the linear evolution and nonlinear evolution both exist, and moreover, they satisfy spacetime translation invariance respectively because their effects cannot counteract each

other in general. Then our derivation only shows that the linear part of free evolution, if satisfying spacetime translation invariance and relativistic invariance, must assume the same form as the free Schrödinger equation in the nonrelativistic domain. Obviously, our derivation cannot exclude the existence of nonlinear quantum evolution. Moreover, since a general nonlinear evolution can readily satisfy spacetime translation invariance, the invariance requirement can no longer determine the concrete form of possible nonlinear evolution.

3.5 On the conservation of energy-momentum

The conservation of energy and momentum is one of the most important principles in modern physics. In this section, we will analyze the basis and physical meaning of this principle, especially its relationship with the linearity of quantum dynamics.

As we have noted in the above derivation of the free Schrödinger equation, the origin of momentum and energy is closely related to spacetime translation; the momentum operator P and energy operator H are defined as the generators of space translation and time translation, respectively. Moreover, it is well known that the conservation of energy and momentum results from spacetime translation invariance. The usual derivation is as follows. The evolution law for an isolated system satisfies spacetime translation invariance due to the homogeneity of space and time. Time translational invariance requires that H has no time dependence, namely $dH/dt = 0$, and space translational invariance requires that the generators of space translation and time translation are commutative, namely $[P, H] = 0$. Then by Ehrenfest's theorem for an arbitrary observable

$$d\langle A \rangle/dt=\langle \partial A/\partial t\rangle - i\langle [A, H]\rangle, \quad (3.28)$$

where $= \int \psi^*(x, t)A\psi(x, t)dx$ is defined as the expectation value of A, we have

$$d\langle H \rangle/dt=0, \quad (3.29)$$

and

$d< P >/dt=0.$ (3.30)

This means that the expectation values of energy and momentum are conserved for the evolution of an isolated system. Moreover, for arbitrary functions f(H) and f(P), we also have

$d< f(H) >/dt= 0,$ (3.31)

and

$d< f(P) >/dt= 0.$ (3.32)

This is equivalent to the constancy of the expectation values of the generating functions or spacetime translation operators $U(a) \equiv e^{-iaH}$ and $T(a) \equiv e^{-iaP}$

$d < U(a) >/dt = 0,$ (3.33)

and

$d< T(a) >/dt = 0.$ (3.34)

By these two equations it follows that the probability distributions of energy eigenvalues and momentum eigenvalues are constant in time. This statement is usually defined as the conservation of energy and momentum in quantum mechanics.

Now let's analyze the implications of this derivation for the meaning of the conservation of energy and momentum. First of all, we point out that the linearity of evolution is an indispensable presupposition in the derivation. As we have stressed in the derivation of the free Schrödinger equation, spacetime translation invariance does not lead to dH/dt = 0 and [P, H] = 0 without assuming the linearity of evolution. Therefore, the common wisdom that invariance or symmetry implies laws of conservation only holds true for linear evolutions; spacetime translation invariance no longer leads to the conservation of energy and momentum for any nonlinear evolution, and the invariance imposes no restriction for the nonlinear evolution either. Moreover, for a general nonlinear evolution H(ψ), energy and momentum will be not conserved by Ehrenfest's theorem[41]:

$$\frac{d\langle H(\psi)\rangle}{dt} = \langle\frac{\partial H(\psi)}{\partial t}\rangle - i\langle[H(\psi), H(\psi)]\rangle = \langle\frac{\partial H(\psi)}{\partial t}\rangle \neq 0, \tag{3.35}$$

and

$$\frac{d\langle P\rangle}{dt} = \langle\frac{\partial P}{\partial t}\rangle - i\langle[P, H(\psi)]\rangle = -i\langle[P, H(\psi)]\rangle = -\langle\frac{\partial H(\psi)}{\partial x}\rangle \neq 0. \tag{3.36}$$

We can see the violation of the conservation of energy and momentum more clearly by analyzing the nonlinear evolution of momentum eigenstates $e^{i(px-Et)}$ and their superpositions. If a nonlinear evolution can conserve energy and momentum for momentum eigenstates, then the momentum eigenstates must be the solutions of the nonlinear evolution equation; otherwise the evolution will change the definite momentum eigenvalues or energy eigenvalues or both, and thus the conservation of energy and momentum will be violated. Some nonlinear evolutions can satisfy this requirement. For example, when $H(\psi) = P^2/2m + \alpha|\psi|^2$, the solutions still include the momentum eigenstates $e^{i(px-Et)}$, where $E = p^2/2m + \alpha$, and thus energy and momentum are conserved for such nonlinear evolutions of momentum eigenstates. However, even if a nonlinear evolution can conserve energy and momentum for momentum eigenstates, it cannot conserve energy and momentum for the superpositions of momentum eigenstates. The reason is obvious. Only for a linear evolution the momentum eigenstates and their superpositions can both be the solutions of the evolution equation. For any nonlinear evolution $H(\psi)$, if the momentum eigenstates are already its solutions, then their linear superpositions cannot be its solutions. This means that the coefficients of the momentum eigenstates in the superposition will change with time during the evolution. The change of amplitudes of the coefficients directly leads to the change of the probability distribution of momentum eigenvalues and energy eigenvalues, while the change of phases of the coefficients leads to the change of the momentum eigenvalues or energy eigenvalues, which also leads to the change of the probability distribution of momentum eigenvalues or energy eigenvalues. In fact, a nonlinear evolution may not only change the probability distributions of energy and momentum eigenvalues, but also change the energy-momentum relation in general cases (e.g. in the above example)[42]. These results are understandable when considering the fact that a nonlinear evolution of the spatial wave function will generally introduce a time-dependent interaction between its different momentum eigenstates, which is equivalent to introducing a time-dependent external potential for its free evolution in some sense. Therefore, it is not beyond expectation that a nonlinear evolution violates the conservation of energy and momentum in general.

Two points needs to be stressed here. First, energy and momentum are still defined as usual for nonlinear evolutions in the above discussions. One may object that they should be re-defined for a nonlinear evolution. However, this may be not the case. The reason is as follows. Momentum is defined as the generator of space translation, and this definition uniquely determines that its eigenstates are e^{ipx}. Similarly, energy is defined as the generator of time

translation, and this definition uniquely determines that its eigenstates satisfy $H(\psi)\psi(x) = E\psi(x)$. Since these definitions are independent of whether the evolution of the state is linear or nonlinear, they should have a fundamental status in any theory formulated in space and time such as quantum mechanics. The second point is that the above argument implicitly assumes that the nonlinear evolution $H(\psi)$ is universal, i.e., that it applies to all possible states. If the nonlinear evolution only applies to some special states, then the evolution may still conserve energy and momentum. For example, suppose the nonlinear evolution $H(\psi) = P^2/2m + \alpha|\psi|^2$ applies only to the momentum eigenstates $e^{i(px-Et)}$ and the linear evolution $H(\psi) = P^2/2m$ applies to the superpositions of momentum eigenstates, then energy and momentum are still conserved during the evolution. On the other hand, it has been argued that the universal nonlinear quantum dynamics has a serious drawback, namely that the description of composite systems depends on a particular basis in a Hilbert space (Czachor 1996). If a nonlinear quantum evolution only applies to certain privileged bases due to some reason, then such nonlinear quantum dynamics may be logically consistent and also conserve energy and momentum (Gao 2004).

The second implication of the above derivation of the conservation laws is that spacetime translation invariance implies the conservation of energy and momentum for individual states, not for an ensemble of identical systems. As in the derivation of the free Schrödinger equation, we only refer to an isolated system and never refer to any ensemble of identical systems in the derivation of the conservation laws. Moreover, the transformations of spacetime translation also apply to a single isolated system. Therefore, what the derivation tells us is that spacetime translation invariance implies the conservation of energy and momentum for the linear evolution of the states of an isolated system. The conservation of energy and momentum for a single system means that the objective probability distributions of energy eigenvalues and momentum eigenvalues are constant during the evolution of the state of the system. As argued before, the objective probability can be well understood according to the suggested interpretation of the wave function in terms of random discontinuous motion. Similarly, our analysis of nonlinear evolutions also shows that a universal nonlinear evolution violates the conservation of energy and momentum for individual systems.

This implication raises a further issue. It is well known that the conservation of energy and momentum in quantum mechanics refers to an ensemble of identical systems, not to individual systems, and its precise statement is that the probability distributions of the measurement results of energy and momentum

for an ensemble of identical isolated systems are the same at every instant during the evolution of the systems in the ensemble. But as we have argued above, the derivation of the conservation laws based on spacetime translation invariance is for individual isolated systems, not for an ensemble of these systems. The derivation never refers to the measurements of these systems either. Therefore, there is still a gap (which maybe very large) between the derivation and the conservation laws in quantum mechanics. Undoubtedly we must analyze the measurement process in order to fill the gap. We will postpone the detailed analysis of the measurement problem to the next section. Here we only want to answer a more general question. If the conservation laws in quantum mechanics are indeed valid as widely thought, then what are their implications for the evolution of individual states?

First of all, the evolution of the state of an isolated system cannot contain a universal deterministic nonlinear evolution, which applies to all possible states; otherwise the evolution will violate the conservation of energy and momentum not only at the individual level but also at the ensemble level. Next, the evolution may contain linear evolutions as well as special deterministic nonlinear evolutions that apply only to certain privileged states. They can both conserve energy and momentum for individual states[43]. Lastly, the evolution may also contain a (universal) stochastic nonlinear evolution, which applies to all possible states. Although the evolution cannot conserve energy and momentum for individual states, it may conserve energy and momentum for an ensemble of identical states. As we will see in the next chapter, the collapse of the wave function may be such a stochastic nonlinear evolution.

To summarize, we have analyzed the relationships between the conservation of energy and momentum, spacetime translation invariance and the linearity of quantum dynamics. It has been often claimed that the conservation of energy and momentum is a conservation law resulting from the requirement of spacetime translation invariance. However, this common-sense view is not wholly right. Only when assuming the linearity of quantum dynamics, can spacetime translation invariance lead to the conservation of energy and momentum. Moreover, the connection between invariance of natural laws and conservation laws is for individual states, not for an ensemble of identical states. Although a nonlinear evolution of the wave function can readily satisfy spacetime translation invariance, the invariance can no longer lead to the conservation of energy and momentum, let alone determining the form of the nonlinear evolution. Rather, a universal nonlinear evolution that applies to all possible states will inevitably violate the conservation of energy and momentum.

Since the conservation of energy and momentum is required by spacetime translation invariance only for the linear evolution of the wave function of an isolated system, the principle cannot exclude the existence of a possible nonlinear evolution that may violate it. In other words, spacetime translation invariance is no longer a reason to require that the evolution of the wave function of an isolated system must conserve energy and momentum. On the other hand, the conservation of energy and momentum may still hold true for an ensemble of identical isolated systems as claimed by the standard quantum mechanics. Therefore, a (universal) stochastic nonlinear evolution of the wave function may exist. Although such evolutions cannot conserve energy and momentum for individual states, it may conserve energy and momentum at the ensemble level. However, unlike the linear evolution, which is natural in the sense that its form can be uniquely determined by the invariance requirements, the stochastic nonlinear evolution must have a physical origin, and its form can only be determined by the underlying mechanism. In the next chapter, we will investigate the possible stochastic nonlinear evolution of the wave function.

Chapter 4
The Solution to the Measurement Problem

Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a Ph.D.? ... Do we not have jumping then all the time?

—John Bell

In standard quantum mechanics, it is postulated that when a wave function is measured by a macroscopic device, it will no longer follow the linear Schrödinger equation, but instantaneously collapse to one of the wave functions that correspond to definite measurement results. However, this collapse postulate is ad hoc[44], and the theory does not tell us why and how a definite measurement result appears (Bell 1990).

There are in general two ways to solve the measurement problem. The first one is to integrate the collapse evolution with the normal Schrödinger evolution into a unified dynamics, *e.g.* in the dynamical collapse theories (Ghirardi 2008). The second way is to reject the collapse postulate and assume that the Schrödinger equation completely describes the evolution of the wave function. There are two main alternative theories for avoiding collapse. The first one is the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952), which takes the wave function as an incomplete description and adds some hidden variables to explain the emergence of definite measurement results. The second is the many-worlds interpretation (Everett 1957; DeWitt and Graham 1973), which assumes the existence of many equally real worlds corresponding to different possible results of quantum experiments and still regards the unitarily evolving wave function as a complete description of the total worlds.

It has been in hot debate which solution to the measurement problem is the right one or in the right direction. One of the main reasons is that the physical meaning of the wave function is not well understood. The failure of making sense of the wave function is partly because the problem is only investigated in the context of conventional impulse measurements. As we have seen in the previous chapters, with the help of protective measurement, the problem of interpreting the wave function can be solved independent of how to solve the measurement problem. Since the principle of protective measurement is based on the established parts of quantum mechanics, namely the linear Schrödinger evolution of the wave function (for microscopic systems) and the Born rule, its implications[45], especially the suggested interpretation of the wave function

based on it, can be used to examine the existing solutions to the measurement problem before experiments give the last verdict (cf. Marshall et al 2003)[46].

In this chapter, we will analyze the implications of protective measurement and the suggested interpretation of the wave function based on it for the solution to the measurement problem. It is first shown that the two main quantum theories without wavefunction collapse, namely the de Broglie-Bohm theory and the many-worlds interpretation, are inconsistent with protective measurement and the picture of random discontinuous motion of particles. This result implies that wavefunction collapse is a real physical process. Next, it is argued that the random discontinuous motion of particles may provide an appropriate random source to collapse the wave function. Moreover, the wavefunction collapse is a discrete process due to the discontinuity of motion, and the collapse states are energy eigenstates when the principle of conservation of energy is satisfied. Based on these analyses, we further propose a discrete model of energy-conserved wavefunction collapse. It is shown that the model is consistent with existing experiments and our macroscopic experience. We also provide a critical analysis of other dynamical collapse models, including Penrose's gravity-induced collapse model and the CSL (Continuous Spontaneous Localization) model.

4.1 The reality of wavefunction collapse

At first sight, the main solutions to the measurement problem, i.e., the de Broglie-Bohm theory, the many-worlds interpretation and dynamical collapse theories, seem apparently inconsistent with the suggested interpretation of the wave function. They all attach reality to the wave function, *e.g.* taking the wave function as a real physical entity on configuration space or assuming the wave function has a field-like spatiotemporal manifestation in the real three-dimensional space (see, *e.g.* Ghirardi 1997, 2008; Wallace and Timpson 2009). But according to our suggested interpretation, the wave function is not a field-like physical entity on configuration space[47]; rather, it is a description of the random discontinuous motion of particles in real space (and at a deeper level a description of the dispositional property of the particles that determines their random discontinuous motion). Anyway, in spite of the various views on the wave function in these theories, they never interpret the wave function as a description of the motion of particles in real space. However, on the one hand, the interpretation of the wave function in these theories is still an unsettled issue, and on the other hand, these theories may be not influenced by the interpretation of the wave function in a significant way. Therefore, they may be consistent with our suggested interpretation of the wave function after certain revision.

4.1.1 Against the de Broglie-Bohm theory

Let's first investigate the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952). According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The wave function follows the linear Schrödinger equation and never collapses. The particles, called Bohmian particles, are guided by the wave function via the guiding equation to undergo deterministic continuous motion. The result of a measurement is indicated by the positions of the Bohmian particles representing the pointer of the measuring device, and thus it is always definite. Moreover, it can be shown that the de Broglie-Bohm theory gives the same predictions of measurement results as standard quantum mechanics by means of a quantum equilibrium hypothesis (so long as the latter gives unambiguous predictions). Concretely speaking, the quantum equilibrium hypothesis provides the initial conditions for the guidance equation which make the de Broglie-Bohm theory obey Born's rule in terms of position distributions. Moreover, since all measurements can be finally expressed in terms of position, *e.g.* pointer positions, this amounts to full accordance with all predictions of quantum mechanics[48]. In this way, it seems that the de Broglie-Bohm theory can succeed in avoiding the collapse of the wave function.

However, although the de Broglie-Bohm theory is mathematically equivalent to quantum mechanics, there is no clear consensus with regard to its physical interpretation. The physical contents of the theory contain three parts: the Bohmian particles, the wave function, and the interaction between them. We first analyze the Bohmian particles and their physical properties. It is fair to say that what physical properties a Bohmian particle has is still an unsettled issue, and different proponents of the theory may have different opinions. For example, it has been often claimed that a Bohmian particle has mass, as the guiding equation for each Bohmian particle of a many-body system obviously contains the mass of each subsystem (Goldstein 2009). Yet it seems unclear whether the mass is inertial mass or (passive or active) gravitational mass or both or neither. On the other hand, it has been argued that the mass of a quantum system should be possessed by its wave function, not by its Bohmian particles (Brown, Dewdney and Horton 1995). It was even claimed (without proof) that a Bohmian particle has no properties other than its position (Hanson and Thoma 2011). In the last analysis, in order to know exactly what physical properties a Bohmian particle has, we need to analyze the guiding equation that defines the laws of motion for them.

In the minimum formulation of the theory, which is usually called Bohmian mechanics (Goldstein 2009)[49], the guiding equation contains an electromagnetic interaction term eA(x,t) for the Bohmian particle of a one-particle system with mass m and charge e in the presence of an external electromagnetic field[50]. According to this equation, the motion of a Bohmian particle is not only guided by the wave function, but also influenced by the external vector potential. In particular, the existence of the electromagnetic interaction term indicates that the Bohmian particle has the charge of the system, and the charge is localized in its position[51]. Similarly, the appearance of the mass of the system in the equation indicates that the Bohmian particle also has the (inertial) mass of the system. Therefore, according to Bohmian mechanics, the Bohmian particle of a one-particle system such as an electron has the mass and charge of the system. For example, in the ground state of a hydrogen atom, the Bohmian particle of the electron in the atom has the mass and charge of the electron, and it is at rest in a random position relative to the nucleus. That the Bohmian particle of a one-particle system has the mass and charge of the system can be seen more clearly from the quantum potential formulation of the de Broglie-Bohm theory. Its guiding equation contains both an electromagnetic interaction term and a gravitational interaction term in the presence of external electromagnetic field and gravitational field, which indicates that the Bohmian particle has the charge and (passive gravitational) mass of the system.

It can be seen that although a Bohmian particle has mass and charge, the functions of these properties are not as complete as usual. For example, in Bohmian mechanics, a charged Bohmian particle responds not to the electric scalar potential, but only to the magnetic vector potential, and it has no gravitational mass but only inertial mass. This apparent abnormality is in want of a reasonable physical explanation. In addition, in the quantum potential formulation, although the Bohmian particles of a quantum system respond to external gravitational and electromagnetic potentials, they don't have gravitational and electromagnetic influences on other charged quantum systems, including their Bohmian particles. Moreover, the Bohmian particles of a quantum system do not have gravitational and electromagnetic interactions with each other. Therefore, the (gravitational) mass and charge of a Bohmian particle are always passive, i.e., a Bohmian particle is only a receptor of gravitational and electromagnetic interactions. This characteristic may lead to some problems. For one, the nonreciprocal interactions will violate the conservation of energy and momentum (except that the Bohmian particles have no momentum and energy). At the worst, it may already suggest that the hypothetical Bohmian

particles are redundant entities in the theory (and their role in solving the measurement problem is ad hoc), since they have no any influence on other entities in the theory such as the wave function. Note that these problems do not exist for the wave function; the evolution of the wave function of a charged quantum system is influenced by both electric scalar potential and magnetic vector potential, as well as by gravitational potential, and the wave functions of two charged quantum systems also have gravitational and electromagnetic interactions with each other.

Another suggestion of the non-existence of Bohmian particles concerns the mass and charge distributions of a one-particle system such as an electron. As we have shown above, the guiding equation in the de Broglie-Bohm theory requires that the Bohmian particle of a one-particle system has the mass and charge of the system, and the mass and charge are localized in a position where the Bohmian particle is. On the other hand, as noted before, protective measurement shows that the mass and charge of a one-particle system such as an electron are not localized in one position but distributed throughout space, and the mass and charge density in each position is proportional to the modulus square of its wave function there. Therefore, the de Broglie-Bohm theory is inconsistent with the results of protective measurement concerning the mass and charge distributions of a quantum system[52]. This poses a serious objection to the de Broglie-Bohm theory.

Now let's turn to the wave function in the de Broglie-Bohm theory. Admittedly, the interpretation of the wave function in the theory has been debated by its proponents. For example, the wave function has been regarded as a field similar to electromagnetic field (Bohm 1952), an active information field (Bohm and Hiley 1993), a field carrying energy and momentum (Holland 1993), a causal agent more abstract than ordinary fields (Valentini 1997), a component of physical law (Durr, Goldstein and Zangh`i 1997), and a dispositional property of Bohmian particles (Belot 2011) *etc.* Notwithstanding the differences between these existing interpretations, they are inconsistent with the meaning of the wave function as implied by the results of protective measurement. To say the least, they fail to explain the existence of the mass and charge density for a charged quantum system, which is measurable by protective measurement and proportional to the modulus square of the wave function of the system. Our previous analysis shows that the mass and charge density of a quantum system is formed by the ergodic motion of a localized particle with the total mass and charge of the system, which is discontinuous and random in nature. Thus the wave function describes the state of random discontinuous motion of particles,

and at a deeper level, it represents the property of the particles that determines their random discontinuous motion. Since the principle of protective measurement is based on the linear Schrödinger evolution of the wave function and the Born rule, which also hold true in the de Broglie-Bohm theory, its implications, especially the resulting interpretation of the wave function, are still valid in the theory.

The realistic interpretation of the wave function poses another serious threat against the Bohmian-particles explanation of the guiding equation imposed by the de Broglie-Bohm theory. The guiding equation is only a mathematical transformation of the relation between the density ρ and the flux density j for the wave function; the relation is $j = \rho v$, while the guiding equation is $v=j/\rho$. Since the wave function of a quantum system is not merely a probability amplitude for the predictions of measurement results, but also a realistic description of the physical state of the system as implied by protective measurement[53], the guiding equation already has a physical explanation relating only to the realistic wave function. Inasmuch as a fundamental mathematical equation in a physical theory has a unique physical explanation, the additional explanation of the guiding equation relating to the hypothetical Bohmian particles will be improper[54]. In addition, the positions of the Bohmian particles as added (hidden) variables seem redundant too[55]. In some sense, there are already additional variables besides the wave function for the random discontinuous motion of particles. They are the definite position, momentum and energy of the particles at each instant. Though these variables are not continuous and deterministic, their random motion might just lead to the stochastic collapse of the wave function and further account for the emergence of random measurement results. We will discuss this possibility in detail later on.

Lastly, we analyze the hypothetical interaction between the Bohmian particles and the wave function in the de Broglie-Bohm theory. It can be seen that the guiding responsibility of the wave function assumed by the theory is inconsistent with the meaning of the wave function. As noted above, the wave function represents the property of particles that determines their random discontinuous motion. Accordingly, the wave function indeed guides the motion of particles in some sense. However, the wave function guides the motion of the particles not in a deterministic and continuous way as assumed by the de Broglie-Bohm theory, but in a probabilistic and discontinuous way; the modulus square of the wave function determines the probability density of the particles appearing in certain positions in space. Moreover, the motion of these particles is ergodic. By contrast, the motion of the Bohmian particles is not ergodic, and the time

averages of the Bohmian particles' positions typically differ remarkably from the ensemble averages (Aharonov, Erez and Scully 2004).

Although one may assume that a quantum system contains additional Bohmian particles besides its non-Bohmian particles that undergo random discontinuous motion, the motion of the Bohmian particles cannot be guided by the wave function of the system. For the wave function of the system represents the property of the non-Bohmian particles of the system, and its efficiency is to guide the motion of *these* particles in a probabilistic way. In particular, the wave function is neither a field-like entity distributing throughout space nor a property of the Bohmian particles that may guide their motion, and at every instant there are only non-Bohmian particles being in positions that are usually far from the positions of the hypothetical Bohmian particles. Note also that the non-Bohmian particles cannot have known interactions such as gravitational and electromagnetic interactions with the Bohmian particles either; otherwise the theory will contradict quantum mechanics and experiments. Without being guided by the wave function in a proper way, the motion of the Bohmian particles will be unable to generate the right measurement results in conventional impulse measurements.

In conclusion, we have argued that the de Broglie-Bohm theory is inconsistent with the results of protective measurement and the meaning of the wave function implied by them when considering its physical contents.

4.1.2 Against the many-worlds interpretation

Now let's turn to the second approach to avoid wavefunction collapse, the many-worlds interpretation. Although this theory is widely acknowledged as one of the main alternatives to quantum mechanics, its many fundamental issues, *e.g.* the preferred basis problem and the interpretation of probability, have not been completely solved yet (see Barrett 1999, 2011; Saunders et al 2010 and references therein). In this subsection, we will mainly analyze whether the existence of many worlds is consistent with the results of protective measurement and the picture of random discontinuous motion of particles.

According to the many-worlds interpretation, each component of the wave function of a measuring device that represents a definite measurement result corresponds to each world among the many worlds (Barrett 2011). This means that in one world there is only one component of the superposed wave function and the other components do not exist, and thus these components that correspond to the other worlds cannot be observed in this world. As a result, in every world the whole superposed wave function of the measuring device cannot be measured. If all components of the superposed wave function of the device can be observed in one world, then they will all exist in this world, which

obviously contradicts the many-worlds interpretation.

It is unsurprising that the existence of such many worlds may be consistent with the results of conventional impulse measurements, as the many-worlds interpretation is just invented to explain the emergence of these results, *e.g.* the definite measurement result in each world always denotes the result of a conventional impulse measurement. However, this does not guarantee consistency for all types of measurements. Indeed, it can be seen that the existence of the many worlds defined above is inconsistent with the results of protective measurements. The reason is that the whole superposed wave function of a quantum system including a measuring device can be measured by a protective measurement[56]. The result of the protective measurement as predicted by quantum mechanics indicates that all components of the superposed wave function of the measuring device exist in the same world where the protective measurement is made. Therefore, according to protective measurement, the components of the superposed wave function of a measuring device, each of which represents a definite measurement result, do not correspond to many worlds, in each of which there is only one such component and a copy of the measuring device that obtains a definite result; rather, the whole superposed wave function of the measuring device, if it exists, only exists in one world, namely our world, and in this world there is only one measuring device that obtains no definite result. In this way, protective measurement provides a strong argument against the many-worlds interpretation[57].

Four points are worth stressing. First of all, the above argument does not depend on how the many worlds are precisely defined in the many-worlds interpretation. For example, it is irrelevant to whether the many worlds are fundamental or emergent, and in particular, it also applies to Wallace's formulation of the many-worlds interpretation based on a structuralist view on macro-ontology. The key point is that all components of the superposed wave function of a measuring device can be detected by protective measurement in one world, namely our world, and thus they all exist in this world. Therefore, it is impossible that the superposed wave function of a measuring device corresponds to many worlds, only one of which is our world[58].

Next, the above argument is not influenced by environment-induced decoherence. On the one hand, even if the superposition state of a measuring device is entangled with the states of other systems, the entangled state of the whole system can also be measured by protective measurement in principle (Anandan 1993). The method is by adding appropriate protection procedure to the whole system so that its entangled state is a nondegenerate eigenstate of the

total Hamiltonian of the system together with the added potential. Then the entangled state can be protectively measured. On the other hand, environment-induced decoherence is not an essential element of the many-worlds interpretation. Even when a measuring device is isolated from environment (and the measured particle is absorbed by the device), the interpretation also requires that each component of the wave function of the measuring device in which there is a definite measurement result corresponds to each world among the many worlds; otherwise the many-worlds interpretation will not give the same predictions of measurement results as standard quantum mechanics (so long as the latter gives unambiguous predictions).

Thirdly, the above argument does not require protective measurement to be able to distinguish the superposed wave function of a measuring device (in each component of which there is a definite measurement result) from one of its components, or whether the superposed wave function collapses or not during a conventional impulse measurement. Since the determination demands the distinguishability of two non-orthogonal states, which is prohibited by quantum mechanics, no measurements consistent with the theory including protective measurement can do this. What protective measurement tells us is that such a superposed wave function, which existence is assumed by the many-worlds interpretation, does not correspond to the many worlds defined by the many-worlds interpretation. In other words, protective measurement reveals inconsistency of the many-worlds interpretation. Lastly, we stress again that the principle of protective measurement is irrelevant to the controversial process of wavefunction collapse and only depends on the linear Schrödinger evolution and the Born rule. As a result, protective measurement can (at least) be used to examine the internal consistency of the no-collapse solutions to the measurement problem, *e.g.* the many-worlds interpretation, before experiments give the last verdict.

In the following, we will further show that the existence of many worlds is not consistent with the picture of random discontinuous motion of particles either. In order to examine the many-worlds interpretation, it is necessary to know exactly what a quantum superposition is. No matter how to define the many worlds, they correspond to some components of a quantum superposition after all (e.g. the components where measuring devices obtain definite results, and in particular, observers have definite conscious experience). According to the picture of random discontinuous motion of particles, a quantum superposition exists in a form of time division. For a superposition of two positions A and B of a quantum system (e.g. the pointer of a measuring device), the system randomly and discontinuously jumps between these two positions. At some random and

discontinuous instants the system is in position A, and at other instants it is in position B. In this picture of quantum superposition, it is obvious that there is only one system all along, which randomly and discontinuously moves throughout all components of the superposition, no matter the system is a microscopic particle or a measuring device or an observer. In other words, there is only one world whose instantaneous state is constantly changing in a random and discontinuous way.

This conclusion is also supported by a comparison between discontinuous motion and continuous motion. For a quantum particle undergoing discontinuous motion, the position of the particle changes discontinuously. For a classical particle, its position changes continuously. There is no essential difference between these two kinds of changes. For both cases the position of the particle is always definite at each instant, and the positions of the particle at different instants may be different. Moreover, the discontinuous change, like the continuous change, does not create the many worlds, because, among other reasons, the change happens all the while but the creating process only happens once. Therefore, if there is only one world in classical mechanics, then there is also one world in quantum mechanics according to the picture of random discontinuous motion of particles, no matter how the many worlds are precisely defined.

We have argued that there are no many (physical) worlds as claimed by the many-worlds interpretation, and in particular, even if the physical state or brain state of an observer is in a quantum superposition, there is still one physical observer. However, the argument does not exclude the variants of the many-worlds interpretation that assume a distinct dynamics for the evolution of an observer's mental state, *e.g.* the many-minds theory (Albert and Loewer 1988) [59]. For example, although the superposed brain state of an observer does not correspond to many physical observers, each of which has a definite measurement record, it may correspond to many minds of a unique observer, each of which has the experience of a definite measurement record, as assumed by the many-minds theory. Since what we can immediately access is not the position of the pointer of a measuring device, but our immediate conscious experience, it is indeed necessary to analyze the conscious experience of an observer during a conventional impulse measurement. In the final analysis, the measurement problem is the problem of explaining the apparent incompatibility of our determinate experience and the existence of indeterminate quantum superpositions.

According to our existing experience, when an observer makes an impulse

measurement (by or not by a measuring device) on a quantum superposition of two states of a measured system, each of which can lead to a determinate conscious perception of the observer, his conscious perception is randomly one of the determinate perceptions corresponding to the two states (with probability being equal to the objective probability of the respective state in the superposition). The question is whether an observer in a quantum superposition of definite brain states, which may be called a quantum observer, can have a determinate conscious perception corresponding to one of these brain states in a probabilistic way consistent with the above experience. We will argue below that the answer is negative.

According to the picture of random discontinuous motion of particles, for a quantum observer there is still one physical observer whose brain state is definite at every instant but undergoes random discontinuous change. There are three possibilities for the conscious perception of such a quantum observer. The first possibility is that the conscious perception of a quantum observer is irrelevant to his superposed brain state. Obviously this possibility is inconsistent with the above experience. The second possibility is that the conscious perception of a quantum observer depends on his superposed brain state, and the observer can instantaneously be conscious of his brain state. In this case, the conscious perception of a quantum observer, parallel to his brain state, will also undergo random and discontinuous change between the determinate conscious perceptions corresponding to the brain states in the superposition[60]. This is not consistent with the above experience either.

The third possibility is that the conscious perception of a quantum observer depends on his superposed brain state, and the observer can be conscious of his brain state only during a finite time interval. Then the conscious perception of the quantum observer will not undergo random and discontinuous change between the conscious perceptions corresponding to the brain states in the superposition, as the time average of his brain state during a finite time interval contains no randomness. In other words, his conscious perception will be not random but fixed[61]. This is also inconsistent with the above experience.

To sum up, the above analysis shows that the de Broglie-Bohm theory and the many-worlds interpretation are inconsistent with protective measurement and the resulting interpretation of the wave function in terms of random discontinuous motion of particles. If there are no hidden variables besides the wave function, then the state of a quantum system including a measuring device will be represented only by its wave function. If there are no many worlds either, then a definite measurement result, which is usually denoted by a definite position of

the pointer of a measuring device, will be represented by a local wave packet of the pointer, rather than by a superposition of local wave packets. As a result, the transition from microscopic uncertainty to macroscopic certainty (e.g. the emergence of definite measurement results) can only be achieved by the collapse of the wave function. In other words, wavefunction collapse will be a real physical process.

As noted earlier, however, the existing ontology of the dynamical collapse theories that admit the reality of wavefunction collapse, such as mass density ontology and flash ontology (Ghirardi, Grassi and Benatti 1995; Ghirardi 1997, 2008; Allori et al 2008), is inconsistent with the picture of random discontinuous motion of particles. Especially, the existence of the effective mass and charge density of a quantum system, which is measurable by protective measurement, seems to already exclude the mass density ontology. In addition, the existing dynamical collapse theories are still phenomenological models, and they are also plagued by some serious problems such as energy non-conservation etc (Pearle 2007, 2009). In particular, the physical origin of the wavefunction collapse, including the origin of the randomness of the collapse process, is still unknown, though there are already some interesting conjectures (see, *e.g.* Di´osi 1989; Penrose 1996). In the following sections, we will try to solve these problems and propose a new dynamical collapse model in terms of the random discontinuous motion of particles. A more detailed review of the existing dynamical collapse theories will be given in the last section.

4.2 The origin of wavefunction collapse

It is well known that a ‘chooser’ and a ‘choice’ are needed to bring the required dynamical collapse of the wave function (Pearle 1999). The chooser is the noise source that collapses the wave function, and the choices are the states toward which the collapse tends. In this section, we will first analyze these two relatively easier problems and then investigate the more difficult problem, the physical origin of wavefunction collapse.

4.2.1 The chooser in discrete time

To begin with, let’s analyze the chooser problem. In the existing dynamical collapse models, the chooser is generally assumed to be an unknown classical noise field independent of the collapsed wave function (Pearle 2007, 2009). If what the wave function describes is the random discontinuous motion of particles, then it seems natural to assume that the random motion of particles is the appropriate noise source to collapse the wave function. This has three merits at least. First, the noise source and its properties are already known. For example, the probability of the particles being in certain position, momentum and energy at each instant is given by the modulus square of their wave function

at the instant. Next, this noise source is not a classical field, and thus the model can avoid the problems introduced by the field such as the problem of infinite energy etc (Pearle 2009). Last but not least, the random discontinuous motion of particles can also manifest itself in the equation of motion by introducing the collapse evolution of the wave function. In the following, we will give a more detailed argument for this claim.

According to the suggested interpretation of the wave function, the wave function of a quantum particle is an instantaneous dispositional property of the particle that determines its random discontinuous motion. However, the wave function is not a complete description of the instantaneous state of the particle. The instantaneous state of the particle at a given instant also includes its random position, momentum and energy at the instant, which may be called the random part of the instantaneous state of the particle. Although the probability of the particle being in each random instantaneous state is completely determined by the wave function, its stay in the state at each instant is a new physical fact independent of the wave function. Therefore, it seems natural to assume that the random stays of the particle may have certain physical efficiency that manifests in the complete equation of motion[62]. Since the motion of the particle is essentially random, its stay at an instant does not influence its stays at other instants in any direct way. Then the random stays of the particle can only manifest themselves in the equation of motion by their influences on the evolution of the wave function[63]. This forms a feedback in some sense; the wave function of a particle determines the probabilities of its stays in certain position, momentum and energy, while its random stay at each instant also influences the evolution of the wave function in a stochastic way[64].

However, the existence of the stochastic influences on the evolution of the wave function relies on an important precondition: the discreteness of time. If time is continuous and instants are durationless, the random stays of a particle can have no stochastic influence on anything. The reason is as follows. First, the duration of each random stay of the particle is zero in continuous time. Due to the randomness of motion, when there are at least two possible instantaneous states a particle can move between, the particle cannot stay in the same instantaneous state throughout a finite time. For the joint probability of the particle being in the same instantaneous state for all infinitely uncountable instants in the finite time interval is obviously zero, and the total probability of the particle being in other instantaneous states is not zero at any instant in between either. In other words, in order that a particle stays in the same instantaneous state for a finite time, the probability of the particle being in this instantaneous state must be one all the

while during the entire interval. This is possible only for the banal case where there is only one instantaneous state the particle can stay and thus there is no motion and its randomness at all throughout the duration[65].

Secondly, the influence of the random stay of a particle at a durationless instant is zero. This can be readily understood. If a physical influence is not zero at each durationless instant, then it may accumulate to infinite during an arbitrarily short time interval, which should be avoided in physics. Lastly, the accumulated influence of the random stays during a finite time interval, even if it can be finite[66], contains no randomness. For the discontinuity and randomness of motion exist only at each durationless instant, during which the influence of the random stay is zero, and they don't exist during a finite time interval or even an infinitesimal time interval. For example, the state of random discontinuous motion in real space, which is defined during an infinitesimal time interval at a given instant, is described by the position density and position flux density, and they are continuous quantities that contain no discontinuity and randomness.

Therefore, if time is continuous and instants are durationless, then the random stays of a particle can have no stochastic effects. This also means that the random stays of a particle can influence the evolution of its wave function in a stochastic way only when instants are not zero-sized but finite-sized, i.e., when time is discrete or quantized. Once the duration of each random stay of a particle is finite, each random stay can have a finite stochastic influence on the evolution of the wave function. It is worth stressing again that if time is not discrete but continuous, a particle cannot stay in one of the infinitely many instantaneous states all through for a finite time; rather, it can only stay there for one zero-sized instant. But if time is discrete and instants are not zero-sized but finite-sized, even if a particle stays in an instantaneous state only for one instant, the duration of its stay is also finite as the instant is finite-sized. In some sense, the discreteness of time prevents a particle from jumping from its present instantaneous state to another instantaneous state and makes the particle stay in the present instantaneous state all through during each finite-sized instant[67]. Since it has been conjectured that the Planck scale is the minimum spacetime scale[68], we will assume that the size of each discrete instant or the quantum of time is the Planck time in our following analysis[69].

To sum up, the realization of the randomness and discontinuity of motion in the laws of motion requires that time is discrete. In discrete time, a particle randomly stays in an instantaneous state with definite position, momentum and energy at each discrete instant, with a probability determined by the modulus

square of its wave function at the instant. Each random, finite stay of the particle may have a finite influence on the evolution of its wave function. As we will show in the next section, the accumulation of such discrete and random influences may lead to the correct collapse of the wave function, which can then explain the emergence of definite measurement results. Accordingly, the evolution of the wave function will be governed by a revised Schrödinger equation, which includes the normal linear terms and a stochastic nonlinear term that describes the discrete collapse dynamics. Note that the wave function (as an instantaneous property of particles) also exists in the discrete time, which means that the wave function does not change during each discrete instant, and the evolution of the wave function including the linear Schrödinger evolution is also discrete.

4.2.2 Energy conservation and the choices

Now let's investigate the choice problem, namely the problem of determining the states toward which the collapse tends. The random stay of a particle may have a stochastic influence on the evolution of its wave function at each discrete instant. Then when the stochastic influences accumulate and result in the collapse of the wave function, what are the states toward which collapse tends? This is the choice problem or preferred basis problem. It may be expected that the stochastic influences of the motion of a particle on its wave function should not be arbitrary but be restricted by some fundamental principles. In particular, it seems reasonable to assume that the resulting dynamical collapse of the wave function should also satisfy the conservation of energy. As a result, the collapse states or choices will be the energy eigenstates of the total Hamiltonian of the system[70]. In the following, we will give a more detailed analysis of the consequences of this assumption. Its possible physical basis will be investigated in the next subsection.

As we have argued in the last section, for a deterministic evolution of the wave function such as the linear Schrödinger evolution, the requirement of energy conservation applies to a single isolated system. However, for a stochastic evolution of the wave function such as the dynamical collapse process, the requirement of energy conservation cannot apply to a single system in general but only to an ensemble of identical systems[71]. It can be proved that only when the collapse states are energy eigenstates of the total Hamiltonian of a given system, can energy be conserved for an ensemble of identical systems for wavefunction collapse (See Pearle 2000 for a more detailed analysis). Note that for the linear Schrödinger evolution under an external potential, energy is conserved but momentum is not conserved even at the ensemble level, and thus

it is not momentum conservation but energy conservation that is a more universal restriction for wavefunction collapse.

The conservation of energy can not only help to solve the preferred basis problem, but also further determine the law of dynamical collapse to a large extent. For each system in the same quantum state in an ensemble, in order that the probability distribution of energy eigenvalues of the state can keep constant for the whole ensemble (i.e. energy is conserved at the ensemble level), the random stay of the system at each discrete instant can only change its (objective) energy probability distribution[72], and moreover, the change must also satisfy a certain restriction. Concretely speaking, the random stay in a definite energy E_i will increase the probability of the energy branch $|E_i>$ and decrease the probabilities of all other energy branches pro rata. Moreover, the increasing amplitude must be proportional to the total probability of all other energy branches, and the coefficient is related to the energy uncertainty of the state. We will demonstrate this result in the next subsection.

A more important problem is whether this energy-conserved collapse model can explain the emergence of definite measurement results and our macroscopic experience. At first sight the answer appears negative. For example, the energy eigenstates being collapse states seems apparently inconsistent with the localization of macroscopic objects. However, a detailed analysis given in the following subsections will demonstrate that the model can be consistent with existing experiments and our macroscopic experience. The key is to realize that the energy uncertainty driving the collapse of the entangled state of a many-body system is not the uncertainty of the total energy of all subsystems, but the sum of the absolute energy uncertainty of every subsystem. As a result, the collapse states are the product states of the energy eigenstates of the Hamiltonian of each subsystem for a non-interacting or weakly-interacting many-body system. This provides a further collapse rule for the superpositions of degenerate energy eigenstates of a many-body system.

4.2.3 In search of a deeper basis

In this subsection, we will investigate the possible physical basis of the energy conservation restriction for wavefunction collapse.

It is well known that the conservation of energy and momentum refers to an ensemble of identical systems in standard quantum mechanics. However, this standard view seems unnatural when assuming an objective interpretation of the wave function of a single system, *e.g.* our suggested interpretation in terms of random discontinuous motion of particles. An ensemble is not an actual system after all, and the conservation of something for an ensemble seems physically

meaningless. Moreover, since a single system in the ensemble does not ‘know’ the other systems and the whole ensemble, there must exist some underlying mechanism that can ensure the conservation of energy for an ensemble. Then the conservation of energy for an ensemble of identical systems is probably a result of the laws of motion for individual systems in the ensemble. Here is a possible scheme. First of all, energy is conserved for the evolution of individual energy eigenstates. Next, a superposition of energy eigenstates will dynamically collapse to one of these energy eigenstates, and the probability of the collapse result satisfies Born’s rule. Then the wavefunction collapse will satisfy the conservation of energy for an ensemble of identical systems.

In the following, we will further suggest a possible physical basis for this scheme of energy-conserved wavefunction collapse. According to the picture of random discontinuous motion, for a particle in a superposition of energy eigenstates, the particle stays in an instantaneous state with a definite energy eigenvalue at a discrete instant, and at another instant it may jump to another instantaneous state with another energy eigenvalue. It seems to be a reasonable assumption that the particle has both the tendency to jump among the instantaneous states with different energies and the tendency to stay in the instantaneous states with the same energy, and their relative strength is determined by the energy probability distribution of the particle. This is satisfactory in logic, as there should exist two opposite tendencies in general, and their relative strength is determined by certain condition. In some sense, the two tendencies of a particle are related to the two parts of its instantaneous state; the jumping tendency is related to the wave function, and it is needed to manifest the superposition of different energy eigenstates, while the staying tendency is related to the random stays. These two opposite tendencies together constitute the complete "temperament" of a particle.

It can be argued that the tendency to stay in the same energy for individual particles might be the physical origin of the energy-conserved wavefunction collapse. For a particle in a superposition of energy eigenstates, the particle stays in an instantaneous state with definite energy at a discrete instant, and the staying tendency of the particle will increase its probability of being in the instantaneous states with the present energy at next instant. In other words, the random stay of a particle in an instantaneous state with an energy eigenvalue will increase the probability of the energy eigenvalue (and correspondingly decrease the probabilities of other energy eigenvalues pro rata). Moreover, the increase of probability may relate to the energy probability distribution of the particle. By the continuity of change of staying tendency, the particle will jump more readily among the instantaneous states with small energy uncertainty and more hardly

among the instantaneous states with large energy uncertainty (which can also be regarded as a restriction of energy change). Thus the larger the energy uncertainty of the superposition is, the larger the increase of probability is during each random stay. A detailed calculation, which will be given in the next subsection, shows that such random changes of energy probability distribution can continuously accumulate to lead to the collapse of the superposition of energy eigenstates to one of them.

It can be further argued that the probability distribution of energy eigenvalues should remain constant during the random evolution of an ensemble of identical systems, and thus the resulting wavefunction collapse will satisfy Born's rule. The reason is as follows. When an initial superposition of energy eigenstates undergoes the dynamical collapse process, the probability distribution of energy eigenvalues should manifest itself through the collapse results for an ensemble of identical systems. At a deeper level, it is very likely that the laws of nature permit nature to manifest itself, or else we will be unable to find the laws of nature and verify them by experiments, and our scientific investigations will be also pointless. This may be regarded as a meta-law. Since the collapse evolution of individual systems is completely random and irreversible, the diagonal density matrix elements for an ensemble of identical systems must be precisely the same as the initial probability distribution at every step of the evolution. Otherwise the frequency distribution of the collapse results in the ensemble cannot reflect the initial probability distribution, or in other words, the probability information contained in the initial state will be completely lost due to the random and irreversible collapse[73]. As a consequence, the collapse evolution will conserve energy at the ensemble level, and the collapse results will also satisfy Born's rule in quantum mechanics.

Certainly, there is still a question that needs to be answered. Why energy? Why not position or momentum? If there is only one property that undergoes the random discontinuous motion (e.g. position), then the above tendency argument for the unique property may be satisfying. But if there are many properties that undergoes the random discontinuous motion, then we need to answer why the tendency argument applies only to energy. A possible answer is that energy is the property that determines the linear evolution of the state of motion, and thus it seems natural and uniform that energy also determines the nonlinear collapse evolution. Moreover, energy eigenstates are the states of motion that no longer evolve (except an absolute phase) for the linear evolution. Then by analogy, it is likely that energy eigenstates are also the states that no longer evolve for the nonlinear collapse evolution, i.e., that the energy eigenstates are the collapse

states. However, we may never be able to reach (and know we reach) the end point of explanation. Another important task is to develop a concrete model and compare it with experiments. We do this in the subsequent sections.

4.3 A discrete model of energy-conserved wavefunction collapse

After giving a speculative analysis of the origin of wavefunction collapse in terms of the random discontinuous motion of particles, we will propose a discrete model of energy-conserved wavefunction collapse based on results obtained from the analysis.

Consider a multi-level system with a constant Hamiltonian. Its initial state is:

$$|\psi(0)\rangle = \sum_{i=1}^{m} c_i(0)\,|E_i\rangle, \qquad (4.1)$$

where $|E_i\rangle$ is the energy eigenstate of the Hamiltonian of the system, E_i is the corresponding energy eigenvalue, and $c_i(0)$ satisfies the normalization relation $\Sigma_i |c_i(0)|^2 = 1$.

According to our conjecture on the origin of wavefunction collapse, this superposition of energy eigenstates will collapse to one of the eigenstates after a discrete dynamical process, and the collapse evolution satisfies the conservation of energy at the ensemble level. The physical picture of the dynamical collapse process is as follows. At the initial discrete instant $t_0 = t_P$ (where t_P is the Planck time), the system randomly stays in a branch $|E_i$ with probability $P_i(0) \equiv |c_i(0)|^2$. [74] This finite stay slightly increases the probability of the staying branch and decreases the probabilities of all other branches pro rata. Similarly, at any discrete instant $t = nt_P$ the system randomly stays in a branch $|E_i$ with probability $P_i(t) \equiv |c_i(t)|^2$, and the random stay also changes the probabilities of the branches slightly. Then during a finite time interval much larger than t_P, the probability of each branch will undergo a discrete and stochastic evolution. In the end, the probability of one branch will be close to one, and the probabilities of other branches will be close to zero. In other words, the initial superposition will randomly collapse to one of the energy branches in the superposition.

Now we will give a concrete analysis of this dynamical collapse process. Since the linear Schrödinger evolution does not change the energy probability distribution, we may only consider the influence of dynamical collapse on the energy probability distribution. Suppose the system stays in branch $|E_i\rangle$ at the discrete instant $t = nt_P$, and the stay changes the probability of this branch, $P_i(t)$, to

$$P_i^i(t + t_P) = P_i(t) + \Delta P_i, \tag{4.2}$$

where the superscript i denotes the staying branch, and ΔP_i is a functional of $P_i(t)$. Due to the conservation of probability, the increase of the probability of one branch can only come from the scale-down of the probabilities of all other branches. This means that the probability of another branch $P_j(t)$ $(j \neq i)$ correspondingly turns to be[30]

$$P_j^i(t + t_P) = P_j(t) - \frac{P_j(t)\Delta P_i}{1 - P_i(t)}, \tag{4.3}$$

where the superscript i also denotes the staying branch. The probability of this random stay at the instant is $p(E_i, t) =$ $P_i(t)$. Then we can work out the diagonal density matrix elements of the evolution[75]:

[30] One can also obtain this result by first increasing the probability of the staying branch and then normalizing the probabilities of all branches. This means that $P_i(t+t_P) = \frac{P_i(t)+\Delta}{1+\Delta}$ and $P_j(t+t_P) = \frac{P_j(t)}{1+\Delta}$ for any $j \neq i$. In this way, we have $\Delta P_i = \frac{\Delta}{1+\Delta}(1 - P_i(t))$ and $\Delta P_j = \frac{\Delta}{1+\Delta}P_j(t)$ for any $j \neq i$.

$$\begin{aligned}
\rho_{ii}(t + t_P) &= \sum_{j=1}^{m} p(E_j, t) P_i^j(t + t_P) \\
&= P_i(t)[P_i(t) + \Delta P_i] + \sum_{j \neq i} P_j(t)[P_i(t) - \frac{P_i(t)\Delta P_j(t)}{1 - P_j(t)}] \\
&= \rho_{ii}(t) + P_i(t)[\Delta P_i - \sum_{j \neq i} P_j(t)\frac{\Delta P_j(t)}{1 - P_j(t)}].
\end{aligned} \tag{4.4}$$

Here we shall introduce the first rule of dynamical collapse, which says that the probability distribution of energy eigenvalues for an ensemble of identical systems is constant during the dynamical collapse process. As we have argued in the last subsection, this rule is required by the principle of energy conservation at the ensemble level, and it may also have a physical basis relating to the manifestability of nature. By this rule, we have $\rho_{ii}(t + t_P) = \rho_{ii}(t)$ for any i. This leads to the following set of equations:

$$\Delta P_1(t) - \sum_{j \neq 1} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} = 0,$$
$$\Delta P_2(t) - \sum_{j \neq 2} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} = 0,$$
$$\ldots$$
$$\Delta P_m(t) - \sum_{j \neq m} \frac{P_j(t)\Delta P_j(t)}{1 - P_j(t)} = 0. \qquad (4.5)$$

By solving this equations set (e.g.by subtracting each other), we find the following relation for any i:

$$\frac{\Delta P_i}{1 - P_i(t)} = k, \qquad (4.6)$$

where k is an undetermined dimensionless quantity that relates to the state $|\psi(t)\rangle$.
By using Eq. (4.6), we can further work out the non-diagonal density matrix elements of the evolution. But it is more convenient to calculate the following variant of non-diagonal density matrix elements:

$$\begin{aligned} \varrho_{ij}(t + t_P) &= \sum_{l=1}^{m} p(E_l, t) P_i^l(t + t_P) P_j^l(t + t_P) \\ &= \sum_{l \neq i,j} P_l(t)[P_i(t) - kP_i(t)][P_j(t) - kP_j(t)] \\ &\quad + P_i(t)[P_i(t) + k(1 - P_i(t))][P_j(t) - kP_j(t)] \\ &\quad + P_j(t)[P_j(t) + k(1 - P_j(t))][P_i(t) - kP_i(t)] \\ &= (1 - k^2)\varrho_{ij}(t). \qquad (4.7) \end{aligned}$$

Since the usual collapse time, τ_c, is defined by the relation $\rho_{ij}(\tau_c) = 1/2\rho_{ij}(0)$, we may use a proper approximation, where k is assumed to be the same as its initial value during the time interval $[0, \tau_c]$, to simplify the calculation of the collapse time. Then we have:

$$\varrho_{ij}(t) \approx (1 - k^2)^n \varrho_{ij}(0). \qquad (4.8)$$

The corresponding collapse time is in the order of:

$$\tau_c \approx \frac{1}{k^2} t_P, \qquad (4.9)$$

In the following, we shall analyze the formula of k defined by Eq.(4.6). To begin with, the probability restricting condition $0 \leq P_i(t) \leq 1$ for any i requires that $0 \leq k \leq 1$. When k = 0, no collapse happens, and when k = 1, collapse happens instantaneously. Note that k cannot be smaller than zero, as this will lead to the negative value of $P_i(t)$ in some cases. For instance, when k is negative and $P_i(t) < |k|/(1+|k|)$, $P_i(t + t_P) = P_i(t) + k[1 - P_i(t)]$ will be negative and violate the probability restricting condition. That k is positive indicates that each random stay increases the probability of the staying branch and decreases the probabilities of other branches, which is consistent with the analysis given in the last subsection.

Next, k is proportional to the duration of stay. The influence of each stay on the probability of the staying branch is an accumulating process. When the duration of stay is zero as in continuous space and time, no influence exists and no collapse happens. When the duration of stay, t_P, is longer, the probability of the staying branch will increase more. Thus we have $k \propto t_P$.

Thirdly, k is also proportional to the energy uncertainty of the superposition of energy eigenstates. First, from a dimensional analysis k should be proportional to an energy term in order to cancel out the dimension of time. Next, the energy term should be the energy uncertainty of the superposition defined in an appropriate way according to the analysis of the last subsection. When the energy uncertainty is zero, i.e., when the state is an energy eigenstate, no collapse happens. When the energy uncertainty is not zero, collapse happens. Moreover, the larger the energy uncertainty is, the larger the increase of the probability of the staying branch for each random stay is, namely the larger k is. Therefore, k will be proportional to the energy uncertainty of the superposition. How to define the energy uncertainty then? Since k is invariant under the swap of any two branches (P_i, E_i) and (P_j, E_j) according to Eq. (4.6), the most natural

definition of the energy uncertainty of a superposition of energy eigenstates is[76]:

$$\Delta E = \frac{1}{2}\sum_{i,j=1}^{m} P_i P_j |E_i - E_j|. \qquad (4.10)$$

For the simplest two-level system, we have

$$\Delta E = P_1 P_2 |E_1 - E_2|. \qquad (4.11)$$

It seems a little counterintuitive that k contains the energy uncertainty term that relates to the whole energy distribution. The puzzle is two-fold. First, this means that the increase of the probability of the staying branch relates not to the energy difference between the staying branch and all other branches, but to the energy uncertainty of the whole state. This is reflected in the formula of ΔE in the existence of the energy difference between any two branches, $|E_i - E_j|$ for any i and j. Next, the increase of the probability of the staying branch relates also to the energy probability distribution that determines the energy uncertainty. This is reflected in the formula of ΔE in the existence of P_iP_j. In fact, these seemingly puzzling aspects are still understandable. The first feature is required by the first rule of dynamical collapse that ensures energy conservation at the ensemble level. This can be clearly seen from Eq. (4.6). If the increase of the probability of the staying branch relates to the difference between the energy of the staying branch and the average energy of all other branches, then Eq. (4.6) will not hold true because the swap symmetry of k will be violated, and as a result, the first rule of dynamical collapse will be broken. The second feature can be understood as follows. In the picture of random discontinuous motion, the probability distribution contains the information of staying time distribution. An energy branch with small probability means that the system jumps through it less frequently. Thus this energy branch only makes a small contribution to the restriction of energy change or the increase of the staying tendency. As a result, k or the increase of the probability of the staying branch will relate not merely to energy difference, but also to the energy probability distribution.
Then after omitting a coefficient in the order of unity, we can get the formula of k in the first order:

$$k \approx \Delta E t_P/\hbar. \qquad (4.12)$$

This is the second rule of dynamical collapse. By inputting Eq. (4.12) into Eq. (4.9), we can further get the collapse time formula:

$$\tau_c \approx \frac{\hbar E_P}{(\Delta E)^2}, \qquad (4.13)$$

where $E_P = h/t_P$ is the Planck energy, and ΔE is the energy uncertainty of the initial state[77].

Here it is worth pointing out that k must contain the first order term of ΔE. For the second order or higher order term of ΔE will lead to much longer collapse time for some common measurement situations, which contradicts experiments (Gao 2006a, 2006b). Besides, a similar analysis of the consistency with experiments may also provide a further support for the energy-conserved collapse model in which the collapse states are energy eigenstates. First of all, if the collapse states are not energy eigenstates but momentum eigenstates, then the energy uncertainty will be replaced by momentum uncertainty in the collapse time formula Eq. (4.13), namely $\tau_c \approx hE_P/(\Delta pc)^2$. As a result, the collapse time will be too short to be consistent with experiments for some situations. For example, for the ground state of hydrogen atom the collapse time will be about several days. Note that the second order or higher order term of Δp will also lead to much longer collapse time for some common measurement situations, which contradicts experiments.

Next, if the collapse states are position eigenstates[78], then the collapse time formula Eq. (4.13) will be replaced by something like $\tau_c \approx l^2 t_P/(\Delta x)^2$, where l is certain length scale relating to the collapsing state. No matter what length scale l is, the collapse time of a momentum eigenstate will be zero as its position uncertainty is infinite. This means that the momentum eigenstates of any quantum system will collapse instantaneously to one of its position eigenstates and thus cannot exist. Moreover, the superposition states with very small momentum uncertainty will also collapse very quickly even for microscopic particles. These results are apparently inconsistent with quantum mechanics. Although it may be possible to adjust the length scale l to make the model consistent with experience, the collapse time formula will be much more complex than that in the above energy-conserved collapse model. Let's give a little more detailed analysis here. There are two universal length scales for a quantum system: its Compton wavelength λ_c and the Planck length l_P. It is obvious that both of them cannot be directly used as the length scale in the collapse time formula $\tau_c \approx l^2 t_P/(\Delta x)^2$. Then the formula can only be written in a

more complex form: $\tau_c \approx (\lambda_c/l_P)^{\alpha} \cdot \lambda_c^2 t_P/(\Delta x)^2$. Moreover, experiments such as the SQUID experiments and our everyday macroscopic experience require $\alpha \approx 8$. It seems very difficult to explain this unusually large exponent in theory. To sum up, the collapse states can hardly be position eigenstates when considering the consistency with experiments and the simplicity of theory.

Based on the above analysis, the state of the multi-level system at instant $t = nt_P$ will be:

$$|\psi(t)\rangle = \sum_{i=1}^{m} c_i(t) e^{-iE_i t/\hbar} |E_i\rangle, \qquad (4.14)$$

Besides the linear Schrödinger evolution, the collapse dynamics adds a discrete stochastic evolution for $P_i(t) \equiv |c_i(t)|^2$:

$$P_i(t + t_P) = P_i(t) + \frac{\Delta E}{E_P}[\delta_{E_s E_i} - P_i(t)], \qquad (4.15)$$

where ΔE is the energy uncertainty of the state at instant t defined by Eq. (4.10), E_s is a random variable representing the random stay of the system, and its probability of assuming E_i at instant t is $P_i(t)$. When $E_s = E_i$, $\delta_{EsEi} = 1$, and when $E_s = E_i$, $\delta_{EsEi} = 0$.

This equation of dynamical collapse can be directly extended to the entangled states of a many-body system. The difference only lies in the definition of the energy uncertainty ΔE. According to our analysis in the last subsection, for a non-interacting or weakly-interacting many-body system in an entangled state, for which the energy uncertainty of each subsystem can be properly defined, ΔE is the sum of the absolute energy uncertainty of all subsystems, namely

$$\Delta E = \frac{1}{2} \sum_{l=1}^{n} \sum_{i,j=1}^{m} P_i P_j |E_{li} - E_{lj}|, \qquad (4.16)$$

where n is the total number of the entangled subsystems, m is the total number of energy branches in the entangled state, and E_{li} is the energy of subsystem l in the i-th energy branch of the state. Correspondingly, the collapse states are the product states of the energy eigenstates of the Hamiltonian of each subsystem. It should be stressed here that ΔE is not defined as the uncertainty of the total energy of all subsystems as in the energy-driven collapse models (see, *e.g.* Percival 1995, 1998a; Hughston 1996). For each subsystem has its own energy uncertainty that drives its collapse, and the total driving “force” for the whole

entangled state should be the sum of the driving “forces” of all subsystems, at least in the first order approximation. Although these two kinds of energy uncertainty are equal in numerical values in some cases (e.g. for a strongly-interacting many-body system), there are also some cases where they are not equal. For example, for a superposition of degenerate energy eigenstates of a non-interacting many-body system, which may arise during a common measurement process, the uncertainty of the total energy of all subsystems is exactly zero, but the absolute energy uncertainty of each subsystem and their sum may be not zero. As a result, the superpositions of degenerate energy eigenstates of a many-particle system may also collapse. As we will see later, this is an important feature of our model, which can avoid Pearle’s (2004) serious objections to the energy-driven collapse models.

It can be seen that the equation of dynamical collapse, Eq.(4.15), has an interesting property, scale invariance. After one discrete instant t_P, the probability increase of the staying branch $|E_i>$ is $\Delta P_i = (1 - P_i)\Delta E/E_p$, and the probability decrease of the neighboring branch $|E_{i+1}>$ is $\Delta P_{i+1} = P_{i+1}\Delta E/E_p$. Then the probability increase of these two branches is

$$\Delta(P_i + P_{i+1}) = \frac{\Delta E}{E_P}[1 - (P_i + P_{i+1})]. \qquad (4.17)$$

Similarly, the equation $\Delta P = (1 - P)\Delta E/E_p$ holds true for the total probability of arbitrarily many branches (one of which is the staying branch). This property of scale invariance may simplify the analysis in many cases. For example, for a superposition of two wavepackets with energy difference, ΔE_{12}, much larger than the energy uncertainty of each wavepacket, $\Delta E_1 = \Delta E_2$, we can calculate the collapse dynamics in two steps. First, we use Eq.(4.15) and Eq.(4.11) with $|E_1 - E_2| = \Delta E_{12}$ to calculate the time of the superposition collapsing into one of the two wavepackets[79]. Here we need not to consider the almost infinitely many energy eigenstates constituting each wavepacket and their probability distribution. Next, we use Eq.(4.15) with $\Delta E = \Delta E_1$ to calculate the time of the wavepacket collapsing into one of its energy eigenstates. In general, this collapse process is so slow that its effect can be ignored.

Lastly, we want to stress another important point. As we have argued before, the discontinuity of motion requires that the collapse dynamics must be discrete in nature, and moreover, the collapse states must be energy eigenstates in order that the collapse dynamics satisfies the conservation of energy at the ensemble level. As a result, the energy eigenstates and their corresponding eigenvalues must be

also discrete for any quantum system. This result seems to contradict quantum mechanics, but when considering that our universe has a finite size (i.e. a finite event horizon), the momentum and energy eigenvalues of any quantum system in the universe may be indeed discrete[80]. The reason is that all quantum systems in the universe are limited by the finite horizon, and thus no free quantum systems exist in the strict sense. For example, the energy of a massless particle (e.g. photon) can only assume

discrete values $E_n = n^2 \frac{hc}{4R_U}$, and the minimum energy is $E_1 = \frac{hc}{4R_U} \approx 10^{-33} eV$, where $R_U \approx 10^{25} m$ is the radius of the horizon of our universe[37]. Besides, for a free particle with mass m_0, its energy also assumes discrete values $E_n = n^2 \frac{h^2}{32 m_0 R_U^2}$. For instance, the minimum energy is $E_1 \approx 10^{-72} eV$ for free electrons, which is much smaller than the minimum energy of photons[38].

It is interesting to see whether this tiny discreteness of energy makes the collapse dynamics more abrupt. Suppose the energy uncertainty of a quantum state is $\Delta E \approx 1eV$, and its energy ranges between the minimum energy E_1 and $1eV$. Then we can get the maximum energy level $l_{max} \approx \sqrt{\frac{1eV}{10^{-33}eV}} \approx 10^{16}$. The probability of most energy eigenstates in the superposition will be about $P \approx 10^{-16}$. During each discrete instant t_P, the probability increase of the staying energy branch is $\Delta P \approx \frac{\Delta E}{E_P}(1-P) \approx 10^{-28}$.

[37] Note that the present upper bound on the photon mass is about $m_\gamma < 10^{-18} eV/c^2$ (Nakamura et al. 2010).

[38] Whether this heuristic analysis is (approximately) valid depends on the application of the final theory of quantum gravity to our finite universe. However, it is worth noting that the existence of discrete energy levels for a free quantum system limited in our universe is also supported by the hypothetical holographic principle, which implies that the total information within a universe with a finite event horizon is finite. If the energy of a quantum system is continuous, then the information contained in the system will be infinite.

This indicates that the probability change during each random stay is still very tiny. Only when the energy uncertainty is larger than 10^{23}eV or $10^{-5}E_P$, will the probability change during each random stay be sharp. Therefore, the collapse evolution is still very smooth for the quantum states with energy uncertainty

much smaller than the Planck energy.

4.4 On the consistency of the model and experiments

In this section, we will analyze whether the discrete model of energy-conserved wavefunction collapse is consistent with existing experiments and our macroscopic experience. Note that Adler (2002) has already presented a detailed consistency analysis in the context of energy-driven collapse models, and as we will see below, most of his analysis also applies to our model.

4.4.1 Maintenance of coherence

First of all, the model satisfies the constraint of predicting the maintenance of coherence when this is observed. Since the energy uncertainty of the state of a microscopic particle is very small in general, its collapse will be too slow to have any detectable effect in present experiments on these particles. For example, the energy uncertainty of a photon emitted from an atom is in the order of 10^{-6}eV , and the corresponding collapse time is 10^{25}s according to Eq. (4.13) of our collapse model, which is much longer than the age of the universe, 10^{17}s. This means that the collapse states (i.e. energy eigenstates) are never reached for a quantum system with small energy uncertainty even during a time interval as long as the age of the universe. As another example, consider the SQUID experiment of Friedman et al (2000), where the coherent superpositions of macroscopic states consisting of oppositely circulating supercurrents are observed. In the experiment, each circulating current corresponds to the collective motion of about 10^9 Cooper pairs, and the energy uncertainty is about 8.6×10^{-6}eV . Eq. (4.13) predicts a collapse time of 10^{23}s, and thus maintenance of coherence is expected despite the macroscopic structure of the state[81]. For more examples see Adler (2002).

4.4.2 Localization in measurement situations

In the following, we will investigate whether the discrete model of energy-conserved wavefunction collapse can account for the emergence of definite measurement results. Let's first see a simple position measurement experiment. Consider an initial state describing a particle in a superposition of two locations (e.g. a superposition of two Gaussian wavepackets separated by a certain distance). After the measurement interaction, the position measuring device evolves to a superposition of two macroscopically distinguishable states:

$$(c_1\psi_1 + c_2\psi_2)\phi_0 \rightarrow c_1\psi_1\phi_1 + c_2\psi_2\phi_2, \quad (4.18)$$

where ψ_1, ψ_2 are the states of the particle in different locations, ϕ_0 is the initial state of the position measuring device, and ϕ_1, ϕ_2 are the different outcome states of the device. For an ideal measurement, the two particle/device states $\psi_1\phi_1$ and

$\psi_2\phi_2$ have precisely the same energy spectrum. Then it appears that this superposition will not collapse according to the energy-conserved collapse model.

However, this is not the case. The key is to see that the two states of the particle in the superposition are detected in different parts of the measuring device, and they interact with the different atoms or molecules in these parts. Thus we should rewrite the device states explicitly as $\phi_0 = \chi_A(0)\chi_B(0)$, $\phi_1 = \chi_A(1)\chi_B(0)$, and $\phi_2 = \chi_A(0)\chi_B(1)$, where $\chi_A(0)$ and $\chi_B(0)$ denote the initial states of the device in the parts A and B, respectively, and $\chi_A(1)$ and $\chi_B(1)$ denote the outcome states of the device in the parts A and B, respectively. Then we have

$$(c_1\psi_1+c_2\psi_2)\chi_A(0)\chi_B(0) \rightarrow c_1\psi_1\chi_A(1)\chi_B(0)+c_2\psi_2\chi_A(0)\chi_B(1) \quad (4.19)$$

This reformulation clearly shows that there exists energy difference between the subsystems in the different outcome states of the device. Since there is always some kind of measurement amplification from the microscopic state to the macroscopic outcome in the measurement process, there will be a large energy difference between the states $\chi_A(0)$, $\chi_B(0)$ and $\chi_A(1)$, $\chi_B(1)$. As a result, the total energy difference $\Delta E = |\Delta E_A| + |\Delta E_B|$ is also very large, and it will result in the rapid collapse of the above superposition into one of its branches according to the energy-conserved collapse model[82].

Let's see a more realistic example, a photon being detected via photoelectric effect (e.g. by a single-photon avalanche diode). In the beginning of the detection, the spreading spatial wave function of the photon is entangled with the states of a large number of surface atoms of the detector. In each local branch of the entangled state, the total energy of the photon is wholly absorbed by the electron in the local atom interacting with the photon. This is clearly indicated by the term $\delta(E_f - E_i - \omega)$ in the transition rate of photoelectric effect. The state of the ejecting electron is a (spherical) wavepacket moving outward from the local atom, whose average direction and momentum distribution are determined by the momentum and polarization of the photon. The small energy uncertainty of the photon will also be transferred to the ejecting electron[83].

This microscopic effect of ejecting electron is then amplified (e.g. by an avalanche process of atoms) to form a macroscopic signal such as the shift of the pointer of a measuring device. During the amplification process, the energy difference is constantly increasing between the branch in which the photon is absorbed and the branch in which the photon is not absorbed near each atom interacting with the photon. This large energy difference will soon lead to the collapse of the whole superposition into one of the local branches, and thus the

photon is only detected locally. Take the single photon detector avalanche photodiode as a typical example[84]. Its energy consumption is sharply peaked in a very short measuring interval. One type of avalanche photodiode operates at 10^5 cps and has a mean power dissipation of 4mW (Gao 2006a). This corresponds to an energy consumption of about 2.5×10^{11}eV per measuring interval 10^{-5}s. By using the collapse time formula Eq. (4.13), where the energy uncertainty is $\Delta E \approx 2.5 \times 10^{11}$eV , we find the collapse time is $\tau_c \approx 1.25 \times 10^{-10}$s. This collapse time is much smaller than the measuring interval.

One important point needs to be stressed here. Although a measured particle is detected locally in a detector (e.g. the spatial size of its collapse state is in the order of the size of an atom), its wave function does not necessarily undergo the position collapse assumed in an ideal position measurement by standard quantum mechanics, and especially, energy can be conserved during the localization process according to our model. The reason can be summarized as follows. The wave function of the measured particle is usually a spherical wave (e.g. a spherically symmetric wave function) in three-dimensional space. Its momentum is along the radial direction, but the local and random measurement result distributes along the sphere, perpendicular to the radial direction. During the detection, the measured particle interacts with a single atom of the detector by an ionizing process in each local branch of the entangled state of the whole system including the particle and the atoms in the detector. The particle is usually absorbed by the atom or bound in the atom, and its energy is wholly transferred to the newly-formed atom and the ejecting electrons during the ionizing process in each branch. Then the amplification process such as an avalanche process of atoms introduces very large energy difference between the detected branch and the empty branch, and as a result, the whole superposition will soon collapse into one of its local branches in a random way according to the energy-conserved collapse model[85]. After the collapse, the state of the measured particle is localized in the spatial region of one atom. Moreover, since each local branch of the entangled state of the particle and the detector has the same energy spectrum, the collapse process also conserves energy at the individual level.

4.4.3 Emergence of the classical world

Now let’s see whether the discrete model of energy-conserved wavefunction collapse is consistent with our macroscopic experience. It seems that there is an apparent inconsistency here. According to the model, when there is a superposition of a macroscopic object in an identical physical state (an approximate energy eigenstate) at two different, widely separated locations, the

superposition does not collapse. The reason is that there is no energy difference between the two branches of the superposition. However, the existence of such superpositions is obviously inconsistent with our macroscopic experience; the macroscopic objects are localized. This common objection has been basically answered by Adler (2002). The crux of the matter lies in the influences of environment. The collisions and especially the accretions of environmental particles will quickly increase the energy uncertainty of the entangled state of the whole system including the object and environmental particles, and thus the initial superposition will soon collapse to one of the localized branches according to our model. Accordingly, the macroscopic objects can always be localized due to the environmental influences. Note that the energy uncertainty here denotes the sum of the absolute energy uncertainty of each subsystem in the entangled state as defined in our model[86].

As a typical example, we consider a dust particle of radius $a \approx 10^{-5}$cm and mass $m \approx 10^{-7}$g. It is well known that localized states of macroscopic objects spread very slowly under the free Schrödinger evolution. For instance, for a Gaussian wave packet with initial (mean square) width Δ, the wave packet will spread so that the width doubles in a time $t = 2m\Delta^2/h$. This means that the double time is almost infinite for a macroscopic object. If the dust particle had no interactions with environment and its initial state is a Gaussian wave packet with width $\Delta \approx 10^{-5}$cm, the doubling time would be about the age of the universe. However, if the dust particle is in interaction with environment, the situation turns out to be very different. Although the different components that couple to the environment will be individually incredibly localised, collectively they can have a spread that is many orders of magnitude larger. In other words, the state of the dust particle and the environment could be a superposition of zillions of very well localised terms, each with slightly different positions, and which are collectively spread over a macroscopic distance (Bacciagaluppi 2008). According to Joos and Zeh (1985), the spread in an environment full of thermal radiation only is proportional to mass times the cube of time for large times, namely $(\Delta x)^2 \approx \Lambda m\tau^3$, where Λ is the localization rate depending on the environment, defined by the evolution equation of density matrix $\rho_t(x, x') = \rho_0(x, x')e^{-\Lambda t(x-x')^2}$. For example, if the above dust particle interacts with thermal radiation at T = 300K, the localization rate is $\Lambda = 10^{12}$, and the overall spread of its state is of the order of 10m after a second (Joos and Zeh 1985). If the dust particle interacts with air molecules, *e.g.* floating in the air, the spread of its state will be much faster.

Let’s see whether the energy-conserved collapse in our model can prevent the

above spreading of the wave packet. Suppose the dust particle is in a superposition of two identical localized states that are separated by 10^{-5}cm in space. The particle floats in the air, and its average velocity is about zero. At standard temperature and pressure, one nitrogen molecule accretes in the dust particle, which area is 10^{-10}cm^2, during a time interval of 10^{-14}s in average (Adler 2002). Since the mass of the dust particle is much larger than the mass of a nitrogen molecule, the velocity change of the particle is negligible when compared with the velocity change of the nitrogen molecules during the process of accretion. Then the kinetic energy difference between an accreted molecule and a freely moving molecule is about $\Delta E = {}^3kT \approx 10^{-2}\text{eV}$. When one nitrogen molecule accretes in one localized branch of the dust particle (the molecule is freely moving in the other localized branch), it will increase the energy uncertainty of the total entangled state by $\Delta E \approx 10^{-2}\text{eV}$. Then after a time interval of 10^{-4}s, the number of accreted nitrogen molecules is about 10^{10}, and the total energy uncertainty is about 10^{8}eV . According to Eq. (4.13) in our collapse model, the corresponding collapse time is about 10^{-4}s. Since the two localized states in the superposition have the same energy spectra, the collapse also conserves energy.

In the energy-conserved collapse model, the collapse states are energy eigenstates, and in particular, they are nonlocal momentum eigenstates for free quantum systems. Thus it is indeed counterintuitive that the energy-conserved collapse can make the states of macroscopic objects local. As shown above, this is due to the constant influences of environmental particles. When the spreading of the state of a macroscopic object becomes larger, its interaction with environmental particles will introduce larger energy difference between its different local branches, and this will then collapse the spreading state again into a more localized state[87]. As a result, the states of macroscopic objects in an environment will never reach the collapse states, namely momentum eigenstates, though they do continuously undergo the energy-conserved collapse. To sum up, there are two opposite processes for a macroscopic object constantly interacting with environmental particles. One is the spreading process due to the linear Schrödinger evolution, and the other is the localization process due to the energy-conserved collapse evolution. The interactions with environmental particles not only make the spreading more rapidly but also make the localization more frequently. In the end these two processes will reach an approximate equilibrium. The state of a macroscopic object will be a wave packet narrow in both position and momentum, and this narrow wave packet will follow approximately Newtonian trajectories (if the external potential is uniform

enough along the width of the packet) by Ehrenfest's theorem (See Bacciagaluppi 2008 for a similar analysis in the context of decoherence)[88]. In some sense, the emergence of the classical world around us is "conspired" by environmental particles according to the energy-conserved collapse model.

Ultimately, the energy-conserved collapse model should be able to account for our definite conscious experience. According to recent neuroscience literature, the appearance of a (definite) conscious perception in human brains involves a large number of neurons changing their states from resting state (resting potential) to firing state (action potential). In each neuron, the main difference of these two states lies in the motion of 10^6 Na^+s passing through the neuron membrane. Since the membrane potential is in the order of 10^{-2}V, the energy difference between firing state and resting state is $\Delta E \approx 10^4$eV. According to the energy-conserved collapse model, the collapse time of a quantum superposition of these two states of a neuron is

$$\tau_c \approx hE_P/(\Delta E)^2 \approx (\ 2.8\text{MeV}/0.01\text{MeV}\ )^2 \approx 10^5\text{s}, \quad (4.20)$$

where the Planck energy $E_P \approx 10^{19}$GeV . When considering the number of neurons that can form a definite conscious perception is usually in the order of 10^7, the collapse time of the quantum superposition of two different conscious perceptions will be

$$\tau_c \approx hE_P/(\Delta E)^2 \approx (\ 2.8\text{MeV}/100\text{GeV}\ )^2 \approx 10^{-9}\text{s}, \quad (4.21)$$

Since the normal conscious time of a human being is in the order of several hundred milliseconds, the collapse time is much shorter than the normal conscious time. Therefore, our conscious perceptions are always definite according to the energy-conserved collapse model.

4.5 Critical comments on other dynamical collapse models

In this section, we will give a critical analysis of other dynamical collapse models. These models can be sorted into two categories. The first one may be called spontaneous collapse models, in which the dynamical collapse of the wave function is assumed to happen even for an isolated system. They include the gravity-induced wavefunction collapse model (Di´osi 1989; Penrose 1996), the GRW model (Ghirardi, Rimini and Weber 1986)[89] *etc.* The second category may be called interaction-induced collapse models, which assume that the dynamical collapse of the wave function of a given system results from its particular interaction with a noise field. One typical example is the CSL model (Pearle 1989; Ghirardi, Pearle and Rimini 1990)[90]. In the following, we will primarily analyze Penrose's gravity-induced wavefunction collapse model and the CSL model, which are generally regarded as two of the most promising

models of wavefunction collapse.

4.5.1 Penrose's gravity-induced wavefunction collapse model

It seems very natural to guess that the collapse of the wave function is induced by gravity. The reasons include: (1) gravity is the only universal force being present in all physical interactions; (2) gravitational effects grow with the size of the objects concerned, and it is in the context of macroscopic objects that linear superpositions may be violated. The gravity-induced collapse conjecture can be traced back to Feynman (1995)[91]. In his Lectures on Gravitation, he considers the philosophical problems in quantizing macroscopic objects and contemplates on a possible breakdown of quantum theory. He said, "I would like to suggest that it is possible that quantum mechanics fails at large distances and for large objects, it is not inconsistent with what we do know. If this failure of quantum mechanics is connected with gravity, we might speculatively expect this to happen for masses such that $GM^2/\hbar c = 1$, of M near 10^{-5} grams."

Penrose (1996) further proposed a concrete gravity-induced collapse argument. The argument is based on a profound and fundamental conflict between the general covariance principle of general relativity and the superposition principle of quantum mechanics. The conflict can be clearly seen by considering the superposition state of a static mass distribution in two different locations, say position A and position B. On the one hand, according to quantum mechanics, the valid definition of such a superposition requires the existence of a definite spacetime background, in which position A and position B can be distinguished. On the other hand, according to general relativity, the spacetime geometry, including the distinguishability of position A and position B, cannot be predetermined, and must be dynamically determined by the position superposition state. Since the different position states in the superposition determine different spacetime geometries, the spacetime geometry determined by the whole superposition state is indefinite, and as a result, the superposition and its evolution cannot be consistently defined. In particular, the definition of the time-translation operator for the superposed spacetime geometries involves an inherent ill-definedness, and this leads to an essential uncertainty in the energy of the superposed state. Then by analogy Penrose argued that this superposition, like an unstable particle in usual quantum mechanics, is also unstable, and it will decay or collapse into one of the two states in the superposition after a finite lifetime. Furthermore, Penrose suggested that the essential energy uncertainty in the Newtonian limit is proportional to the gravitational self-energy E_Δ of the difference between the two mass distributions, and the collapse time, analogous to the half-life of an unstable particle, is

$T \approx h/E_{\Delta}$. (4.22)

This criterion is very close to that put forward by Di´ osi (1989) earlier, and it is usually called the Di´osi-Penrose criterion. Later, Penrose (1998) further suggested that the collapse states are the stationary solutions of the Schrödinger-Newton equation.

Let's now analyze Penrose's argument. The crux of the matter is whether the conflict between quantum mechanics and general relativity requires that a quantum superposition of two spacetime geometries must collapse after a finite time. We will argue in the following that the answer is negative. First of all, although it is widely acknowledged that there exists a fundamental conflict between the general covariance principle of general relativity and the superposition principle of quantum mechanics, it is still a controversial issue what the exact nature of the conflict is and how to solve it. For example, it is possible that the conflict may be solved by reformulating quantum mechanics in a way that does not rely on a definite spacetime background (see, *e.g.* Rovelli 2011).

Next, Penrose's argument seems too weak to establish a necessary connection between the conflict and wavefunction collapse. Even though there is an essential uncertainty in the energy of the superposition of different spacetime geometries, this kind of energy uncertainty is different in nature from the energy uncertainty of unstable particles or unstable states in usual quantum mechanics (Gao 2010). The former results from the ill-definedness of the time-translation operator for the superposed spacetime geometries (though its nature seems still unclear), while the latter exists in a definite spacetime background, and there is a well-defined time-translation operator for the unstable states. Moreover, the decay of these unstable states is a natural result of the linear Schrödinger evolution, and the process is not random but deterministic. By contrast, the hypothetical spontaneous decay or collapse of the superposed spacetime geometries is nonlinear and random. In addition, the decay of an unstable state (e.g. excited state of an atom) is actually not spontaneous but caused by the background field constantly interacting with it. In some extreme situations, the state may not decay at all when in a very special background field with bandgap (Yablonovitch 1987). In short, there exists no convincing analogy between a superposition of different spacetime geometries and an unstable state in usual quantum mechanics. Accordingly, one cannot argue for the decay or collapse of the superposition of different spacetime geometries by this analogy. Although an unstable state in quantum mechanics may decay after a very short time, this does not imply that a superposition of different spacetime geometries should also

decay - and, again, sometimes an unstable state does not decay at all under special circumstances. To sum up, Penrose's argument by analogy only has a very limited force, and especially, it is not strong enough to establish a necessary connection between the conflict between quantum mechanics and general relativity and wavefunction collapse.

Thirdly, it can be further argued that the conflict does not necessarily lead to the wavefunction collapse. The key is to realize that the conflict also needs to be solved before the wavefunction collapse finishes, and when the conflict has been solved, the wavefunction collapse will lose its basis relating to the conflict. As argued by Penrose, the quantum superposition of different spacetime geometries and its evolution are both ill-defined due to the fundamental conflict between the general covariance principle of general relativity and the superposition principle of quantum mechanics. The ill-definedness seems to require that the superposition must collapse into one of the definite spacetime geometries, which has no problem of ill-definedness. However, the wavefunction collapse seems too late to save the superposition from the "suffering" of the ill-definedness during the collapse. In the final analysis, the conflict or the problem of ill-definedness needs to be solved before defining a quantum superposition of different spacetime geometries and its evolution. In particular, the possible collapse evolution of the superposition also needs to be consistently defined, which again indicates that the wavefunction collapse does not solve the problem of ill-definedness. On the other hand, once the problem of ill-definedness is solved and a consistent description obtained (however this is still an unsolved issue in quantum gravity), the wavefunction collapse will completely lose its connection with the problem[92]. Therefore, contrary to Penrose's expectation, it seems that the conflict between quantum mechanics and general relativity does not entail the existence of wavefunction collapse.

Even though Penrose's gravity-induced collapse argument is debatable, the wavefunction collapse may still exist due to other reasons, and thus Penrose's concrete suggestions for the collapse time formula and collapse states also need to be further examined as some aspects of a phenomenological model. First of all, let's analyze Penrose's collapse time formula Eq. (4.22), according to which the collapse time of a superposition of two mass distributions is inversely proportional to the gravitational self-energy of the difference between the two mass distributions. As we have argued above, the analogy between such a superposition and an unstable state in quantum mechanics does not exist, and gravity does not necessarily induce wavefunction collapse either. Thus this collapse time formula, which is based on a similar application of Heisenberg's

uncertainty principle to unstable states, will lose its original physical basis. In particular, the appearance of the gravitational self-energy term in the formula is in want of a reasonable explanation. In fact, it has already been shown that this gravitational self-energy term does not represent the ill-definedness of time-translation operator (or the fuzziness of the identification between two spacetimes) in the strictly Newtonian regime (Christian 2001). In this regime, the time-translation operator can be well defined, but the gravitational self-energy term is not zero. In addition, as Di´osi (2007) pointed out, the microscopic formulation of the collapse time formula is unclear and still has some problems (e.g. the cut-off difficulty).

Next, let's examine Penrose's suggestion for the collapse states. According to Penrose (1998), the collapse states are the stationary solutions of the Schrödinger-Newton equation, namely Eq. (2.31) given in Chapter 2. The equation describes the gravitational self-interaction of a single quantum system, in which the mass density $m|\psi(x, t)|^2$ is the source of the classical gravitational potential. As we have argued in Chapter 2, although a quantum system has mass density that is measurable by protective measurement, the density is not real but effective, and it is formed by the ergodic motion of a localized particle with the total mass of the system. Therefore, there does not exist a gravitational self-interaction of the mass density. This conclusion can also be reached by another somewhat different argument. Since charge always accompanies mass for a charged particle such as an electron[93], the existence of the gravitational self-interaction, though which is too weak to be excluded by present experiments, may further entail the existence of a remarkable electrostatic self-interaction of the particle[94], which already contradicts experiments as we have shown in Chapter 2. This analysis poses a serious objection to the Schrödinger-Newton equation and Penrose's suggestion for the collapse states[95].

Lastly, we briefly discuss another two problems of Penrose's collapse scheme. The first one is the origin of the randomness of collapse results. Penrose did not consider this issue in his collapse scheme. If the collapse is indeed spontaneous as implied by his gravity-induced collapse argument, then the randomness cannot result from any external influences such as an external noise field, and it can only come from the studied quantum system and its wave function. The second problem is energy non-conservation. Although Penrose did not give a concrete model of wavefunction collapse, his collapse scheme requires the collapse of superpositions of different positions, while this kind of space collapse inevitably violates energy conservation[96]. Since the gravitational

energy of a quantum system is much smaller than the energy of the system, Penrose's collapse scheme still violates energy conservation even if the gravitational field is counted[97]. As we have noted earlier, for an isolated system only the collapse states are energy eigenstates can energy conserve (at the ensemble level) during the collapse. If the principle of conservation of energy is indeed universal as widely thought, then the spontaneous collapse models that violate energy conservation will have been excluded. By contrast, although the interaction-induced collapse models such as the CSL model also violate energy conservation in their present formulations, there is still hope that when counting the energy of external noise field the total energy may be conserved in these models (Pearle 2000; Bassi, Ippoliti and Vacchini 2005). Let's turn to the CSL model now.

4.5.2 The CSL model

In the CSL model, the collapse of the wave function of a quantum system is assumed to be caused by its interaction with a classical scalar field, w(x, t). The collapse states are the eigenstates of the smeared mass density operator, and the mechanism leading to the suppression of the superpositions of macroscopically different states is fundamentally governed by the integral of the squared differences of the mass densities associated to the superposed states. It may be expected that the introduction of the noise field can help to solve the problems plagued by the spontaneous collapse models, *e.g.* the problems of energy non-conservation and the origin of randomness *etc.* However, one must first answer what field the noise field is and especially why it can collapse the wave functions of all quantum systems. The validity of the CSL model strongly depends on the existence of this hypothetical noise field. In this subsection, we will mainly analyze this important legitimization problem of the CSL model[98]. Whatever the nature of the noise field w(x, t) is, it cannot be quantum in the usual sense since its coupling to a quantum system is not a standard coupling between two quantum systems. The coupling is anti-Hermitian (Bassi 2007), and the equation of the resulting dynamical collapse is not the standard Schrödinger equation with a stochastic potential either. According to our current understandings, the gravitational field is the only universal field that might be not quantized, though this possibility seems extremely small in the view of most researchers. Therefore, it seems natural to identify this noise field with the gravitational field. In fact, it has been argued that in the CSL model the w-field energy density must have a gravitational interaction with ordinary matter (Pearle and Squires 1996; Pearle 2009). The argument of Pearle and Squires (1996) can be summarized as follows[99].

There are two equations which characterize the CSL model. The first equation is a modified Schrödinger equation, which expresses the influence of an arbitrary field w(x, t) on the quantum system. The second equation is a probability rule which gives the probability that nature actually chooses a particular w(x, t). This probability rule can also be interpreted as expressing the influence of the quantum system on the field. As a result, w(x, t) can be written as follows:

$w(x, t) = w_0(x, t) + < A(x, t) >$, (4.23)

where A(x, t) is the mass density operator smeared over the GRW scale a, < A(x, t) > is its quantum expectation value, and $w_0(x, t)$ is a Gaussian randomly fluctuating field with zero drift, temporally white noise in character and with a particular spatial correlation function. Then the scalar field w(x, t) that causes collapse can be interpreted as the gravitational curvature scalar with two sources, the expectation value of the smeared mass density operator and an independent white noise fluctuating source. This indicates that the CSL model is based on the semiclassical gravity, and the smeared mass density is the source of the gravitational potential. Note that the reality of the field w(x, t) requires that the smeared mass density of a quantum system is real[58].

According to our previous analysis, however, a quantum system does not have a real mass density distribution in space, no matter it is smeared or not. Moreover, although the approach of semiclassical gravity may be consistent in the context of dynamical collapse models (Pearle and Squires 1996; Ghirardi 2008), it may have been excluded as implied by the analysis. Besides, as we have pointed out in Section 2, protective measurement shows that a quantum system has an effective mass density proportional to the modulus square of its wave function. Thus the assumed existence of the smeared mass density in the CSL model, even if it is effective, also contradicts protective measurement. Note that it is crucial that the mass density be smeared over the GRW scale a in the CSL model; without such a smearing the energy excitation of particles undergoing collapse would be beyond experimental constraints (Pearle and Squires 1996). In conclusion, it seems that the noise field introduced in the CSL model cannot have a gravitational origin required by the model, and this may raise strong doubts about the reality of the field.

[58] Note that Ghirardi, Grassi and Benatti (1995) and Ghirardi (1997) already explicitly proposed the so-called mass density ontology in the context of dynamical collapse theories. According to Ghirardi (2008), "what the theory is about, what is real ‘out there’ at a given space point x, is just a field, *i.e.* a variable m(x, t) given by the expectation value of the mass density operator M(x) at x obtained by multiplying the mass of any kind of particle times the number

density operator for the considered type of particle and summing over all possible types of particles.

On the other hand, even though the approach of semiclassical gravity is viable and the noise field in the CSL model can be the gravitational field, one still need to answer why the gravitational field has the very ability to collapse the wave functions of all quantum systems as required by the model. It is worth noting that the randomly fluctuating field in the model, $w_0(x, t)$, is not the gravitational field of the studied quantum system but the background gravitational field. Thus Penrose's gravity-induced wavefunction collapse argument, even if valid, does not apply to the CSL model, which is essentially an interaction induced model of wavefunction collapse. The fluctuations of the background gravitational field can readily lead to the decoherence of the wave function of a quantum system, but it seems that they have no ability to cause the collapse of the wave function.

Lastly, let's briefly discuss another two problems of the CSL model. The first one is the well-known problem of energy non-conservation. The collapse in the model narrows the wave function in position space, thereby producing an increase of energy[100]. A possible solution is that the conservation laws may be satisfied when the contributions of the noise field w(x, t) to the conserved quantities are taken into account. It has been shown that the total mean energy can be conserved (Pearle 2004), and the energy increase can also be made finite when further revising the coupling between the noise field and the studied quantum system (Bassi, Ippoliti and Vacchini 2005). But a complete solution has not been found yet, and it is still unknown whether such a solution indeed exists. The second problem is to make a relativistic quantum field theory which describes collapse (Pearle 2009). Notwithstanding a good deal of effort, a satisfactory theory has not been obtained at present (see Bedingham 2011 for a recent attempt). The main difficulty is that the hypothetical interaction responsible for collapse will produce too many particles out of the vacuum, amounting to infinite energy per sec per volume, in the relativistic extension of these interaction-induced collapse models. Note that the spontaneous collapse models without collapse interaction (e.g. the energy-conserved collapse model) don't face this difficulty. We will discuss the problem of compatibility between wavefunction collapse and the principle of relativity in the next Chapter.

Chapter 5
On the Unification of Quantum Mechanics and Special Relativity

We have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory ... It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal.

—John Bell

In this chapter, we will briefly analyze random discontinuous motion of particles and its collapse evolution in the relativistic domain[101]. It is first shown that the Lorentz transformation seriously distorts the picture of random discontinuous motion of particles, and the distortion results from the relativity of simultaneity. We then argue that absolute simultaneity is not only possible in the relativistic domain, but also necessitated by the existence of random discontinuous motion of particles and its collapse evolution. This leads to the existence of a preferred Lorentz frame when combined with the requirement of the constancy of speed of light. It is further shown that the collapse dynamics may provide a way to detect the frame according to the energy-conserved collapse model. If quantum mechanics indeed describes random discontinuous motion of particles as protective measurement suggests, then this analysis may be helpful for solving the problem of the incompatibility of quantum mechanics with special relativity[102].

5.1 Distorted picture of motion

Let's first see how the picture of random discontinuous motion is distorted by the Lorentz transformation that leads to the relativity of simultaneity.

5.1.1 Single particle picture

For the random discontinuous motion of a particle, the particle has a propensity to be in any possible position at a given instant, and the probability density of the particle appearing in each position x at a given instant t is determined by the modulus square of its wave function, namely $\rho(x, t) = |\psi(x, t)|^2$. The physical picture of the motion of the particle is as follows. At a discrete instant the particle randomly stays in a position, and at the next instant it will still stay there or randomly appear in another position, which is probably not in the neighborhood of the previous position. In this way, during a time interval much larger than the duration of one instant, the particle will move discontinuously throughout the whole space with position probability density $\rho(x, t)$. Since the distance between the locations occupied by the particle at two neighboring instants may be very large, this jumping process is obviously nonlocal. In the

nonrelativistic domain where time is absolute, the nonlocal jumping process is the same in every inertial frame. But in the relativistic domain, the jumping process will look different in different inertial frames due to the Lorentz transformation. Let's give a concrete analysis.

Suppose a particle is in position x_1 at instant t_1 and in position x_2 at instant t_2 in an inertial frame S. In another inertial frame S with velocity v relative to S, the Lorentz transformation leads to:

$$t_1' = \frac{t_1 - x_1 v/c^2}{\sqrt{1 - v^2/c^2}}, \qquad (5.1)$$

$$t_2' = \frac{t_2 - x_2 v/c^2}{\sqrt{1 - v^2/c^2}}, \qquad (5.2)$$

$$x_1' = \frac{x_1 - v t_1}{\sqrt{1 - v^2/c^2}}, \qquad (5.3)$$

$$x_2' = \frac{x_2 - v t_2}{\sqrt{1 - v^2/c^2}}. \qquad (5.4)$$

Since the jumping process of the particle is nonlocal, the two events (t_1, x_1) and (t_2, x_2) may readily satisfy the spacelike separation condition $|x_2 - x_1| > c|t_2 - t_1|$. Then we can always select a possible velocity ($v < c$) that leads to $t_2 = t_1$:

$$v = \frac{t_2 - t_1}{x_2 - x_1} c^2. \qquad (5.5)$$

But obviously the two positions of the particle in frame S , namely x_1 and x_2, are not equal. This means that in frame S the particle will be in two different positions x_1 and x_2 at the same time at instant t_1. In other words, it seems that there are two identical particles at instant t_1 in frame S . Note that the velocity of S relative to S may be much smaller than the speed of light, and thus the appearance of the two-particle picture is irrelevant to the high-energy processes described by relativistic quantum field theory, *e.g.* the creation and annihilation of particles.

The above result shows that for any pair of events in frame S that satisfies the spacelike separation condition, there always exists an inertial frame in which the two-particle picture will appear. Since the jumping process of the particle in

frame S is essentially random, it can be expected that the two-particle picture will appear in the infinitely many inertial frames in an even way. Then during an arbitrary finite time interval, in each inertial frame the measure of the instants at which there are two particles in appearance, which is equal to the finite time interval divided by the total number of the frames that is infinite, will be zero. Moreover, there may also exist situations where the particle is at arbitrarily many positions at the same time at an instant in an inertial frame, though the measure of these situations is also zero. Certainly, at nearly all instants which measure is one, the particle is still in one position at an instant in all inertial frames. Therefore, the many-particle appearance of the random discontinuous motion of a particle cannot be measured in principle.

However, for the random discontinuous motion of a particle, in any inertial frame different from S, the Lorentz transformation will inevitably make the time order of the random stays of the particle in S reversal and disorder, as the discontinuous motion of the particle is nonlocal and most neighboring random stays are spacelike separated events. In other words, the time order is not Lorentz invariant. Moreover, the set of the instants at which the time order of the random stays of the particle is reversed has finite measure, which may be close to one. As we will see below, this reversal and disorder of time order will lead to more distorted pictures for quantum entanglement and wavefunction collapse.

5.1.2 Picture of quantum entanglement

Now let's analyze the motion of two particles in quantum entanglement. For the random discontinuous motion of two particles in an entangled state, the two particles have a joint propensity to be in any two possible positions, and the probability density of the two particles appearing in each position pair x_1 and x_2 at a given instant t is determined by the modulus square of their wave function at the instant, namely $\rho(x_1, x_2, t) = |\psi(x_1, x_2, t)|^2$.

Suppose two particles are in an entangled state $\psi_u\phi_u + \psi_d\phi_d$, where ψ_u and ψ_d are two spatially separated states of particle 1, ϕ_u and ϕ_d are two spatially separated states of particle 2, and particle 1 and particle 2 are also separated in space. The physical picture of this entangled state is as follows. Particles 1 and 2 are randomly in the state $\psi_u\phi_u$ or $\psi_d\phi_d$ at an instant, and then they will still stay in this state or jump to the other state at the next instant. During a very short time interval, the two particles will discontinuously move throughout the states $\psi_u\phi_u$ and $\psi_d\phi_d$ with the same probability 1/2. In this way, the two particles form an inseparable whole, and they jump in a precisely simultaneous way. At an arbitrary instant, if particle 1 is in the state ψ_u or ψ_d, then particle 2 must be in

the state ϕ_u or ϕ_d, and vice versa. Moreover, when particle 1 jumps from ψ_u to ψ_d or from ψ_d to ψ_u, particle 2 must simultaneously jump from ϕ_u to ϕ_d or from ϕ_d to ϕ_u, and vice versa. Note that this kind of random synchronicity between the motion of particle 1 and the motion of particle 2 is irrelevant to the distance between them, and it can only be explained by the existence of joint propensity of the two particles as a whole.

The above picture of quantum entanglement is assumed to exist in one inertial frame. It can be expected that when observed in another inertial frame, this perfect picture will be distorted in a similar way as for the single particle case. Let's give a concrete analysis below. Suppose in an inertial frame S, at instant t_a particle 1 is at position x_{1a} and in state ψ_u and particle 2 at position x_{2a} and in state ϕ_u, and at instant t_b particle 1 is at position x_{1b} and in state ψ_d and particle 2 at position x_{2b} and in state ϕ_d. Then according to the Lorentz transformation, in another inertial frame S with velocity v relative to S, where v satisfies:

$$v' = \frac{t_a - t_b}{x_{1a} - x_{2b}} c^2, \qquad (5.6)$$

the instant at which particle 1 is at position x_{1a} and in state ψ_u is the same as the instant at which particle 2 is at position x_{2b} and in state ϕ_d, namely

$$t'_{1a} = t'_{2b} = \frac{1}{\sqrt{1 - v'^2/c^2}} \cdot \frac{x_{1a} t_b - x_{2b} t_a}{x_{1a} - x_{2b}}. \qquad (5.7)$$

This means that in S there exists an instant at which particle 1 is in state ψ_u but particle 2 is in state ϕ_d. Similarly, in another inertial frame S with velocity v relative to S, there also exists an instant t at which particle 1 is in state ψ_d but particle 2 is in state ϕ_u, where v and t satisfy the following relations:

$$v'' = \frac{t_a - t_b}{x_{2a} - x_{1b}} c^2, \qquad (5.8)$$

$$t'' = \frac{1}{\sqrt{1 - v''^2/c^2}} \cdot \frac{x_{2a} t_b - x_{1b} t_a}{x_{2a} - x_{1b}}. \qquad (5.9)$$

Note that since the two particles are well separated in space, the above two velocities can readily satisfy the restricting conditions $v < c$ and $v < c$ when the time interval $|t_a - t_b|$ is very short.

In fact, since the two particles in the above entangled state are separated in space and their motion is essentially random, in any inertial frame different from S, the instantaneous correlation between the motion of the two particles in S can only keep half the time, and the correlation will be reversed for another half of time, during which the two particles will be in state $\psi_u\phi_d$ or $\psi_d\phi_u$ at each instant. For a general entangled state $\sqrt{a}\psi_u\phi_u + \sqrt{b}\psi_d\phi_d$, the proportion of correlation-reversed time will be 2ab, and the proportion of correlation-kept time will be $a^2 + b^2$. Moreover, the instants at which the original correlation is kept or reversed are discontinuous and random. This means that the synchronicity between the jumpings of the two particles is destroyed too.

To sum up, the above analysis indicates that the instantaneous correlation and synchronicity between the motion of two entangled particles in one inertial frame is destroyed in other frames due to the Lorentz transformation[103]. As we will see below, however, this distorted picture of quantum entanglement cannot be measured either.

5.1.3 Picture of wavefunction collapse

We have shown that the picture of the instantaneous motion of particles is distorted by the Lorentz transformation due to the nonlocality and randomness of motion. In the following, we will further show that the nonlocal and random collapse evolution of the state of motion (defined during an infinitesimal time interval) will be influenced by the Lorentz transformation more seriously.

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Consider a particle being in a superposition of two Gaussian wavepackets $\frac{1}{\sqrt{2}}\psi_1+\frac{1}{\sqrt{2}}\psi_2$ in an inertial frame S. The centers of the two wavepackets are located in x_1 and x_2 ($x_1 < x_2$), respectively, and the width of each wavepacket is much smaller than the distance between them. After being measured, this superposition state will randomly collapse to ψ_1 or ψ_2 with the same probability 1/2. Suppose the collapse happens at different locations at the same time in frame S. This means that when the superposition state collapses to the branch ψ_1 near position x_1, the other branch ψ_2 near position x_2 will disappear simultaneously. The simultaneity of wavefunction collapse ensures

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stants. As noted above, however, these instants are discontinuous and random, and thus the correlation and synchronicity can hardly be identified.

that the sum of the probabilities of the particle being in all branches is 1 at every instant.

According to the picture of random discontinuous motion of particles, the above collapse process can be described as follows. Before the collapse of the superposition state $\frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$, the particle jumps between the two branches ψ_1 and ψ_2 or the two regions near x_1 and x_2 in a discontinuous and random way[4]. At each instant, the particle is either in a position near x_1 or in a position near x_2, and its probability of being in each region is the same 1/2. This means that at every instant there is always one particle, which spends half the time near x_1 and half the time near x_2. After the superposition state collapses to one of its branches, e.g. ψ_1, the particle only jumps in the region near x_1 in a discontinuous and random way, and its probability of being in this region is 1. This means that at every instant there is always one particle in a position inside the region.

Now let's see the picture of the above collapse process in another inertial frame S' with velocity v relative to S. Suppose the superposition state $\frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$ collapses to the branch ψ_1 near position x_1 at instant t in frame

[4]In other words, each branch exists in a set of discontinuous and random instants, which measure is 1/2, and the two instant sets constitute the whole continuous time flow.

S. This process contains two events happening simultaneously in two spatially separated regions. One event is the disappearance of the branch $\frac{1}{\sqrt{2}}\psi_2$ near position x_2 at instant t, and the other is the change from $\frac{1}{\sqrt{2}}\psi_1$ to ψ_1 happening near position x_1 at instant t[5]. According to the Lorentz transformation, the happening times of these two events in S' are

$$t'_1 = \frac{t - x_1 v/c^2}{\sqrt{1 - v^2/c^2}}, \tag{5.10}$$

$$t'_2 = \frac{t - x_2 v/c^2}{\sqrt{1 - v^2/c^2}}. \tag{5.11}$$

It can be seen that $x_1 < x_2$ leads to $t'_1 > t'_2$. Then during the period between t'_1 and t'_2, the branch $\frac{1}{\sqrt{2}}\psi'_2$ near position x'_2 already disappeared, but the branch $\frac{1}{\sqrt{2}}\psi'_1$ near position x'_1 has not changed to ψ'_1. This means that at any instant between t'_1 and t'_2, there is only a non-normalized state $\frac{1}{\sqrt{2}}\psi'_1$. According to the picture of random discontinuous motion of particles, for a particle in the state $\frac{1}{\sqrt{2}}\psi'_1$, the probability of the particle being in the branch ψ'_1 is

[5]Strictly speaking, since the collapse time is always finite, these events happen not at a precise instant but during a very short time, which may be much shorter than the time of light propagating between x_1 and x_2.

1/2, and the particle is in the branch ψ'_1 or in the region near x_1 only at some discontinuous and random instants, which total measure is 1/2. At other instants, which measure is also 1/2, the particle does not exist anywhere. In other words, at each instant the particle either exists in a position near x_1 or disappears in the whole space with the same probability, 1/2. This result indicates that in the inertial frame S', the particle only exists half the time during the period between t'_1 and t'_2. By contrast, the particle always exists in certain position in space at any time in the inertial frame S.

Similarly, if the superposition state $\frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{2}}\psi_2$ collapses to the branch ψ_2 near position x_2 at instant t in frame S, then in frame S', during the period between t'_1 and t'_2, the branch $\frac{1}{\sqrt{2}}\psi'_2$ near position x'_2 already turns to ψ'_2, while the branch $\frac{1}{\sqrt{2}}\psi'_1$ near position x'_1 has not disappeared and is still there. Therefore, there is only a non-nomalized state $\frac{1}{\sqrt{2}}\psi'_1 + \psi'_2$ at any instant between t'_1 and t'_2. According to the picture of random discontinuous motion of particles, this means that during the period between t'_1 and t'_2, there are more than one particle in S': the first particle is in the branch ψ'_2 all the time, and the second identical particle exists half the time in the branch ψ'_1 (and it exists nowhere in space for another half of time).

However, although the state of the particle in S' is not

nomalized, the total probability of *detecting* the particle in the whole space is still 1, not 1/2 or 3/2, in the frame[6]. In other words, although the collapse process is seriously distorted in S', the distortion cannot be measured. The reason is that in S' the collapse resulting from measurement happens at different instants in different locations[7], and the superposition of the branches in these locations and at these instants are always nomalized. In the following, we will give a more detailed explanation.

As noted above, in frame S' the collapse first happens at t'_2 for the branch $\frac{1}{\sqrt{2}}\psi'_2$ near position x'_2, and then happens at t'_1 for the branch $\frac{1}{\sqrt{2}}\psi'_1$ near position x'_1 after a delay. If we measure the branch $\frac{1}{\sqrt{2}}\psi'_2$, then the resulting collapse will influence the other branch $\frac{1}{\sqrt{2}}\psi'_1$ only after a delay of $\Delta t' = \frac{|x_1-x_2|v/c^2}{\sqrt{1-v^2/c^2}}$, while if we measure the branch $\frac{1}{\sqrt{2}}\psi'_1$, then the resulting collapse will influence the other branch $\frac{1}{\sqrt{2}}\psi'_2$ in advance by the same time interval $\Delta t'$, and the influence is backward in time. Now suppose we

[6]This does not contradict the usual Born rule, which only applies to the situations where collapse happens simultaneously at different locations in space.

[7]Concretely speaking, the time order of the collapses happening at different locations in S' is connected with that in S by the Lorentz transformation.

make a measurement on the branch $\frac{1}{\sqrt{2}}\psi'_2$ near position x'_2 and detect the particle there (i.e. the collapse state is ψ'_2). Then before the other branch $\frac{1}{\sqrt{2}}\psi'_1$ disappears, which happens after a delay of $\Delta t'$, we can make a second measurement on this branch near position x'_1. It seems that the probability of detecting the particle there is not zero but 1/2, and thus the total probability of finding the particle in the whole space is larger than one and it is possible that we can detect two particles. However, this is not the case. Although the second measurement on the branch $\frac{1}{\sqrt{2}}\psi'_1$ near position x'_1 is made later than the first measurement, it is the second measurement that collapses the superposition state $\frac{1}{\sqrt{2}}\psi'_1 + \frac{1}{\sqrt{2}}\psi'_2$ to ψ'_2 near position x'_2; the local branch $\frac{1}{\sqrt{2}}\psi'_1$ near position x'_1 disappears immediately after the measurement, while the influence of the resulting collapse on the other branch $\frac{1}{\sqrt{2}}\psi'_2$ near position x'_2 is backward in time and happens before the first measurement on this branch. Therefore, the second measurement near position x'_1 can only obtain a null result, and why the first measurement detects the particle near position x'_2 is because the superposition state already collapses to ψ'_2 near position x'_2 before the measurement due to the second measurement.

By a similar analysis, we can also demonstrate that the measurements on an entangled state of two particles, e.g. $\psi_u\phi_u + \psi_d\phi_d$, can only obtain correlated results in every inertial frame. If a measurement on particle 1 obtains the result u or d, indicating the state of the particle collapses to the state ψ_u or ψ_d after the measurement, then a second measurement on particle 2 can only obtain the result u or d, indicating the state of particle 2 collapses to the state ϕ_u or ϕ_d after the measurement. Accordingly, although the instantaneous correlation and synchronicity between the motion of two entangled particles is destroyed in all but one inertial frame, the distorted picture of quantum entanglement cannot be measured.

5.2 On the absoluteness of simultaneity

The above analysis clearly demonstrates the apparent conflict between the random discontinuous motion of particles and the Lorentz transformation in

special relativity. The crux of the matter lies in the relativity of simultaneity. If simultaneity is relative as required by the Lorentz transformation, then the picture of random discontinuous motion of particles will be seriously distorted except in one preferred frame, though the distortion is unobservable in principle. Only when simultaneity is absolute, can the picture of random discontinuous motion of particles be kept perfect in every inertial frame. In the following, we will show that absolute simultaneity is not only possible, but also necessitated by the existence of random discontinuous motion of particles and its collapse evolution.

Although the relativity of simultaneity has been often regarded as one of the essential concepts of special relativity, it is not necessitated by experimental facts but a result of the choice of standard synchrony (see, *e.g.* Reichenbach 1958; Gr¨unbaum 1973)[104]. As Einstein (1905) already pointed out in his first paper on special relativity, whether or not two spatially separated events are simultaneous depends on the adoption of a convention in the framework of special relativity. In particular, the choice of standard synchrony, which is based on the constancy of one-way speed of light and results in the relativity of simultaneity, is only a convenient convention. Strictly speaking, the speed constant c in special relativity is two-way speed, not one-way speed, and as a result, the general spacetime transformation required by the constancy of two-way speed of light is not the Lorentz transformation but the Edwards-Winnie transformation (Edwards 1963; Winnie 1970):

$$x' = \eta(x - vt), \qquad (5.12)$$

$$t' = \eta[1 + \beta(k + k')]t + \eta[\beta(k^2 - 1) + k - k']x/c, \quad (5.13)$$

where x, t and x', t' are the coordinates of inertial frames S and S', respectively, v is the velocity of S' relative to S, c is the invariant two-way speed of light, $\beta = v/c$, and $\eta = 1/\sqrt{(1 + \beta k)^2 - \beta^2}$. k and k' represent the directionality of one-way speed of light in S and S', respectively, and they satisfy $-1 \leqslant k, k' \leqslant 1$. Concretely speaking, the one-way speeds of light along x and $-x$ directions in S are $c_x = \frac{c}{1-k}$ and $c_{-x} = \frac{c}{1+k}$, respectively, and the one-way speeds of light along x' and $-x'$ directions in S' are $c_{x'} = \frac{c}{1-k'}$ and $c_{-x'} = \frac{c}{1+k'}$, respectively.

If adopting the standard synchrony convention, namely assuming the one-way speed of light is isotropic and constant in every inertial frame, then $k, k' = 0$ and the Edwards Winnie transformation will reduce to the Lorentz transformation, which leads to the relativity of simultaneity. Alternatively, one can also adopt the nonstandard synchrony convention that makes simultaneity absolute. In order to do this, one may first synchronize the clocks at different locations in an arbitrary inertial frame by Einstein's standard synchrony, that is, one assumes the one-way speed of light is isotropic in this frame, and then let the clocks in other frames directly regulated by the clocks in this frame

when they coincide in space. The corresponding spacetime transformation can be derived as follows. Let S be the preferred Lorentz frame in which the one-way speed of light is isotropic, namely let $k = 0$. Then we get

$$k' = \beta(k^2 - 1) + k = -\beta. \tag{5.14}$$

Besides, since the synchrony convention leads to the absoluteness of simultaneity, we also have in the Edwards-Winnie transformation:

$$\beta(k^2 - 1) + k - k' = 0. \tag{5.15}$$

Thus the spacetime transformation that restores absolute simultaneity is:

$$x' = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot (x - vt), \tag{5.16}$$

$$t' = \sqrt{1 - v^2/c^2} \cdot t. \tag{5.17}$$

where x, t are the coordinates of the preferred Lorentz frame, x', t' are the coordinates of another inertial frame, and v is the velocity of this frame relative to the preferred frame. In this frame, the one-way speed of light along x' and $-x'$ direction are $c_{x'} = \frac{c^2}{c-v}$ and $c_{-x'} = \frac{c^2}{c+v}$, respectively.

The above analysis demonstrates the possibility of keeping simultaneity absolute within the framework of special relativity. One can adopts the standard synchrony that leads to the relativity of simultaneity, and one can also adopts the nonstandard synchrony that restores the absoluteness of simultaneity. This is permitted because there is no causal connection between two spacelike separated events in special relativity. However, if there is a causal influence connecting two distinct events, then the claim that they are not simultaneous will have a nonconventional basis (Reichenbach 1958, 123-135; Gr¨basis (Reichenbach 1958, 123-135; Gr¨ 368). In particular, if there is an arbitrarily fast causal influence connecting two spacelike separated events, then these two events will be simultaneous. In the following, we will show that random discontinuous motion and its collapse evolution just provide a nonconventional basis for the

absoluteness of simultaneity.

Consider a particle being in a superposition of two well separated spatial branches. According to the picture of random discontinuous motion, the particle jumps between these two branches in a random and discontinuous way. At an instant the particle is in one branch, and at the next instant it may be in the other spatially-separated branch. The disappearance of the particle in the first branch can be regarded as one event, and the appearance of the particle in the second branch can be regarded as another event. Obviously there is an instantaneous causal connection between these two spacelike separated events; if the particle did not disappear in the first branch, it could not appear in the second branch. Therefore, these two events should be regarded as simultaneous. Note that this conclusion is irrelevant to whether the two events and their causal connection are observable. Furthermore, simultaneity cannot be relative but be absolute, otherwise these two distinct events will be not simultaneous in all but one inertial frame[105].

Let's further consider the collapse evolution of random discontinuous motion during a measurement. It can be seen that the measurement on one branch of the superposition has a causal influence on the other branch (as well as on the measured branch) via the collapse process, and this nonlocal influence is irrelevant to the distance between the two branches. Accordingly, the time order of the measurement and the collapse of the superposition happening in the two separated regions cannot be conventional but must be unique. Since the collapse time can be arbitrarily short, the measurement and the collapse of the superposition can be regarded as simultaneous. Moreover, the collapses of the superposition in the two regions, which are spacelike separated events, are also simultaneous[106]. The simultaneity is irrelevant to the selection of inertial frames, which again means that simultaneity is absolute.

Certainly, the collapse of an individual superposition cannot be measured within the framework of the existing quantum mechanics. However, on the one hand, the above conclusion is irrelevant to whether the collapse events can be measured or not, and on the other hand, the collapse of an individual superposition may be observable when the quantum dynamics is deterministic nonlinear (Gisin 1990), *e.g.* when the measuring device is replaced with a conscious observer (Squires 1992; Gao 2004).

5.3 Collapse dynamics and preferred Lorentz frame

The random discontinuous motion of particles and its collapse evolution requires that simultaneity is absolute. If the collapse of the wave function happens simultaneously at different locations in space in every inertial frame, then the

one-way speed of light will be not isotropic in all but one inertial frame. In other words, if the absolute simultaneity is restored, then the non-invariance of the one-way speed of light will single out a preferred Lorentz frame, in which the one-way speed of light is isotropic[107]. The detectability of this frame seems to depend on the measurability of individual collapse. Once the collapse of an individual wave function can be measured, the clocks at different locations in space can be synchronized with the help of the instantaneous wavefunction collapse in every inertial frame, and the preferred Lorentz frame can then be determined by measuring the one-way speed of light, which is isotropic in the frame.

However, even if the collapse of an individual wave function cannot be measured, the preferred Lorentz frame may also be determined by measuring the (average) collapse time of the wave functions of identical systems in an ensemble according to our energy-conserved collapse model[108]. The reason is that the law of collapse dynamics in our model, like the time order of the collapses in different positions, is not relativistically invariant either. Let's give a more detailed analysis below.

According to the energy-conserved collapse model, the (average) collapse time formula for an energy superposition state, denoted by Eq. (4.13), can be rewritten as

$$\tau_c \approx \frac{\hbar^2}{t_P(\Delta E)^2}, \qquad (5.18)$$

where t_P is the Planck time, ΔE is the energy uncertainty of the state. It can be seen that this collapse time formula is not relativistically invariant, and thus there exists a preferred Lorentz frame according to the collapse model. We assume the formula is valid in the preferred Lorentz frame, denoted by S_0, in the relativistic domain[109]. Then in another inertial frame the collapse time will depend on the velocity of the frame relative to S_0. According to the Lorentz transformation[110], in an inertial frame S with velocity v relative to the frame S_0 we have:

$$\tau_c' = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \tau_c, \qquad (5.19)$$

$$t_P' = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot t_P, \qquad (5.20)$$

$$\Delta E' \approx \frac{1 - v/c}{\sqrt{1 - v^2/c^2}} \cdot \Delta E. \qquad (5.21)$$

Here we only consider the situation where the particle has very high energy, namely E ≈ pc, and thus Eq. (5.21) holds. Besides, we assume the Planck time t_P is the minimum time in the preferred Lorentz frame, and in another frame the minimum time (i.e. the duration of a discrete instant) is connected with the Planck time t_P by the time dilation formula required by special relativity. Then by inputting these equations into Eq. (5.22), we can obtain the relativistic collapse time formula for an arbitrary experimental frame with velocity v relative to the frame S_0:

$$\tau_c \approx (1 + v/c)^{-2} \frac{\hbar^2}{t_P (\Delta E)^2}. \qquad (5.22)$$

This formula contains a term relating to the velocity of the experimental frame relative to the preferred Lorentz frame. It can be expected that this velocity-dependent term originates from the relativistic equation of collapse dynamics. Indeed, the equation of collapse dynamics, which nonrelativistic form is denoted by Eq. (4.15), does contain a velocity term in order to be relativistic invariant[111]:

$$P_i(t + t_P) = P_i(t) + f(v) \frac{\Delta E}{E_P} [\delta_{E_s E_i} - P_i(t)]. \qquad (5.23)$$

where f(v) ≈ 1 + v/c when E ≈ pc, and v is the velocity of the experimental frame relative to the preferred Lorentz frame. From this equation we can also derive the above relativistic collapse time formula.

Therefore, according to our energy-conserved collapse model, the collapse time of a given wave function will differ in different inertial frames[112]. For example, when considering the maximum difference of the revolution speed of the Earth with respect to the Sun is Δv ≈ 60km/s, the maximum difference of the

collapse time measured in different times (e.g. spring and fall respectively) on the Earth will be $\Delta\tau_c \approx 4 \times 10^{-4}\tau_c$. As a result, the collapse dynamics will single out a preferred Lorentz frame in which the collapse time of a given wave function is longest, and the frame can also be determined by comparing the collapse time of a given wave function in different frames. It may be expected that this preferred Lorentz frame is the CMB-frame in which the cosmic background radiation is isotropic, and the one-way speed of light is also isotropic in this frame[113].

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Notes

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[1] Note that the proponents of protective measurement did not give an explanation of the charge density. According to them, this type of measurement implies that the wave function of a single quantum system is ontological, i.e., that it is a real physical wave (Aharonov, Anandan and Vaidman 1993).

[2] The Hilbert space is a compete vector space with scalar product. The state vector in a Hilbert space contains proper vectors normalizable to unity as well as improper vectors normalizable only to the Dirac delta functions. The exact nature of the Hilbert space depends on the system; for example, the state space for position and momentum states is the space of square-integrable functions.

[3] For a continuous property such as position, $P(x) = |\langle x|\psi\rangle|^2$ is the probability density at x, and P(x)dx is the probability of obtaining measurement result between x and x + dx.

[4] By contrast, in a conventional impulse measurement the initial position of the pointer is well localized around zero, and thus the conjugate momentum P has a very large uncertainty which leads to a very large uncertainty in the Hamiltonian of the measurement (2.1).

[5] In order to read the position of pointer, an impulse position measurement needs to be made after the weak measurement, and this will lead to a partial collapse of the measured wave function. For a helpful discussion see Miller (2010).

[6] For a review of earlier objections to the validity and meaning of protective measurements and the answers to them see Aharonov, Anandan and Vaidman (1996), Dass and Qureshi (1999) and Vaidman (2009).

[7] The change in the total Hamiltonian during these processes is smaller than PA=T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

[8] As in conventional impulse measurements, there is also an issue of retrieving the information about the center of the wave packet of the pointer (Dass and Qureshi 1999). One strategy is to consider adiabatic coupling of a single quantum system to an ensemble of measuring devices and make impulse position measurements on the ensemble of devices to determine the pointer

position. For example, the ensemble of devices could be a beam of atoms interacting adiabatically with the spin of the system. Although such an ensemble approach inevitably carries with it uncertainty in the knowledge of the position of the device, the pointer position, which is the average of the result of these position measurements, can be determined with arbitrary accuracy. Another approach is to make repeated measurements (e.g. weak quantum nondemolition measurements) on the single measuring device. This issue does not affect the principle of protective measurements. In particular, retrieving the information about the position of the pointer only depends on the Born rule and is irrelevant to whether the wave function collapses or not during a conventional impulse measurement.

[9] Anandan (1993) and Dickson (1995) gave some initial analyses of the implications of this result for quantum realism. According to Anandan (1993), protective measurement refutes an argument of Einstein in favor of the ensemble interpretation of quantum mechanics. Dickson's (1995) analysis was more philosophical. He argued that protective measurement provides a reply to scientific empiricism about quantum mechanics, but it can neither refute that position nor confirm scientific realism, and the aim of his argument is to place realism and empiricism on an even score in regards to quantum mechanics.

[10] This point was discussed and stressed by Dass and Qureshi (1999).

[11] Quoted in Moore (1994), p.148.

[12] This important point was also stressed by Aharonov, Anandan and Vaidman (1993).

[13] It is worth stressing that the added protection procedure depends on the measured state, and different states need different protection procedures in general.

[14] Whether the charge is real or effective will be investigated in the next section.

[15] Any physical measurement is necessarily based on some interaction between the measured system and the measuring system. One basic form of interaction is the electrostatic interaction between two electric charges as in our example, and the existence of this interaction during a measurement, which is indicated by the deviation of the trajectory of the charged measuring system such as an electron, means that the measured system also has the charge responsible for the interaction. If one denies this point, then it seems that one cannot obtain any information about the measured system by the measurement. Note that the arguments against the naive realism about operators and the eigenvalue realism

in the quantum context are irrelevant here (Daumer et al 1997; Valentini 2010).

[16] Strictly speaking, the mass density is $m|\psi(x)|^2+\psi^*H\psi/c^2$ in the nonrelativistic domain, but the second term is very small compared with the first term and can be omitted.

[17] Alternatively one might simply insist that even if the mass and charge distributions of a charged quantum system are real, they still have no gravitational and electrostatic self-interactions. One may further argue that this is because the system is of quantum nature (for a classical charged system these self-interactions do exist), and the superposition principle of quantum mechanics prohibits the existence of these self-interactions. However, this view is untenable. On the one hand, even if the superposition principle may be used to explain the absence of self-interactions for a charged quantum system, it does not tell us whether the mass and charge distributions of the quantum system are real or not. One cannot simply stipulate that these distributions are real, because they may be effective and formed by the ergodic motion of a localized particle with the total mass and charge of the system, and especially, the effective mass and charge distributions have no gravitational and electrostatic self-interactions, which is consistent with the superposition principle. Thus this view begs the question and leaves the origin of mass and charge density as a mystery. On the other hand, the assumption that real mass and charge distributions have gravitational and electrostatic self-interactions has been confirmed not only in the classical domain but also in the quantum domain for many-body systems. For example, two charged quantum systems such as two electrons, which represent two real charge distributions, do have electrostatic interactions. Thus it is reasonable to expect that this assumption also holds true for individual quantum systems. Our following analysis will show that this assumption, when combining with the superposition principle, can help to reveal the physical origin of the mass and charge density of a quantum system.

[18] It has been argued that the existence of a gravitational self-interaction term in the Schrödinger-Newton equation does not have a consistent Born rule interpretation (Adler 2007). The reason is that the probability of simultaneously finding a particle in different positions is zero.

[19] By contrast, the potential strength of the gravitational self-interaction for a free electron is about 4×10^{-89}.

[20] Note that even if there are only two masses and charges in space at a given instant, the densities formed by their motion also have gravitational and electrostatic interactions. Therefore, the mass and charge density of a quantum

system can only be formed by the ergodic motion of one localized particle with the total mass and charge of the system.

[21] At a particular time the charge density is either zero (if the electron is not there) or singular (if the electron is inside the infinitesimally small region including the space point in question).

[22] Note that in Nelson's stochastic mechanics, the electron, which is assumed to undergo a Brownian motion, moves only within a region bounded by the nodes (Nelson 1966). This ensures that the theory can be equivalent to quantum mechanics in a limited sense. Obviously this sort of motion is not ergodic and cannot generate the required charge density distribution. Likewise, some variants of stochastic mechanics (Bell 1986b; Vink 1993; Barrett, Leifer and Tumulka 2005), which assume that the motion of particles is discrete random jump but still nonergodic, cannot be consistent with protective measurement either. In addition, it has been argued that stochastic mechanics is inconsistent with quantum mechanics (Glabert, H¨anggi and Talkner 1979; Wallstrom 1994). Glabert, H¨anggi and Talkner (1979) argued that the Schrödinger equation is not equivalent to a Markovian process, and the various correlation functions used in quantum mechanics do not have the properties of the correlations of a classical stochastic process. Wallstrom (1994) further showed that one must add by hand a quantization condition, as in the old quantum theory, in order to recover the Schrödinger equation, and thus the Schrödinger equation and the Madelung hydrodynamic equations are not equivalent. In fact, Nelson (2005) also showed that there is an empirical difference between the predictions of quantum mechanics and his stochastic mechanics when considering quantum entanglement and nonlocality. For example, for two widely-separated but entangled harmonic oscillators, the two theories predict totally different statistics; stochastic mechanics predicts that measurements of the position of the first one at time T (oscillation period) and the position of the second one at time 0 do not interfere with each other, while quantum mechanics predicts that there exists a strong correlation between them.

[23] The word "cause" used here only denotes a certain instantaneous condition determining the change of position, which may appear in the laws of motion. Our analysis is irrelevant to whether the condition has causal power or not.

[24] This deterministic instantaneous condition has been often called intrinsic velocity (Tooley 1988).

[25] In discrete space and time, the motion will be a discrete jump across space along a fixed direction at each time unit, and thus it will become continuous

motion with constant velocity in the continuous limit.

[26] In the next chapter, we will derive this equation of free motion from fundamental physical principles. This will make the argument given here more complete. Besides, the derivation itself may also provide an argument for discontinuous motion that does not resort to direct experience, as the equation of free motion does not permit the persisting existence of the local state of continuous motion. For details see Section 3.4.

[27] However, the analysis cannot tell us the precise size and possible structure of an electron.

[28] Recall that a function x(t) is continuous if and only if for every t and every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that whenever a point t_0 has distance less than δ to t, the point $x(t_0)$ has distance less than ε to x(t).

[29] However, there is an exception. When the probability density function is a special δ-function such as $\delta(x - x(t))$, where x(t) is a continuous function of t, the motion of the particle is deterministic and continuous. In addition, even for a general probability density function it is still possible that the random position series forms a continuous trajectory, though the happening probability is zero.

[30] The existence of this limit relies on the continuity of the evolution of the probabilistic instantaneous condition or propensity of a particle that determines its random discontinuous motion.

[31] Note that the relation between $j(x, t)$ and $\psi(x, t)$ depends on the concrete evolution under an external potential such as electromagnetic vector potential. By contrast, the relation $\rho(x, t) = |\psi(x, t)|^2$ holds true universally, independent of the concrete evolution.

[32] For a many-particle system in an entangled state, the propensity property is possessed by the whole system. See Chapter 5 for a detailed analysis of the physical picture of quantum entanglement.

[33] Note that for random discontinuous motion the properties (e.g. position) of a quantum system in a superposed state are indeterminate in the sense of usual hidden variables, though they do have definite values at each instant. This makes the theorems that restrict hidden variables such as the Kochen-Specker theorem (Kochen and Specker 1967) irrelevant.

[34] But if the spin state of a particle is entangled with its spatial state and the branches of the entangled state are well separated in space, the particle in different branches will have different spin, and it will also undergo random

discontinuous motion between these different spin states. This is the situation that usually happens during a spin measurement.

[35] This is an important presupposition in our derivation. We will consider the possible case of nonlinearity of H in the next section.

[36] Different from the derivation given below, most existing “derivations” of the energy-momentum relation are based on the somewhat complex analysis of an elastic collision process. Moreover, they resort to either some Newtonian limit (e.g. $p = mv$) or some less fundamental relation (e.g. $p = Eu/c^2$) or even some mathematical intuition (e.g. four-vectors) (see Sonego and Pin 2005 and references therein).

[37] Alternatively we can obtain the transformations of momentum and energy by directly requiring the relativistic invariance of momentum eigenstate $e^{i(px-Et)}$, which leads to the relation $px - Et = p_0x_0 - E_0t_0$. Note that any superposition of momentum eigenstates is also invariant under the coordinates transformation. The reason is that it is a scalar that describes the physical state of a quantum system, and when observed in different reference frames it should be the same (except an absolute phase). This also means that the state evolution equation must be relativistically invariant in nature. However, if the relativistic invariant equation is replaced by the nonrelativistic approximation such as the Schrödinger equation, the state will no longer satisfy the relativistic invariance.

[38] According to the analysis here, it seems that we can in principle avoid talking about mass in modern physics from a more fundamental point of view (cf. Okun 2009).

[39] This also means that the Klein-Gordon equation can be derived in the relativistic domain when assuming that the wave function is a number function.

[40] In order to derive the complete Schrödinger equation in a fundamental way, we need a fundamental theory of interactions such as quantum field theory.

[41] In order to ensure that the nonlinear evolution is unitary and thus the total probability is conserved in time, the Hamiltonian $H(\psi)$ must be also Hermitian. Besides, this property is also required to ensure that the energy eigenvalues (which satisfy the equation $H(\psi)\psi(x) = E\psi(x)$) are real. When the Hamiltonian $H(\psi)$ is Hermitian, the Ehrenfest theorem still holds true.

[42] This will violate the relativistic invariance of momentum eigenstates.

[43] For more discussions about the arguments for linear quantum dynamics see Holman (2006) and references therein.

[44] For example, the collapse to a position eigenstate during an ideal position measurement is obviously unphysical, as the position eigenstate has infinite average energy.

[45] As we have shown in Chapter 2, there are at least three levels of implications. First, protective measurement can measure the mass and charge density of a quantum system, which is proportional to the modulus square of the wave function of the system. This indicates that the mass and charge of a quantum system are attributes of its wave function. Next, when assuming that real mass and charge distributions have gravitational and electrostatic interactions, which has been confirmed not only in the classical domain but also in the quantum domain for many-body systems, it can be shown that the mass and charge density of a quantum system is formed by the time average of the ergodic motion of a localized particle with the total mass and charge of the system. This indicates that the wave function is a description of the ergodic motion of particles. Lastly, it can be further argued that the ergodic motion is not continuous but discontinuous and random. This leads to our suggested interpretation of the wave function, according to which the wave function in quantum mechanics is a description of random discontinuous motion of particles. Most of our critical analysis of the existing solutions to the measurement problem only depends on the first two implications.

[46] In other words, the principle of protective measurement and its implications hold true in any formulation of quantum mechanics that keeps the linear Schrödinger evolution of the wave function (for microscopic systems) and the Born rule, such as the de Broglie-Bohm theory and the many-worlds interpretation. Thus it is legitimate to use them to examine these alternatives to quantum mechanics. Note that the possible existence of very slow collapse of the wave function for microscopic systems does not influence the principle of protective measurement and its implications.

[47] It has been argued that the wave function living on configuration space can hardly be considered as a real physical entity due to its multi-dimensionality (see, *e.g.* Monton 2002, 2006 and references therein). However, it seems that this common objection is not conclusive, and one can still insist on the reality of the wave function living on configuration space by resorting to some metaphysical arguments. For example, a general strategy is to show how a many-dimensional world can appear three-dimensional to its inhabitants, and then argue on that basis that a wavefunction ontology is adequate to explain our experience (Albert 1996; Lewis 2004). As we argued earlier, the existence of the effective mass and

charge density of a quantum system, which is measurable by protective measurement, poses a more serious objection to the wavefunction ontology; even for a single quantum system the wave function cannot be taken as a field-like entity in three-dimensional space either. Moreover, the reason is not metaphysical but physical, i.e., the field-like interpretation contradicts both quantum mechanics and experimental observations.

[48] Certainly, as Albert (1992) noted, no theory can have exactly the same empirical content as quantum mechanics does, as the latter (in the absence of any satisfactory account of wavefunction collapse) does not have any exact empirical content.

[49] For a critical analysis of this minimal formal interpretation see Belousek (2003).

[50] Note that for spin 1/2 particles there is also a spin-dependent term (Holland and Philippidis 2003).

[51] That a Bohmian particle has no properties other than its position is possible only when the mass and charge terms disappear in the guiding equation, but the resulting theory will contradict quantum mechanics and experiments.

[52] This conclusion relies on the common-sense assumption that an electron indeed has the charge of an electron (and the mass of an electron). A possible way to avoid the inconsistency is to assume that an electron has twice the charge of an electron: one for its wave function and the other for its Bohmian particle. In this case, since what protective measurement measures is the mass and charge distributions relating to the wave function, not the masses and charges of the Bohmian particles, the above inconsistency can be avoided. However, this theory seems too clumsy and unnatural to be true. Moreover, it will introduce more problems. For one, there is a dilemma concerning the electromagnetic interaction between the wave function and the Bohmian particle of an electron. If they do have usual electromagnetic interaction, then the theory will be inconsistent with quantum mechanics and experiments. If they have no electromagnetic interaction, then this will add more problems. For instance, the manifestation of the charge of a Bohmian particle will be much stranger; it is not only passive but also selective. One needs to explain why the charged Bohmian particle of an electron responds not to the magnetic vector potential generated by the wave function of this electron, but to the magnetic vector potential generated by the wave function of another electron. As we will see later, a more serious objection concerns the guiding responsibility of the wave function.

[53] This is also admitted by most interpretations of the de Broglie-Bohm theory.

[54] This conclusion may not hold true if the guiding equation is not exactly the same as the above, *e.g.* the guiding equation contains an additional stochastic damping term (Valentini and Westman 2005). Although such revised theories make predictions different from quantum mechanics, they may be consistent with existing experiments.

[55] The reality of the trajectories of the Bohmian particles has been questioned based on analysis of weak measurement and protective measurement (Englert, Scully, Sussmann and Walther 1992; Aharonov and Vaidman 1996; Aharonov, Englert and Scully 1999; Aharonov, Erez and Scully 2004). However, these objections may be answered by noticing what protective measurement measures is the wave function, not the Bohmian particles (see also Drezet 2006). For a comprehensive answer to these objections see Hiley, Callaghan and Maroney (2000).

[56] Note that protective measurement in general requires that the measured wave function is known beforehand so that an appropriate protective interaction can be added. But this requirement does not influence our argument, as the superposed wave function of a measuring device can be prepared in a known form before the protective measurement.

[57] This objection does not apply to the de Broglie-Bohm theory, according to which the wave function of a measuring device does not collapse either, but it exists only in one world.

[58] Note that this objection is more serious than the problem of approximate decoherence for the many-worlds interpretation. The interference between the nonorthogonal components of a quantum state can not be detected for individual states, but only be detected for an ensemble of identical states. Moreover, the presence of tiny interference terms in a (local) quantum state does not imply that all components of the state wholly exist in one world.

[59] According to these theories, the physical state always evolves in a deterministic way and may be superposed and indefinite, while the mental state is always definite but evolves randomly. In some sense, these theories can be regarded as hidden-variable theories like the de Broglie-Bohm theory. The latter assumes the definite positions of Bohmian particles provide observers with definite measurement records, while the former assumes the definite mental states of the observers, though which are non-physical parameters, directly provide observers with definite measurement records.

[60] As in the many-worlds case, the random discontinuous motion does not result in the emergence of many minds either. Since the brain state of a quantum

observer is definite and only assumes one brain state in the superposition at a given instant, even if there are many minds with different conscious perceptions at the instant, these perceptions are irrelevant to those corresponding to the brain states in the superposition except the present brain state. Thus such a theory of many minds cannot be consistent with the above experience. In addition, although the quantum observer has a dispositional property relating to his superposition state, the property is still a definite property of the unique observer and thus cannot correspond to the existence of many minds.

[61] Moreover, it can be expected that the conscious perception of the observer is none of the perceptions corresponding to the brain states in the superposition because these states have the same status.

[62] This is distinct from the case of continuous motion. For the latter, the position of a particle at each instant is completely determined by the deterministic instantaneous condition at the instant, and thus the position of the particle has no influence on the deterministic instantaneous condition.

[63] In fact, since the random stays of a particle as one part of its instantaneous state are completely random, the complete evolution equation of the instantaneous state of the particle is only about the evolution of the wave function. Therefore, the random stays of the particle can only manifest themselves in the complete equation of motion by their stochastic influences on the evolution of the wave function.

[64] In other words, the wave function of a particle determines its random discontinuous motion, while the motion also influences the evolution of the wave function reciprocally.

[65] Unfortunately, this banal case does not exist. Due to the uncertainty relation between position and momentum in quantum mechanics, there are always infinitely many different instantaneous states (with definite position and momentum) where a particle can stay at any time.

[66] Our analysis of a concrete model in the next section will show that under some reasonable assumptions the accumulated influence of the random stays during a finite time interval is still zero when time is continuous.

[67] This means that the minimum duration of the random stay of a particle in a definite position or momentum or energy is always a discrete instant. It can be imagined that the duration of the random stay of a particle in an eigenvalue of energy is a discrete instant, but the duration of its random stay in each position is still zero as in continuous space and time. In this case, however, the position probability distribution of the particle cannot be uniquely determined during its

stay in the definite energy for a general state of motion where the energy branches are not wholly separated in space. Moreover, it seems that only the duration of the random stay of a particle in the eigenvalue of every property is the same can the (objective) probability distributions of all these properties be consistent with those given by the modulus square of the wave function in quantum mechanics.

[68] Note that the existing arguments, which are based on some sort of combination of quantum theory and general relativity (see, *e.g.* Garay 1995 for a review), do not imply but only suggest that space and time are discrete. Moreover, the meanings and realization of discrete spacetime are also different in the existing models of quantum gravity.

[69] It has been conjectured that a fundamental theory of physics may be formulated by three natural constants: the Planck time (t_P), the Planck length (l_P) and the Planck constant (h), and all other physical constants are expressed by the combinations of them (Gao 2006b). For example, the speed of light is $c = l_P/t_P$, and the Einstein gravitational constant is $\kappa = 8\pi l_P t_P/h$. In this sense, the quantum motion in discrete space and time, represented by the above three constants, is more fundamental than the phenomena described by the special and general theory of relativity, represented by the speed of light and the gravitational constant, respectively. However, even if this conjecture turns out to be right, it is still a big challenge how to work out the details (see Gao 2011c for an initial attempt).

[70] For the superpositions of degenerate energy eigenstates of a many-particle system, a further collapse rule is needed. We will discuss this issue later on.

[71] As we will see later, the conservation of energy may also hold true at the individual level for the collapse evolution of some special wave functions.

[72] If the phase of an energy eigenstate also changes with time, then the probability distribution of energy eigenvalues will in general be changed for each identical system in the ensemble, and as a result, energy will be not conserved even at the ensemble level.

[73] Note that the reversible Schrödinger evolution conserves the information even for individual isolated systems.

[74] Strictly speaking, the description “branch” should be replaced by “instantaneous state”, *e.g.* the branch $|E_i>$ should be replaced by the instantaneous state with energy E_i. Yet the branch description may be more succinct and visual, and we will use it in the following discussions.

[75] The density matrix describes the ensemble of states which arise from all possible random stays.

[76] Note that the common RMS (mean square root) uncertainty also satisfies the swap symmetry. Thus it still needs to be studied what the exact form of k is.

[77] This collapse time formula indicates that there is no wavefunction collapse in continuous time because $t_P \to 0$ leads to $\tau_c \to \infty$. One premise of this conclusion is that the influence of each random stay is proportional to the duration of stay.

[78] In continuous space and time, a position eigenstate has infinite average energy and cannot be physically real. But in discrete space and time, position eigenstates will be the states whose spatial dimension is about the Planck length, and they may exist.

[79] Note that most collapse states in an ensemble of identical systems keep the shape of the wavepacket almost precisely.

[80] There might exist a subtle connection here. It seems that the energy-conserved wavefunction collapse in discrete time requires a finite event horizon to ensure the energy eigenvalues of any system are discrete. On the other hand, it seems that discrete spacetime permits the existence of dark energy as quantum fluctuations of spacetime to lead to acceleration and finite event horizon (Gao 2005). In any case, the existence of a cosmological constant also leads to the existence of a finite event horizon.

[81] A potentially more promising case is provided by certain long-lived nuclear isomers, which have large energy gaps from their ground states (see Adler 2002 and references therein). For example, the metastable isomer of ^{180}Ta, the only nuclear isomer to exist naturally on earth, has a half-life of more than 10^{15} years and an energy gap of 75keV from the ground state. According to Eq. (4.13), a coherent superposition of the ground state and metastable isomer of ^{180}Ta will spontaneously collapse to either the isomeric state or the ground state, with a collapse time of order 20 minutes. It will be a promising way to test our collapse model by examining the maintenance of coherence of such a superposition.

[82] Since the uncertainty of the total energy of the whole entangled system is still zero, the energy-driven collapse models will predict that no wavefunction collapse happens and no definite measurement result appears for the above measurement process, which contradicts experimental observations (Pearle 2004).

[83] In more general measurement situations, the measured particle (e.g. electron) is not annihilated by the detector. However, in each local branch of the entangled state of the whole system, the particle also interacts with a single atom of the detector by an ionizing process, and its total energy is also wholly transferred to the atom and the ejecting electrons.

[84] We take the widely-used Geiger counter as another illustration of the amplification process during measurement. A Geiger counter is an instrument used to detect particles such as α particles, β particles and γ rays *etc.* It consists of a glass envelope containing a low-pressure gas (usually a mixture of methane with argon and neon) and two electrodes, with a cylindrical mesh being the cathode and a fine-wire anode running through the centre of the tube. A potential difference of about 10^3V relative to the tube is maintained between the electrodes, therefore creating a strong electric field near the wire. The counter works on the mechanism of gas multiplication. Ionization in the gas is caused by the entry of a particle. The ions are attracted to their appropriate electrode, and they gain sufficient energy to eject electrons from the gas atoms as they pass through the gas. This further causes the atoms to ionize. Therefore, electrons are produced continuously by this process and rapid gas multiplication takes place (especially in the central electrode because of its strong electric field strength). Its effect is that more than 10^6 electrons are collected by the central electrode for every ion produced in the primary absorption process. These "electron avalanches" create electric pulses which then can be amplified electronically and counted by a meter to calculate the number of initial ionization events. In this way, a Geiger counter can detect low-energy radiation because even one ionized particle produces a full pulse on the central wire. It can be estimated that the introduced energy difference during a detection is $\Delta E \approx 10^9$eV , and the corresponding collapse time is $\tau_c \approx 10^{-5}$s according to our collapse model.

[85] In a similar way, a spherically symmetric wave function will be detected as one linear track in a cloud chamber (cf. Mott 1929).

[86] The uncertainty of the total energy of the whole system is still very small even if the influences of environment are counted. Thus no observable collapse happens for the above situation according to the energy-driven collapse models (Pearle 2004).

[87] It is interesting to note that the state of a macroscopic object can also be localized by the linear Schrödinger evolution via interactions with environment, *e.g.* by absorbing an environmental particle with certain energy uncertainty. For example, if a macroscopic object absorbs a photon (emitted from an atom) with

momentum uncertainty of $\Delta p \approx 10^{-6}$eV/c, the center-of-mass state of the object, even if being a momentum eigenstate initially, will have the same momentum uncertainty by the linear Schrödinger evolution, and thus it will become a localized wavepacket with width about 0.1m. Note that there is no vicious circle here. The energy spreading state of a microscopic particle can be generated by an external potential (e.g. an electromagnetic potential in general) via the linear Schrödinger evolution, and especially they don't necessarily depend on the localization of macroscopic objects such as measuring devices. Thus we can use the existence of these states to explain the localization of macroscopic objects.

[88] When assuming the energy uncertainty of an object is in the same order of its thermal energy fluctuation, we can estimate the rough size of its wavepacket. For instance, for a dust particle of mass $m = 10^{-7}$g, its root mean square energy fluctuation is about 10^{3}eV at room temperature T = 300K (Adler 2002), and thus the width of its wavepacket is about 10^{-10}m.

[89] The GRW model was originally referred to as QMSL (Quantum Mechanics with Spontaneous Localizations). In this model, it is assumed that each elementary constituent of any physical system is subjected, at random times, to random and spontaneous localization processes (or hittings) around appropriate positions. The random hittings happen much less frequently for a microscopic system, *e.g.* an electron undergoes a hitting, on average, every hundred million years. If these hittings are assumed to be brought about by an external system, then the GRW model should be regarded not as a spontaneous collapse model but as an interaction-induced collapse model.

[90] If the involved noise field in the CSL model is not taken as real, then the model should be regarded as a spontaneous collapse model.

[91] It is interesting to note that Feynman considered this conjecture even earlier at the 1957 Chapel Hill conference (see DeWitt and Rickles 2011, ch.22).

[92] Note that if the problem of ill-definedness cannot be solved in principle for the superpositions of very different spacetime geometries, then the wavefunction collapse may be relevant here. Concretely speaking, if the superpositions of very different spacetime geometries cannot be consistently defined in nature, then it is very likely that these superpositions cannot exist, which means that they must have collapsed into one of the definite spacetime geometries before formed from the superpositions of minutely different spacetime geometries. In this case, the large difference of the spacetime geometries in the superposition will set a upper limit for wavefunction collapse. Though the limit may be loose, it does imply the existence of wavefunction collapse. However, this possibility might be very

small, as it seems that there is always some kind of approximate sense in which two different spacetimes can be pointwise identified.

[93] However, the concomitance of mass and charge in space for a charged particle does not necessarily require that they must satisfy the same law of interaction. For example, the fact that electromagnetic fields are quantized in nature does not necessarily imply that gravitational fields must be also quantized.

[94] If there is a gravitational self-interaction but no electrostatic self-interaction for a charged particle, *e.g.* an electron, then the charge and mass of an electron will be located in different positions and have different density distributions in space, though they are described by the same wave function. Concretely speaking, the mass density of an electron is $m_e|\psi(x, t)|^2$ as in the Schrödinger-Newton equation, whereas its charge density is not $e|\psi(x, t)|^2$ but only localized in a single position (which permits no electrostatic self-interaction). This result seems very unnatural and has no experimental support either.

[95] Since the Schrödinger-Newton equation is the nonrelativistic realization of the typical model of semiclassical gravity, in which the source term in the classical Einstein equation is taken as the expectation of the energy momentum operator in the quantum state (Rosenfeld 1963), our analysis also presents a serious objection to the approach of semiclassical gravity. Although the existing arguments against the semiclassical gravity models seem so strong, they are still not conclusive (Carlip 2008; Boughn 2009). This new analysis of the Schrödinger-Newton equation may shed some new light on the solution of the issue.

[96] Di´osi (2007) explicitly pointed out that the von-Neumann Newton equation, which may be regarded as one realization of Penrose's collapse scheme, obviously violates conservation of energy. Another way to understand this conclusion is to realize that the energy-conserved wavefunction collapse cannot result from the spacetime geometry difference between the branches in a superposition as suggested by Penrose's collapse scheme. The reason is that there is no difference of spacetime geometries for two different momentum eigenstates. A momentum eigenstate does not influence its background spacetime geometry, as its energy density is zero throughout the whole space. Thus if a superposition of two momentum eigenstates does collapse into one of them, the collapse cannot result from the difference of spacetime geometries in the superposition. As a result, Penrose's gravity-induced collapse argument does not lead to the energy-conserved wavefunction collapse, and if it does lead to

some sort of wavefunction collapse, the collapse cannot conserve energy.

[97] This is contrary to Penrose's own expectation. According to Penrose (2004), "There is the advantage with the gravitational OR scheme put forward above that the energy uncertainty in E_G would appear to cover such a potential non-conservation, leading to no actual violation of energy conservation. This is a matter that needs further study, however. It would seem that there is some kind of trade-off between the apparent energy difficulties in the OR process and the decidedly nonlocal (and curiously slippery) nature of gravitational energy...".

[98] As admitted by Pearle (2009), "When, over 35 years ago, ... I had the idea of introducing a randomly fluctuating quantity to cause wave function collapse, I thought, because there are so many things in nature which fluctuate randomly, that when the theory is better developed, it would become clear what thing in nature to identify with that randomly fluctuating quantity. Perhaps ironically, this problem of legitimizing the phenomenological CSL collapse description by tying it in a natural way to established physics remains almost untouched." Related to this legitimization problem is that the two parameters which specify the model are ad hoc (Pearle 2007). These two parameters, which were originally introduced by Ghirardi, Rimini and Weber (1986), are a distance scale, $a \approx 10^5$cm, characterising the distance beyond which the collapse becomes effective, and a time scale, $\lambda^{-1} \approx 10^{16}$sec, giving the rate of collapse for a microscopic system. If wavefunction collapse is a fundamental physical process related to other fundamental processes, the parameters should be able to be written in terms of other physical constants.

[99] Pearle (2009) further argued that compatibility with general relativity requires a gravitational force exerted upon matter by the w-field. However, as Pearle (2009) admitted, no convincing connection (for example, identification of metric fluctuations, dark matter or dark energy with w(x, t)) has yet emerged, and the legitimization problem (i.e. the problem of endowing physical reality to the noise field) is still in its infancy.

[100] Note that with appropriate choice for the parameters in the CSL model, such a violation of energy conservation is very tiny and hardly detectable by present day technology.

[101] Our analysis is in the low-energy regime and does not consider the high-energy processes described by relativistic quantum field theory, *e.g.* the creation and annihilation of particles.

[102] There is no consensus among contemporary philosophers and physicists

concerning the solution to this incompatibility problem. For a comprehensive discussion of this issue see Maudlin (2002) and references therein.

[103] Certainly, in these frames there are still correlation and synchronicity between the jumpings of the two particles at different instants. As noted above, however, these instants are discontinuous and random, and thus the correlation and synchronicity can hardly be identified.

[104] For more discussions about this issue see Janis (2010) and references therein.

[105] Why does each instantaneous jump of a particle in one inertial frame last much long time in another inertial frame? The lapse of time cannot be explained in physics, and it can only result from the inappropriate synchrony of clocks at different locations in the later frame.

[106] Note that there exists no causal influence between these two events, and they both result from the measurement of the local measuring device, which is the common cause.

[107] Similarly, if the invariance of the one-way speed of light or standard synchrony is assumed as by the Lorentz transformation, then the collapse evolution of random discontinuous motion will also single out a preferred Lorentz frame, in which the collapse of the wave function happens simultaneously at different locations in space, no matter whether the frame can be actually determined. In the final analysis, the emergence of a preferred Lorentz frame is the inevitable result of the combination of the constancy of two-way speed of light and the existence of random discontinuous motion and its collapse evolution. Thus, no matter which assumption is adopted, the preferred Lorentz frame can always be defined as the inertial frame in which the one-way speed of light is isotropic and the collapse of the wave function happens simultaneously in the whole space.

[108] It has been argued that quantum nonlocality and special relativity are incompatible, and a consistent description of wavefunction collapse demands the existence of a preferred Lorentz frame (see, *e.g.* Bell 1986a; Percival 1998b). But it is widely thought that the preferred Lorentz frame cannot be measured even within the framework of dynamical collapse theories.

[109] This assumption seems reasonable, as the collapse time formula in our model already contains the speed of light c via the Planck time t_p. By contrast, the dynamical collapse theories in which the collapse time formula does not contain c are not directly applicable in the relativistic domain.

[110] Here we still use the standard synchrony for the convenience of practical realization.

[111] This seems to be an inevitable consequence of the requirement of energy conservation for wavefunction collapse.

[112] In general, we can measure the collapse time of a wave function through measuring the change of the interference between the corresponding collapse branches for an ensemble of identical systems. The main difficulty of this approach is to exclude the influence of environmental decoherence (cf. Marshall et al 2003).

[113] A further analysis is needed to determine whether this is true in theory.