CBSE 2022 Term II Examinations

(Series ABCD5/5 Code No. 65/5/1)

Class XII • MATHEMATICS (041)

Time Allowed : 2 Hours **Max. Marks : 40**

General Instructions :

1. This question paper contains three Sections - **Section A, B and C**.
2. **Each** section is compulsory.
3. **Section A** carries **6 Short Answer type questions (SA1)** of **2 marks** each.
4. **Section B** carries **4 Short Answer type questions (SA2)** of **3 marks** each.
5. **Section C** carries **4 Long Answer type questions (LA)** of **4 marks** each.
6. There is **an internal choice** in some of the questions.
7. Question 14 is a **case study based question** with **2 sub-parts** of **2 marks** each.

Section A

Question numbers 1 to 6 carry 2 marks each.

Q01. Find : $\int \frac{dx}{\sqrt{4x - x^2}}$.

Sol. $\int \frac{dx}{\sqrt{4x - x^2}} = \int \frac{dx}{\sqrt{2^2 - (x-2)^2}} = \sin^{-1}\left(\frac{x-2}{2}\right) + c$.

Q02. Find the general solution of the following differential equation :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}.$$

Sol. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\Rightarrow \frac{dy}{dx} = (e^x + x^2)e^{-y}$

$\Rightarrow \frac{dy}{e^{-y}} = (e^x + x^2)dx$

$\Rightarrow \int e^y dy = \int (e^x + x^2)dx$

$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$.

Q03. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.

Find the probability distribution of X.

Sol. Note that, $\sum P(X) = 1$

So, $P(X = x_1) + P(X = x_2) + P(X = x_3) + P(X = x_4) = 1$

$\Rightarrow \frac{1}{2}P(X=x_3)+\frac{1}{3}P(X=x_3)+P(X=x_3)+\frac{1}{5}P(X=x_3)=1$

$\Rightarrow \frac{61}{30}P(X=x_3)=1$

$\Rightarrow P(X=x_3)=\frac{30}{61}$

So, $P(X=x_1)=\frac{15}{61}$, $P(X=x_2)=\frac{10}{61}$, $P(X=x_4)=\frac{6}{61}$

Hence, the probability distribution table is :

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

Q04. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then find $|\vec{b}|$.

Sol. Here $|\vec{a}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$ and $|\vec{a}\times\vec{b}|=\sqrt{0^2+1^2+(-1)^2}=\sqrt{2}$

As $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$ so, $(\sqrt{2})^2+(1)^2=(\sqrt{3})^2|\vec{b}|^2$

$\Rightarrow 2+1=3\times|\vec{b}|^2$

$\therefore |\vec{b}|=1$ $\{\because |\vec{b}|\neq -1$.

Q05. If a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$.

Sol. $\cos 2\alpha+\cos 2\beta+\cos 2\gamma = 2\cos^2\alpha-1+2\cos^2\beta-1+2\cos^2\gamma-1$

$= 2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)-3$ $(\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$= 2\times 1-3=-1$.

Q06. (a) Events A and B are such that $P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$ and $P(\bar{A}\cup\bar{B})=\frac{1}{4}$.

Find whether the events A and B are independent or not.

OR

(b) A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour.

Sol. (a) $P(\bar{A}\cup\bar{B})=\frac{1}{4}$ $\Rightarrow P(\overline{A\cap B})=\frac{1}{4}$

$\Rightarrow 1-P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A\cap B)=\frac{3}{4}\ldots$(i)

Note that, $P(A).P(B)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}\neq P(A\cap B)$ [By (i)

Hence, A and B are not independent events.

OR

(b) Required probability $=$ P(both balls are white) $+$ P(both balls are red)

$$=\frac{1}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}=\frac{11}{20}.$$

Section B

Question numbers 7 to 10 carry 3 marks each.

Q07. Evaluate : $\int_0^{\pi/4}\frac{dx}{1+\tan x}$.

Sol. Let $I=\int_0^{\pi/4}\frac{dx}{1+\tan x}$

$$\Rightarrow I=\int_0^{\pi/4}\frac{dx}{1+\tan\left(\frac{\pi}{4}-x\right)}=\int_0^{\pi/4}\frac{dx}{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$\Rightarrow I=\int_0^{\pi/4}\frac{dx}{1+\frac{1-\tan x}{1+\tan x}}=\int_0^{\pi/4}\frac{(1+\tan x)\,dx}{1+\tan x+1-\tan x}=\int_0^{\pi/4}\frac{(1+\tan x)\,dx}{2}$$

$$\Rightarrow I=\frac{1}{2}\left[x+\log|\sec x|\right]_0^{\pi/4}$$

$$\Rightarrow I=\frac{1}{2}\left[\left(\frac{\pi}{4}+\log\left|\sec\frac{\pi}{4}\right|\right)-\left(0+\log|\sec 0|\right)\right]$$

$$\Rightarrow I=\frac{1}{2}\left[\left(\frac{\pi}{4}+\log\left|\sqrt{2}\right|\right)-\left(\log|1|\right)\right]$$

$$\therefore I=\frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\log 2\right)\text{ or, }\frac{1}{4}\left(\frac{\pi}{2}+\log 2\right).$$

Q08. (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}|=|\vec{b}|$, then prove that $(\vec{a}+2\vec{b})$ is perpendicular to $\vec{a}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then prove that $\sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$.

Sol. (a) Consider $|\vec{a}+\vec{b}|^2=|\vec{b}|^2$

$$\Rightarrow(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} = |\vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 + 2\,\vec{a}.\vec{b} = 0$$

$$\Rightarrow \vec{a}.\vec{a} + 2\,\vec{a}.\vec{b} = 0$$

$$\Rightarrow \vec{a}.(\vec{a} + 2\,\vec{b}) = 0$$

$$\therefore \vec{a} \perp (\vec{a} + 2\,\vec{b}) = 0$$

Hence, $(\vec{a} + 2\vec{b})$ is perpendicular to $\vec{a}$.

OR

(b) Consider $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}).(\vec{a} - \vec{b})$

$$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2$$

$$\Rightarrow |\vec{a} - \vec{b}|^2 = 1^2 - 2|\vec{a}||\vec{b}|\cos\theta + 1^2$$ $\left[\because \vec{a}\right.$ and $\vec{b}$ are unit vectors and θ is the angle betwen them

$$\Rightarrow |\vec{a} - \vec{b}|^2 = 2(1 - \cos\theta) = 4\sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin^2\frac{\theta}{2} = \frac{1}{4}|\vec{a} - \vec{b}|^2$$

$$\therefore \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|.$$

Q09. Find the equation of the plane passing through the line of intersection of planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 10$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and passing through the point (–2, 3, 1).

Sol. Required plane $\pi : \vec{r}.(\hat{i} + \hat{j} + \hat{k}) - 10 + \lambda\left[\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4\right] = 0$

That is, $\vec{r}.\left[(\hat{i} + \hat{j} + \hat{k}) + (2\lambda\hat{i} + 3\lambda\hat{j} - \lambda\hat{k})\right] - 10 + 4\lambda = 0 \ldots$(i)

As (i) passes through (–2, 3, 1) so, $(-2\hat{i} + 3\hat{j} + \hat{k}).\left[(\hat{i} + \hat{j} + \hat{k}) + (2\lambda\hat{i} + 3\lambda\hat{j} - \lambda\hat{k})\right] - 10 + 4\lambda = 0$

$$\Rightarrow (-2 + 3 + 1) + (-4\lambda + 9\lambda - \lambda) - 10 + 4\lambda = 0$$

$$\Rightarrow \lambda = 1$$

Replacing value of λ in (i), we get $\vec{r}.\left[(\hat{i} + \hat{j} + \hat{k}) + (2\hat{i} + 3\hat{j} - \hat{k})\right] - 10 + 4 = 0$

$$\Rightarrow \vec{r}.(3\hat{i} + 4\hat{j}) - 6 = 0.$$

Q10. (a) Find : $\int e^x.\sin 2x\,dx$.

OR

(b) Find : $\int \frac{2x}{(x^2 + 1)(x^2 + 2)}dx$.

Sol. (a) Let $I = \int e^x.\sin 2x\,dx \ldots$(i)

$$\Rightarrow I = \sin 2x \int e^x\,dx - \int\left(\frac{d}{dx}(\sin 2x)\int e^x\,dx\right)dx$$

$$\Rightarrow I = e^x \sin 2x - \int 2\cos 2x \times e^x \, dx$$

$$\Rightarrow I = e^x \sin 2x - 2\left[\cos 2x \int e^x \, dx - \int\left(\frac{d}{dx}(\cos 2x)\int e^x \, dx\right)dx\right]$$

$$\Rightarrow I = e^x \sin 2x - 2\left[e^x \cos 2x + 2\int e^x \sin 2x \, dx\right]$$

By (i), $I = e^x \sin 2x - 2\left[e^x \cos 2x + 2I\right]$

$$\Rightarrow 5\,I = e^x \sin 2x - 2e^x \cos 2x$$

$$\Rightarrow I = \frac{e^x}{5}(\sin 2x - 2\cos 2x) + c.$$

OR

(b) Put $(x^2+1) = t \Rightarrow 2x\,dx = dt$

$$\int \frac{2x}{(x^2+1)(x^2+2)}dx = \int \frac{dt}{t(t+1)} = \int\left(\frac{1}{t} - \frac{1}{(t+1)}\right)dt$$

$$\Rightarrow \qquad = \log|t| - \log|t+1| + c$$

$$\Rightarrow \qquad = \log|x^2+1| - \log|x^2+2| + c \text{ or, } \log\left|\frac{x^2+1}{x^2+2}\right| + c.$$

Section C

Question numbers 11 to 14 carry 4 marks each.

Q11. Three persons A, B and C apply for a job of manager in a private company. Chances of their selection are in the ratio 1:2:4. The probability that A, B and C can introduce changes to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A.

Sol. Let E : the change doesn't take place

Let E_1 : person A is appointed, E_2 : person B is appointed, E_3 : person C is appointed.

Clearly we have $P(E_1) = \frac{1}{7}$, $P(E_2) = \frac{2}{7}$, $P(E_3) = \frac{4}{7}$,

$P(E|E_1) = \frac{2}{10}$, $P(E|E_2) = \frac{5}{10}$, $P(E|E_3) = \frac{7}{10}$.

By Bayes' theorem, $P(E_1|E) = \dfrac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3)}$

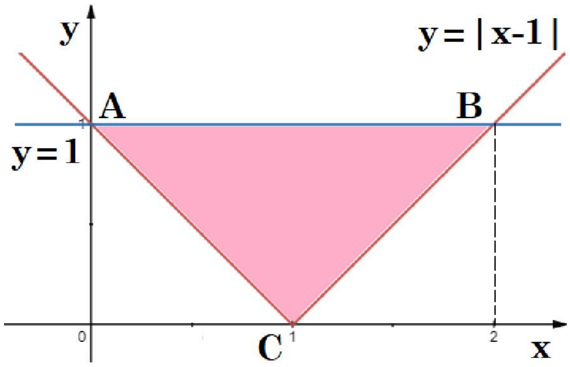
$$\Rightarrow P(E_1|E) = \frac{\frac{2}{10}\times\frac{1}{7}}{\frac{2}{10}\times\frac{1}{7} + \frac{5}{10}\times\frac{2}{7} + \frac{7}{10}\times\frac{4}{7}}$$

$$\Rightarrow P(E_1|E) = \frac{2}{2+10+28}$$

$$\therefore P(E_1|E) = \frac{2}{40} \text{ or, } \frac{1}{20}.$$

Q12. Find the area bounded by the curve $y = |x-1|$ and $y = 1$, using integration.

Sol. Consider the diagram.



Required area, is ar(ABCA).

So, $\text{ar(ABCA)} = \int_0^2 1dx - \int_0^2 |x-1| dx$

$\Rightarrow \text{ar(ABCA)} = [x]_0^2 - \left[\int_0^1 -(x-1)dx + \int_1^2 (x-1)dx \right]$

$\Rightarrow \text{ar(ABCA)} = [2-0] - \left\{ -\left[\frac{(x-1)^2}{2} \right]_0^1 + \left[\frac{(x-1)^2}{2} \right]_1^2 \right\}$

$\Rightarrow \text{ar(ABCA)} = 2 - \left\{ -\left[0 - \frac{1}{2} \right] + \left[\frac{1}{2} - 0 \right] \right\}$

$\Rightarrow \text{ar(ABCA)} = 1$ Sq. units .

Q13. (a) Solve the following differential equation :

$$(y - \sin^2 x)dx + \tan x dy = 0 .$$

OR

(b) Find the general solution of the differential equation :

$$(x^3 + y^3)dy = x^2 y dx .$$

Sol. (a) Re-writing the D.E., we get $\frac{dy}{dx} + \cot x . y = \sin x \cos x$

On comparing with $\frac{dy}{dx} + P(x).y = Q(x)$, we get $P(x) = \cot x,\ Q(x) = \sin x \cos x$

Integration factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

So, the solution is $y(\sin x) = \int (\sin x)(\sin x \cos x)dx + c$

Put $\sin x = t \Rightarrow (\cos x)dx = dt$ in the integral of RHS.

$\therefore y(\sin x) = \int (t)^2 dt + c$

$\Rightarrow y(\sin x) = \frac{t^3}{3} + c$

$\Rightarrow y(\sin x) = \frac{\sin^3 x}{3} + c$ or, $y = \frac{\sin^2 x}{3} + \frac{c}{\sin x}$.

OR

(b) Re-writing the D.E., we get $\dfrac{dy}{dx}=\dfrac{x^2y}{x^3+y^3}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{\frac{y}{x}}{1+\left(\frac{y}{x}\right)^3}$

Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

That is, $v+x\dfrac{dv}{dx}=\dfrac{v}{1+v^3}$

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{v}{1+v^3}-v$

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{v-v-v^4}{1+v^3}$

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{-v^4}{1+v^3}$

$\Rightarrow \dfrac{1+v^3}{v^4}dv=-\dfrac{dx}{x}$

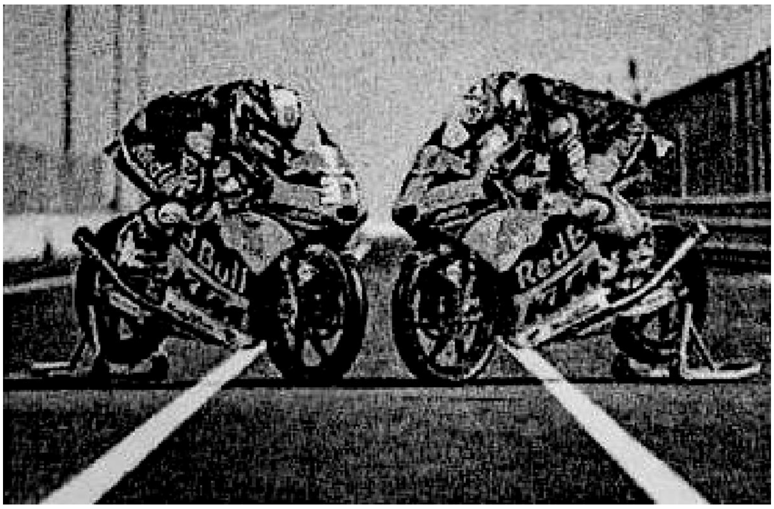
$\Rightarrow \int\left(\dfrac{1}{v^4}+\dfrac{1}{v}\right)dv=-\int\dfrac{dx}{x}$

$\Rightarrow -\dfrac{1}{3v^3}+\log|v|=-\log|x|+c$

$\Rightarrow -\dfrac{x^3}{3y^3}+\log\left|\dfrac{y}{x}\right|=-\log|x|+c$ or, $\log|y|-\dfrac{x^3}{3y^3}=c$.

CASE STUDY BASED QUESTION

Q14. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r}=\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(3\hat{i}+3\hat{j})+\mu(2\hat{i}+\hat{j}+\hat{k})$ respectively.



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines.

(b) Find the point at which the motorcycles may collide.

Sol. (a) For the given lines $\vec{r}=\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(3\hat{i}+3\hat{j})+\mu(2\hat{i}+\hat{j}+\hat{k})$, we have

$\vec{a}_1=\vec{0},\ \vec{b}_1=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{a}_2=3\hat{i}+3\hat{j},\ \vec{b}_2=2\hat{i}+\hat{j}+\hat{k}$

Now $\vec{a}_2-\vec{a}_1=3\hat{i}+3\hat{j}$, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 2 & 1 & 1\end{vmatrix}=3\hat{i}-3\hat{j}-3\hat{k}$

As $\text{S.D.}=\dfrac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$\Rightarrow \text{S.D.}=\dfrac{\left|(3\hat{i}+3\hat{j}).(3\hat{i}-3\hat{j}-3\hat{k})\right|}{\left|3\hat{i}-3\hat{j}-3\hat{k}\right|}$

$\Rightarrow \text{S.D.}=\dfrac{|9-9|}{\sqrt{9+9+9}}=0$ units.

That is, the lines will intersect each other.

(b) Re-writing the given lines in Cartesian form, $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{-1}=\lambda,\ \dfrac{x-3}{2}=\dfrac{y-3}{1}=\dfrac{z}{1}=\mu$

The coordinates of random point on these lines are $P(\lambda,2\lambda,-\lambda)$ and $Q(2\mu+3,\mu+3,\mu)$.

As the lines intersect so, points P and Q must coincide.

That is, $\lambda=2\mu+3$...(i), $2\lambda=\mu+3$...(ii), $-\lambda=\mu$...(iii)

On solving (i) and (iii), we get : $-\mu=2\mu+3 \Rightarrow \mu=-1$

Therefore, the required point of intersection : (1, 2, –1).

So, the motorcycles may collide at (1, 2, –1).

(Series ABCD5/5 Code No. 65/5/2)

Q01. Find the vector equation of a line passing through a point with position vector $2\hat{i}-\hat{j}+\hat{k}$ and parallel to the line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$.

Sol. The d.r.'s of a line joining the points $-\hat{i}+4\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+2\hat{k}$ are 2, –2, 1.

Therefore, the required equation of line passing through the point $2\hat{i}-\hat{j}+\hat{k}$ and with the d.r.'s 2, –2, 1 is : $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$.

Q09. (a) Let $\vec{a}=\hat{i}+\hat{j},\ \vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{a}.\hat{n}=0$ and $\vec{b}.\hat{n}=0$, then find $|\vec{c}.\hat{n}|$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle 30° to each other, then find the area of the parallelogram with $(\vec{a}+3\vec{b})$ and $(3\vec{a}+\vec{b})$ as adjacent sides.

Sol. As $\vec{a}.\hat{n}=0$ and $\vec{b}.\hat{n}=0$ so, it means $\vec{a}\perp\hat{n}$ and $\vec{b}\perp\hat{n}$.

That means, $\hat{n}$ must be in the direction of $\vec{a}\times\vec{b}$.

That is, $\hat{n}=\lambda(\vec{a}\times\vec{b})=\lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}=-2\lambda\hat{k}$

Also $\hat{n}$ is a unit vector so, $\left|-2\lambda\hat{k}\right|=1$ i.e., $\sqrt{4\lambda^2}=1$ i.e., $2\lambda=\pm1$.

$\therefore\ \hat{n}=-(\pm1)\hat{k}=\mp\hat{k}$

Now $\left|\vec{c}.\hat{n}\right|=\left|(\hat{i}+\hat{j}+\hat{k}).(\mp\hat{k})\right|=\left|\mp(1)\right|=1$.

OR

$$\text{Area of parallelogram}=\left|(\vec{a}+3\vec{b})\times(3\vec{a}+\vec{b})\right|$$
$$=\left|\vec{0}+\vec{a}\times\vec{b}+9\vec{b}\times\vec{a}+\vec{0}\right|$$
$$=\left|\vec{a}\times\vec{b}-9\vec{a}\times\vec{b}\right|$$
$$=\left|-8\,\vec{a}\times\vec{b}\right|=8\left|\vec{a}\right|\left|\vec{b}\right|\sin 30^\circ$$
$$=8\times1\times1\times\frac{1}{2}=4\text{ Sq.units}\qquad\left[\because\ \left|\vec{a}\right|=1,\ \left|\vec{b}\right|=1\right.$$

Q10. Evaluate : $\displaystyle\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx$.

Sol. Let $I=\displaystyle\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx$ …(i)

$$\Rightarrow I=\int_0^{\pi/2}\frac{1}{1+\left[\tan\left(\frac{\pi}{2}-x\right)\right]^{2/3}}dx=\int_0^{\pi/2}\frac{1}{1+(\cot x)^{2/3}}dx$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{1}{1+\frac{1}{(\tan x)^{2/3}}}dx$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{(\tan x)^{2/3}}{(\tan x)^{2/3}+1}dx \ \ldots\text{(ii)}$$

Adding (i) and (ii), we get $2I=\displaystyle\int_0^{\pi/2}\frac{1}{1+(\tan x)^{2/3}}dx+\int_0^{\pi/2}\frac{(\tan x)^{2/3}}{(\tan x)^{2/3}+1}dx$

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{1+(\tan x)^{2/3}}{1+(\tan x)^{2/3}}dx=\int_0^{\pi/2}1\,dx$$

$$\Rightarrow 2I=[x]_0^{\pi/2}=\frac{\pi}{2}-0$$

$$\therefore I=\frac{\pi}{4}.$$

Q13. In a factory, machine A produces 30% of total output, machine B produces 25% and the machine C produces the remaining output. The defective items produced by machines A, B and C are 1%, 1.2%, 2% respectively. An item is picked at random from a day's output and found to be defective. Find the probability that it was produced by machine B.

Sol. Let E : the chosen item is defective.

Also let E_1, E_2, E_3 be the events that the item was produced by machines A, B, C respectively.
We have $P(E_1)=30\%,\ P(E_2)=25\%,\ P(E_3)=45\%$,
$P(E|E_1)=1\%,\ P(E|E_2)=1.2\%,\ P(E|E_3)=2\%$.

Using Bayes' Theorem, $P(E_2|E)=\dfrac{P(E|E_2)P(E_2)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)+P(E|E_3)P(E_3)}$

$$\Rightarrow P(E_2|E)=\frac{\frac{1.2}{100}\times\frac{25}{100}}{\frac{1}{100}\times\frac{30}{100}+\frac{1.2}{100}\times\frac{25}{100}+\frac{45}{100}\times\frac{2}{100}}$$

$$\Rightarrow P(E_2|E)=\frac{1.2\times 5}{1\times 6+1.2\times 5+9\times 2}$$

$$\Rightarrow P(E_2|E)=\frac{6}{6+6+18}=\frac{6}{30}\ \text{ or, }\ \frac{1}{5}.$$

(Series ABCD5/5 Code No. 65/5/3)

Q01. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12}=\frac{y+2}{2}=\frac{z-3}{3}.$$

Find the direction cosines of a line parallel to line AB.

Sol. Re-writing the equation of line : $\dfrac{x-\frac{1}{2}}{6}=\dfrac{y+2}{2}=\dfrac{z-3}{3}$.

The d.r.'s of the line are 6, 2, 3.

Therefore, the d.c.'s are $\pm\dfrac{6}{\sqrt{6^2+2^2+3^2}},\ \pm\dfrac{2}{\sqrt{36+4+9}},\ \pm\dfrac{3}{\sqrt{49}}$ i.e., $\pm\dfrac{6}{7},\ \pm\dfrac{2}{7},\ \pm\dfrac{3}{7}$.

Q09. Evaluate : $\displaystyle\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx$.

Sol. Let $\displaystyle I=\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx \ldots$(i)

$$\Rightarrow I=\int_1^3 \frac{\sqrt{1+3-x}}{\sqrt{1+3-x}+\sqrt{4-(1+3-x)}}\,dx$$

$$\Rightarrow I=\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\,dx \ldots\text{(ii)}$$

Adding (i) and (ii), we get $\displaystyle 2I=\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx+\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}\,dx$

$$\Rightarrow 2I=\int_1^3 \frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx=\int_1^3 1\,dx=[x]_1^3=3-1=2$$

$\therefore I=1$.

Q10. Find the distance of the point (2, 3, 4) measured along the line $\dfrac{x-4}{3}=\dfrac{y+5}{6}=\dfrac{z+1}{2}$ from the plane $3x+2y+2z+5=0$.

Sol. Equation of a line PQ (see diagram) passing through P(2, 3, 4) and parallel to (along) the given line L : $\frac{x-4}{3}=\frac{y+5}{6}=\frac{z+1}{2}$ is, $\frac{x-2}{3}=\frac{y-3}{6}=\frac{z-4}{2}=\lambda ...(i)$.

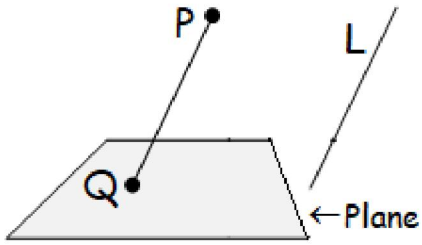
Any random point on line (i) is $Q(3\lambda+2,\ 6\lambda+3,\ 2\lambda+4)$.

If Q lies on the given equation of plane $3x+2y+2z+5=0$,

then, $3(3\lambda+2)+2(6\lambda+3)+2(2\lambda+4)+5=0$

$\Rightarrow \lambda=-1$.

So, coordinates of the point Q are $Q(-1,\ -3,\ 2)$.



$\therefore$ Required distance, $PQ=\sqrt{(2+1)^2+(3+3)^2+(4-2)^2}$

$\therefore PQ=7$ units.

Q13. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

Sol. Let E_1 : box-I is selected, E_2 : box-II is selected, A : getting a red ball.

Here $P(E_1)=P(E_2)=\frac{1}{2}$, $P(A|E_1)=\frac{3}{9}=\frac{1}{3}$, $P(A|E_2)=\frac{5}{10}$.

By Bayes' theorem, $P(E_2|A)=\dfrac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$

$$\Rightarrow P(E_2|A)=\frac{\frac{1}{2}\times\frac{5}{10}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{5}{10}}=\frac{\frac{5}{10}}{\frac{1}{3}+\frac{5}{10}}=\frac{3}{5}.$$